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GLUON BREMSSTRAHLUNG K-FACTORS IN N=1 SUSY QCD

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§ 1. Introduction

Recently the radiative corrections to the basic processes of QCD are calculated (see e.g.[1] and the literature cited therein). It is interesting to make analogous calculations in Supersymmetric (SUSY) QCD.

Here, we use the results of [2], where the exact formulas are presented for the scalar quark pair production and bremsstrahlung gluon in the quark-(anti)quark collisions and gluon fusion processes:

$$q_\alpha(p_1)+\bar{q}_\beta(p_2)\to \Phi_\alpha(p_3)+\bar{\Phi}_\beta(p_4)+G(k) \qquad (1)$$

$$q_\alpha(p_1)+q_\beta(p_2)\to \Phi_\alpha(p_3)+\Phi_\beta(p_4)+G(k) \qquad (2)$$

$$G(p_1)+G(p_2)\to \Phi(p_3)+\bar{\Phi}(p_4)+G(k). \qquad (3)$$

In the note we limit ourselves to removing of the infrared (IR) singularities and estimate corresponding IR finite corrections to the Born cross-sections, i.e. the so-called K-factors

$$K=1+|M(ab\to cd)|^2_{\alpha_s^3}\,/\,|M(ab\to cd)|^2_{\alpha_s^2}\,, \qquad (4)$$

where $|M(ab\to cd)|^2_{\alpha_s^n}$ is the spin-color averaged matrix element squared for the processes $ab\to cd$ in α_s^n order of the perturbation theory. Thus

$$K=1+\frac{\alpha_s}{2\pi}\cdot\hat{a}\,,$$

$\alpha_s=g^2/(4\pi)$ is the strong interaction running constant and $\hat{a}$ is a constant number which we receive after the integration and adding of the real and virtual gluon emission graph contributions in the soft limit $(k\to 0)$.

§ 2. Soft limit calculations

As is known the desired corrections to the Born cross-sections are the virtual gluon loop diagrams (V-graphs) and the real gluon bremsstrahlung (R-graphs) /3/.

In this section we give the detailed calculations for the R- and V- graphs of the process (1). Firstly we make the real gluon phase-space integration in small momentum limit of the emitted gluon $K \to 0$.

The gauge-invariant set of the corresponding diagrams is given in Fig.1.

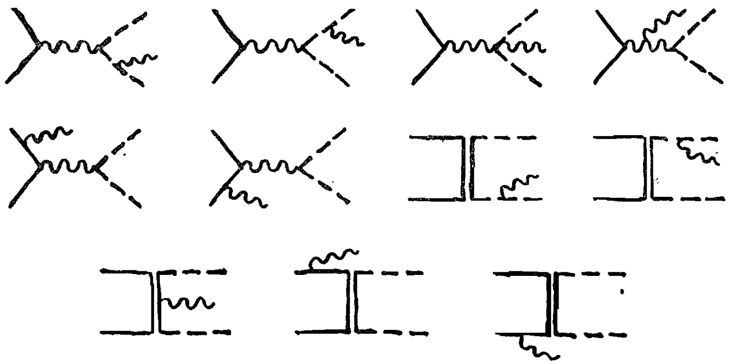

Fig.1. Gauge-invariant set of Feynman diagrams of the process $q\bar{q} \to \Phi\bar{\Phi}G$. Full line is fermion quark, wave is gluon, dashed is scalar quark.

It is easy to verify, that in soft limit we are left with 24 squared IR R-graphs, depicted on Fig.2. Here the diagrams 1-6 correspond to the s-channel gluon exchange, 7-12 to t-channel and 13-24 are the contributions of the interference terms. Integration is suitable to perform in $n = 4 - 2\varepsilon$ -dimensional space. In this case we have the integral of the type /3/

$$R(p_i, xp_j) = \mu^{2\varepsilon}\int \frac{d^{n-1}K}{(2\pi)^{n-1}\, 2K_0}\, \frac{(p_i p_j)}{(K+p_i)^2 (K+xp_j)^2} = \frac{1}{(4\pi)^2}\frac{G(\varepsilon)}{2x\varepsilon^2}\left(\frac{K^2_{max}}{\pi\mu^2}\right)^{-\varepsilon}, \quad (5)$$

$$G(\varepsilon) = \frac{\Gamma(1-\varepsilon)}{\Gamma(1-2\varepsilon)}, \quad K^2_{max} = (p_i + xp_j)^2/4, \quad x = \pm 1 .$$

Here p_i, p_j are the arbitrary momenta of the massless particles. Note, that the $K \to 0$ limit means the following replacements

$$1/(K-p_i-p_j)^2 \to 1/(p_i+p_j)^2, \quad 1/(K+p_i-p_j)^2 \to 1/(p_i-p_j)^2.$$

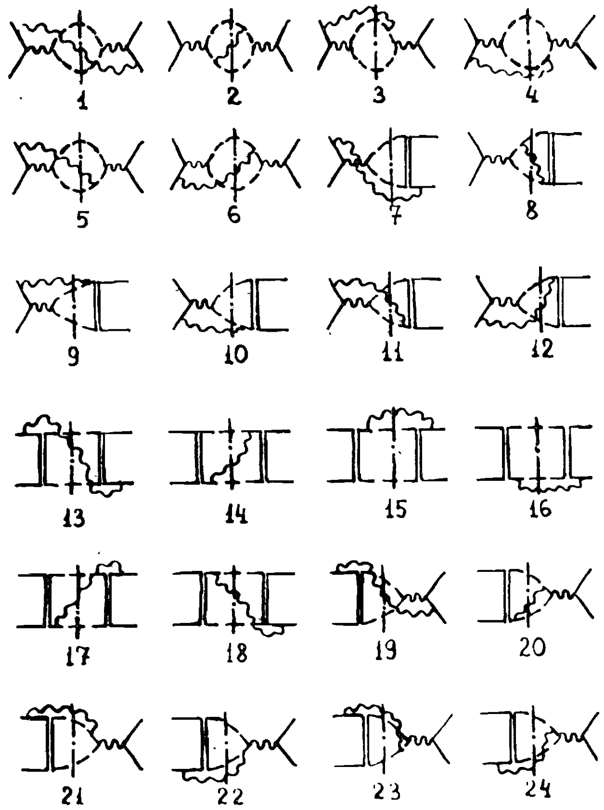

Fig.2. Infrared R-graphs in the soft limit $K \to 0$. The dash-dotted line —·— denotes the unitarity cut of the amplitude.

We notice also that in the calculations Feynman gauge was used. It is convenient because in the limit $K \to 0$ there exist 24 squared V-graphs (Fig.3) such that the singularities of the R- and V-graphs cancel each other in each pair (i, i').

It is easy to see that each pair (i, i') has the same topology and differs only in cutting rule.

The following integral corresponds to the V-graphs:

$$V(p_i, xp_j) = \mu^{2\varepsilon}\int \frac{d^n K}{(2\pi)^n}\, \frac{(p_i p_j)}{K^2 (K+p_i)^2 (K+p_j x)^2} = -\frac{i\, G(\varepsilon)}{(4\pi)^2\, 2x\varepsilon^2}\left(\frac{x(p_i p_j)}{2\pi\mu^2}\right)^{-\varepsilon} \quad (6)$$

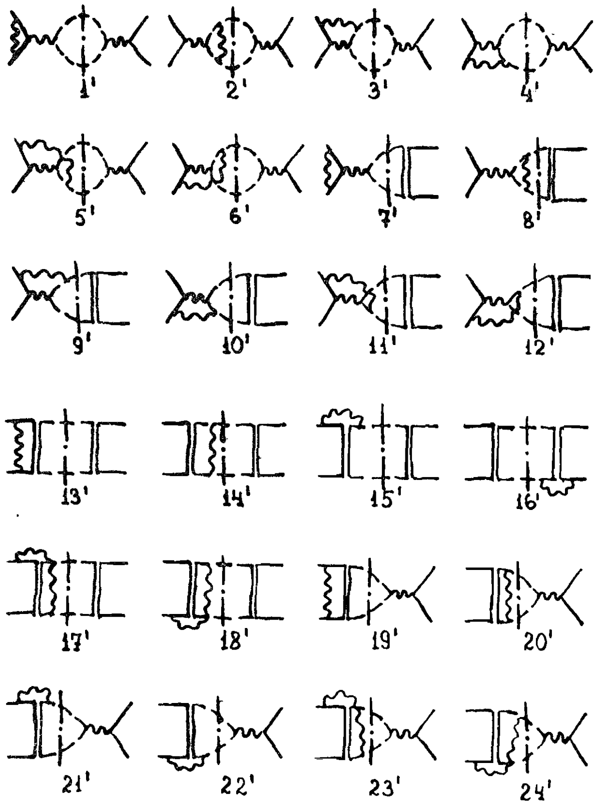

Fig.3. Infrared singular V-graphs, which differ from the corresponding graphs of Fig.2 by the cut rule.

Some results. Firstly we observe the cancellation of the ε^{-2} poles in each pair (i,i'), e.g., for diagrams (1) and (1') in the limit $K\to 0$ we have

$$2\cdot(1)=-\frac{4}{27}\,\frac{ut}{s}\,\frac{1}{(Kp_1)(Kp_2)}\,,\qquad 2\cdot(1')=-\frac{16}{27}\,\frac{ut}{s}\,\frac{i}{K^2(K-p_1)^2(K+p_2)^2}\,.$$

Table 1. Spin and color averaged matrix elements squared for 2→2 processes of N=1 SUSY QCD /4,5/ and QCD /11/ in the α_s^2 order of the perturbation theory.

$\lvert M(q\bar q\to\Phi\bar\Phi)\rvert^2$	$g^4\,\frac{N_c^2-1}{2N_c^2}\cdot\left(2\frac{ut}{s^2}+\frac{u}{t}-\frac{2}{N_c}\,\frac{u}{s}\right)$	SUSY QCD
$\lvert M(qq\to\Phi\Phi)\rvert^2$	$g^4\,\frac{N_c^2-1}{2N_c^2}\left(\frac{u}{t}+\frac{t}{u}\right)$	" — "
$\lvert M(GG\to\Phi\bar\Phi)\rvert^2$	$g^4\,\frac{1}{N_c}\cdot\left(1-\frac{2N_c^2}{N_c^2-1}\,\frac{tu}{s^2}\right)$	" — "
$\lvert M(qq'\to qq')\rvert^2$	$g^4\,\frac{N_c^2-1}{2N_c^2}\cdot\frac{s^2+u^2}{t^2}$	QCD
$\lvert M(qq\to qq)\rvert^2$	$g^4\,\frac{N_c^2-1}{2N_c^2}\cdot\left(\frac{s^2+u^2}{t^2}+\frac{s^2+t^2}{u^2}-\frac{2}{N_c}\,\frac{s^2}{tu}\right)$	" — "
$\lvert M(q\bar q\to GG)\rvert^2$	$g^4\,\frac{(N_c^2-1)^2}{N_c^3}\,\frac{t^2+u^2}{tu}\left(1-\frac{2N_c^2}{N_c^2-1}\,\frac{tu}{s^2}\right)$	" — "
$\lvert M(GG\to GG)\rvert^2$	$g^4\,\frac{4N_c^2}{N_c^2-1}\cdot\left(3-\frac{ut}{s^2}-\frac{us}{t^2}-\frac{st}{u^2}\right)$	" — "

Table 2. $K=1+\frac{\alpha_s}{2\pi}\cdot\hat a$ factors in N=1 SUSY QCD and QCD

$\hat a_{q\bar q\to\Phi\bar\Phi}$	$\frac{16}{9}\pi^2$	
$\hat a_{qq\to\Phi\Phi}$	$\frac{16}{9}\pi^2$	
$\hat a_{GG\to\Phi\bar\Phi}$	$\frac{26}{9}\pi^2$	
$\hat a_{qq'\to qq'}$	$\frac{16}{9}\pi^2$	$\frac{28}{9}\pi^2$ *)
$\hat a_{qq\to qq}$	$\frac{16}{9}\pi^2$	$\frac{28}{9}\pi^2$ *)
$\hat a_{q\bar q\to GG}$	$\frac{26}{9}\pi^2$	$\frac{31}{12}\pi^2$ *)
$\hat a_{GG\to GG}$	$4\pi^2$	$3\pi^2$ *)
$\hat a_{q\bar q\to\gamma^* G}$	$\frac{17}{9}\pi^2$	$\frac{4}{3}\pi^2$ *)

*) A.P.Contogouris e.a. /3/ results.

After the corresponding integration we have

$$2\cdot(1)\Rightarrow-\frac{32}{27}\,\frac{ut}{s^2}\,R(p_1,p_2)\,,\qquad 2\,(1')\Rightarrow-\frac{32}{27}\,\frac{ut}{s^2}\,i\,V(p_1,-p_2)\,,$$

$$s=2(p_1p_2)\,,\quad t=-2(p_1p_3)\,,\quad u=-2(p_1p_4)\,,$$

$$R(p_1,p_2)\underset{\varepsilon\to 0}{\simeq}\frac{1}{32\pi^2\varepsilon^2}\,,\qquad V(p_1,-p_2)\underset{\varepsilon\to 0}{\simeq}\frac{i\,e^{-i\pi\varepsilon}}{32\pi^2\varepsilon^2}\left(1+\frac{\pi^2\varepsilon^2}{6}\right),$$

so

$$g^6\left[2\cdot(1)+2\,\mathrm{Re}\,(1')\right]\Rightarrow\frac{g^6}{27}\,\frac{ut}{s^2}\left[-\frac{1}{\varepsilon^2\pi^2}+\frac{1}{\varepsilon^2\pi^2}\left(1-\frac{\pi^2\varepsilon^2}{2}\right)\left(1+\frac{\varepsilon^2\pi^2}{6}\right)\right]=$$

$$=-\frac{g^6}{81}\,\frac{ut}{s^2}\,.$$

Now we sum the s-channel, t-channel and interference contributions

$$2\sum_{i=1}^{6}\left[(i)+\mathrm{Re}\,(i')\right]\Rightarrow g^6\,\frac{16}{81}\,\frac{ut}{s^2}\,, \qquad (7)$$

$$2\sum_{i=7}^{12}\left[(i)+\mathrm{Re}\,(i')\right]\Rightarrow g^6\,\frac{16}{162}\,\frac{u}{t}\,, \qquad (8)$$

$$2\sum_{i=13}^{24}\left[(i)+\mathrm{Re}\,(i')\right]\Rightarrow -g^6\,\frac{16}{243}\,\frac{u}{s}\,. \qquad (9)$$

Comparing (7-9) with the corresponding Born cross-sections[4,5] (Table 1) for the process $q\bar{q}\to\Phi\bar{\Phi}$ we get the following values for K-factors (see Table 2)

$$K_{q\bar{q}\to\Phi\bar{\Phi}}=K_s=K_t=K_{int}=1+\frac{\alpha_s}{2\pi}\left(\frac{16}{9}\pi^2\right). \qquad (10)$$

§3. Exact formulas for the gluon bremsstrahlung cross-sections and K-factors

Note, the picture presented here for the K-factor estimation is realized in Feynman gauge and breaks in other (axial, lightlike) gauges. The latter are very useful[2] because the large number diagrams of SUSY QCD vanish. This breaking is due to the existence of the equal topology R- and V-graphs but they have different IR behaviour. In light-like gauges it is impossible to have the pair of diagrams which cancels the IR divergencies in ε^{-2}. Besides, there arise new propagators with $(nK)^{-1(2)}$ and correspondingly new type of integrals with the axial vector n , which need additional analysis[6]. Below we propose a simple method for K-factor evaluation based on the exact cross-sections of the processes (1-3) in an arbitrary gauge without V-graphs.

Recently, some computing algorithms[7] are developed for the bremsstrahlung cross-section evaluations by the analytical

programming methods REDUCE[8], which allow to obtain the cross-sections of the (1-3) processes in factorized form with the bremsstrahlung factor pick out[2,9].

In order to single out the IR structure it is necessary to rewrite the matrix element squared in the form of the sum of certain eikonal factors[10]. E.g. for the process $q\bar{q}\to\Phi\bar{\Phi}G$ (see Fig.1) we have the exact result

$$\left|M(q_\alpha\bar{q}_\beta\to\Phi_\alpha\bar{\Phi}_\beta G)\right|^2=g^6\sum_{i=s,t,int}A_i\left|M_i^{(0)}\right|^2\cdot I_i\,, \qquad (11)$$

where the indices i denote the s-, t- and interference channel contributions,

$$A_s=4\,,\ \left|M_s^{(0)}\right|^2=\frac{ut+u_1t_1}{ss_1}\,,\ s_1=2(p_3p_4),\ t_1=-2(p_2p_4),\ u_1=-2(p_2p_3),$$

$$I_s=\{C_1([14]+[23])+C_2([13]+[24]-[14]-[23])+C_3([12]+[34])\}\,\delta_{\alpha\beta}\,, \qquad (12)$$

$$A_t=4\,,\ \left|M_t^{(0)}\right|^2=\frac{ut+u_1t_1}{2tt_1}\,,$$

$$I_t=C_1([14]+[23])+C_2([12]+[34]-[14]-[23])+C_3([13]+[24])\,, \qquad (13)$$

$$A_{int}=2\,,\ \left|M_{int}^{(0)}\right|^2=\frac{(ut+u_1t_1)(ss_1+tt_1-uu_1)}{2ss_1tt_1}\,,$$

$$I_{int}=\{C_4(2[14]+2[23]-[14,32])+C_5[14,32]\}\,\delta_{\alpha\beta} \qquad (14)$$

Here $[ij]$ denotes the eikonal factor

$$[ij]=\frac{(p_ip_j)}{(p_iK)(Kp_j)}\,,$$

$$[ij,k\ell]=2[ij]+2[k\ell]-[ik]-[i\ell]-[jk]-[j\ell]\,,$$

and the color Casimires are

$$C_1=\frac{N_c^2-1}{8N_c}\,,\quad C_2=\frac{(N_c^2-2)(N_c^2-1)}{8N_c^3}\,,\quad C_3=-\frac{N_c^2-1}{8N_c^3}\,,$$

$$C_4=\frac{(N_c^2-1)^2}{8N_c^4}\,,\quad C_5=\frac{N_c^4-1}{8N_c^4}$$

In the case of the color $SU(3)$ group $N_c=3$.

The analysis carried in preceding section shows that the results obtained above (7-9) can be reproduced without drawing the virtual graphs. To see this we need the following replacement in (12-14):

$$S=S_1,\ t=t_1,\ u=u_1,\ [ij,kl]\Rightarrow 0,$$
$$[ij]\Rightarrow 4\,[R(p_i,xp_j)+\mathrm{Re}\,V(p_i,-xp_j)]=\frac{1}{24}. \tag{15}$$

We make similar calculations for processes (2) and (3). For the matrix element squared of process (2) we have

$$|M(q_\alpha q_\beta\to\Phi_\alpha\Phi_\beta G)|^2=g^6\sum_{i=u,t}A_i\,|M_i^{(0)}|^2\cdot I_i\,, \tag{16}$$

Here indices i stand for t- and u-channel contributions

$$A_t=4,\quad |M_t^{(0)}|^2=\frac{ut+u_1t_1}{2tt_1}\,,$$
$$I_t=C_1([12]+[34])-C_2([12]+[34]-[14]-[23])+$$
$$+C_3([13]+[24])\,, \tag{17}$$

$$A_u=4,\quad |M_u^{(0)}|^2=\frac{ut+u_1t_1}{2uu_1}\,,$$
$$I_u=\{C_1([12]+[34])-C_2([12]+[34]-[13]-[24])+$$
$$+C_3([14]+[23])\}\delta_{\alpha\beta}\,. \tag{18}$$

Inserting (15) into (17-18) and comparing with the corresponding Born cross-sections (see Table 1) we obtain

$$K_{qq\to\Phi\Phi}=1+\frac{\alpha_s}{2\pi}\left(\frac{16}{9}\pi^2\right).$$

Similarly, for reaction (3) we have

$$|M(GG\to\Phi\bar\Phi G)|^2=\frac{g^6}{32}\,\frac{N_c^2-1}{4N_c^2}\left[\frac{(s+t+u_1)^2(s_1+t+u_1)^2+u^2t^2+u_1^2t_1^2}{ut\,u_1t_1}\right]\times$$
$$\times\Big\{[34]+N_c^2\Big([34]-[13]-[14]-[23]-[24]-\frac{tt_1+uu_1}{ss_1}[34]\Big)+$$
$$+\frac{N_c^4}{ss_1}\Big[\big([12]+[14]+[23]\big)tt_1+\big([12]+[13]+[24]\big)uu_1\Big]\Big\}$$

and for corresponding K-factor ($N_c=3$)

$$K_{GG\to\Phi\bar\Phi}=1+\frac{\alpha_s}{2\pi}\left(\frac{26}{9}\pi^2\right).$$

Thus, Table 2 represents calculated values of K-factors for the basic processes of SUSY QCD and QCD[1,9] too. We use some results of papers[10,11].

§4. Conclusion

We must note that the results for QCD are in disagreement with the Contogouris e.a.[3] results.

In the formulas of the type (11) and (16) in[2,9] the first factor $|M^{(0)}|^2$ is connected with the lowest order, elastic process, and the second one is related to the IR factor which describes soft quantum emission and defines the corresponding K-factor. However, in the case of the reaction (3) (and also for reactions $q\bar q\to GGG$ and $GG\to GGG$ in QCD) for the soft limit $\kappa\to 0$ the first factor does not reproduce the elastic cross-section and can have another physical interpretation[12]. It is remarkable that for processes mentioned after integration (15) the values of the elastic cross-sections are fully reproduced from the product of both the factors.

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References

1. Ellis R.K., Sexton J.C..Nucl.Phys.,1986,v. B269, N2, 445.
2. Matveev V.A., Darbaidze Ya.Z., Merebashvili Z.V., Slepchenko L.A. Phys.Lett., 1986, v.B177, N2, 188; Phys.Lett., 1987, v. B191, N1/2, 179.
3. Contogouris A.P. e.a. Phys.Rev., 1984, v.D29, 1354; Phys.Rev., 1986,v.D33, 1265;
   Contogouris A.P. Phys.Rev., 1982, v.D26, 1618.
4. Matveev V.A., Slepchenko L.A. Teor.Mat.Fiz. 1984, v.59, 224;
   Dawson S., Eichten E., Quigg C. Phys.Rev., 1985, v.D31, 1581.
5. Glück M., Reya E. Phys.Rev.Lett., 1982, v.48, 662;
   Avaliani I., Matveev V.A., Slepchenko L.A. Teor.Mat.Fiz. 1984, v.59, 91.
6. Leibrandt G. Phys.Rev., 1984, v.29, 1699;
   Khelashvili A.A., Khmaladze V.Yu., Natroshvili K.R. In: Proceedings of workshop "Infrared singularities problem in QCD", TSU, Tbilisi, 1985.
7. Möhring H.J. and Schiller A., In: Intern. Conf. on systems and techniques of analytical computing and applications in theore-

tical physics, JINR, Dubna, D11-80-13, 127, 1980,
Darbaidze Ya.Z. e.a. In: Europe Conf. on Computer Algebra, Leipzig, June, 1987.
8. Hearn A.C., REDUCE User's Manual Version 3.3, Rand Corporation, Santa Monica, CA, 1987.
9. Berends F.A. e.a. Phys.Lett., 1981, v.B103, N2, 124.
10. Ellis R.K., Marchesini G., Webber B.R. Nucl.Phys., 1987, v.B286, 643.
11. Combridge B.L., Kripfganz J., Ranft J. Phys.Lett. 1977, v.B70, 234;
Cutler R., Sivers D. Phys.Rev., 1978, v.D17, 196.
12. CALCUL Collaboration. Nucl.Phys., 1982, v.B206, 61;
Parke S.J., Taylor T.R. Phys.Lett., 1985, v.B157, 81;
Herzog F., Kunzt Z. Phys.Lett., 1985, v.B157, 430.

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Дарбаидзе Я.З. и др. E2-87-711
К-факторы в N=1 суперсимметричной КХД

Вычисляется К-фактор для процесса $q\bar{q} \to \Phi\bar{\Phi}$ в N=1 суперсимметричной КХД. Получены точные формулы для тормозного излучения глюона с явно выделенной инфракрасной структурой, позволяющей непосредственно вычислить соответствующий К-фактор. Для иллюстрации приведены расчеты для сечений процессов $qq \to \Phi\Phi G$ и $GG \to \Phi\bar{\Phi}G$.

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Gluon Bremsstrahlung K-Factors in N=1 Susy QCD

K-factor for the $q\bar{q} \to \Phi\bar{\Phi}$ process in N=1 susy QCD is calculated. The exact gluon bremsstrahlung matrix elements squared allowing for computation of the corresponding K-factors are obtained. As an illustration the cross-sections for the processes $qq \to \Phi\Phi G$ and $GG \to \Phi\bar{\Phi}G$ are presented.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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