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QUANTUM INTERFERENCE ON GRAPHS CONTROLEED
by an external electric field

## 1. Introduction

The rapid development of the techniques of fabricating, small metallic or semiconductor structures has opened an entirely new field of research which is sometimes dubbed mesoscopic physics. The term expresses the fact that while the structures involved are designed by an experimentalist (that is, in a macroscopic way), they are small enough to exhibit typical quantum effects. Conductivity measurements have been performed on various structures : rings, squares and their sequences, honeycomb networks, etc. $1 / 5,7,30,39-41 /$. Most attention has been paid . to the Aharonov-Bohm effect manifested by magnetoresistance oscillations which represents a suitable object for experimental investigetion being stable with respect to variations of the pattern geometry.

There are other interesting situations, however. It is natural to ask, e.g., whet happens if the structure is placed into an electric field, in particular, how its resistance depends on the field intensity. The possibility is technically attractive, because it might open way to a new type of switching devices. In order to decide whether such a prospect is realistic, a careful analysis is needed. In this paper, we address ourselves with this problem.

From the microscopic point of view, the system under conisideration is complicated enough to make its complete description very difficult. We shall not attempt to do it. Instead, we are going to construct a model which, as we believe, reproduces the essential features of the system, and at the same time it is solvable. It is based on the assumption that the "wires" which are building elements of the structure are infinitely thin. Actually they represent a band of atoms whose width can be made as small as $200 \stackrel{\circ}{\mathrm{~A}}$ by the technologies mentioned above. Once we replace them by lines, our problem reduces to the analysis of motion of a quantum particle which is confined to an appropriate planar graph and subjected to the electric field. We limit ourselves to the simplest non-trivial case when the graph consists of a loop with


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two external leads. We shall assume also that the external field is weak enough so that the semiclassical approximation may be used. The results of the present analysis have been announced in Ref. 22.

Let us describe briefly the contents of the paper. In the next section we show, how and under which assumptions the conductivity can be calculated. The key element of the model is to describe how the electron wavefuction "splits" at the junctions. This is discussed in Section 3 ; it allows us to choose the Hamiltonian for a charged particle whose motion is confined to a planar curve in presence of an external field. In the next section, the transmission coefficient is calculated. Its evaluation requires the knowledge of transfer matrices for the corresponding Schrbdinger equations ; we calculate them in Section 5 using the semiclassical approximation. In conclusion, we give some examples of conductivity vs. field intensity plots which show that the prospect of constructing the above-mentioned switching devices is fully realistic.

## 2. The physical background

As we have said, our model is intended to describe a charged quantum particle (an electron) moving on a loop with two external leads under influence of (a) "a voltage $U$ applied to the leads and (b) a homogeneous electric field of intensity parallel to the graph plane and perpendicular to the leads. "The motion on the loop is assumed to be ballistic, i.e., both the elastic and inelastic scattering is negligible.

Let us first say a few words about the ways in which such systems can be prepared. The metallic rings


Fig. 1. Scheme of the model and similar stuctures are usually produced lithographically, by an atomic beam drawing a required graph on a substrate (cf. the references quoted in the introduction). The technique is fairly elaborated at present ; its appealing feature is that in principle an arbitrary planar graph can be drawn.

The situation is not so simple with semiconductors. For them, too, interference experiments have been proposed and performed, but in distinction to the above named ones, they employed heterostructures grown by the epitaxial technique ${ }^{12 /}$. It was even suggested to use the
interference in heterostructures in switching devices/11/ ; our analysis shows, however, that the "graphs" are more suitable for that purpose. Let us remark in this connection that though our nodel is not applicable to the heterostructures, since the channel thickness cannot be neglected in this case, there is a way how this difficulty might be overcome. It is connected with a solution to the so-called Injection problem $/ 27 /$ which we are going to discuss in a separate paper.

Very recently a new technique has been reported which opens way ta fabricating semiconductor graph structures $/ 38 /$. In a sense, it combines the procedures mentioned above. Orie prepares first a semiconductor layer on a substrate and draws lithographically metallic lines on it. The graph is afterwards used as a mask when the layer is removed by ion bombarding.

The essential assumption of our model is that the electrons move ballistically on the loop. It can be fulfilled if the electron mean free path is much longer than the loop size. In semiconductors this quantity depends heavily on how much the material is doped. It ean be mede as long as $\leqslant \mathbb{T} \mu \mathrm{m}$ so the assumption can be justified in this case. The situation is not so good in metals, but even here experiments with ballistic electrons could be expected in the nearest future $/ 4$ \% .

Under this assumption, one is able to solve the one-particle problem, i.e., to deduce the transmission coefficient $T(E)$ for an electron moving towards the loop with an energy $E$. It allows us further ta calculate the conductivity. For a bulk metal, the current corresponding to the apllied voltage $U$ is given by/29/

$$
\begin{equation*}
I=\frac{e}{\pi h} \int_{0}^{\infty}[f(E+e U)-f(E)] \frac{T(E)}{1-T(E)} d E \tag{2.1a}
\end{equation*}
$$

where $f^{*}$ denotes the Fermi-Dirac distribution,

$$
\begin{equation*}
f(E)=\left(1+\exp \left(\frac{E-E_{F}}{k T}\right)\right)^{-1} \tag{2.1b}
\end{equation*}
$$

A typical Fermi energy in metals is a. few eV so $k T \ll E_{F}$ (recail that 1 eV corresponds to the temperature $11605^{\circ} \mathrm{K}$ ). The voltage $\dot{U}$ in the above quated experiments was of order of microvolts so eU $\ll E_{F}$ too. If the transmission coefficient is a slowly varying function of energy in this scale, we get for the conductivity $G=I / U$ from (2.ia) the following approximative expression

$$
\begin{equation*}
G=\frac{e^{2}}{\pi h} \frac{T(E)}{T-T(E)}, \tag{2.2}
\end{equation*}
$$

where $E \approx \mathrm{E}_{\text {P }}$. Notice that in fact the distribution of electrons in the conductivity band is not given by (2.1b), since the wire does not represent a bulk metal, and therefore $E_{F}$ should be replaced by another value. However, the correction again does not exceed kT.

In semiconductors the situation is completely different. There are much less electrons in the conductivity band. On the other hand, they have typically smali effective mass (for GaAs, e.g., we have $m^{*}=$ $=0.067 \mathrm{~m}_{e}$ ), and therefore only a few transwersal modes are contained - In the conductivity band $/ 24 /$. The conductivity will be then calculated from the formula (2.2) again, but now $E$ will be the lowest-transversal mode energy. It depenås not only on the material, but also on the shape of the wire ; for 200 A thick wires, it is of order of $10^{-2} \mathrm{eV}$.

## 3. The mathematical backgroud

For the sake of simplicity, the electrons will be assumed to be spinless. The state Hilbert space of our problem is then of the form

$$
\begin{equation*}
H=I^{2}(0, \infty) \oplus L^{2}\left(0,1_{2}\right) \oplus L^{2}\left(0, I_{3}\right) \oplus L^{2}(-\infty, 0), \tag{3.1}
\end{equation*}
$$

where the orientation of axes is chosen in a way that allows to describe the two junction in a similar fashion. The central problem is how to choose the Hamiltonian $H$ of the model. Two requirements must be fulfilled :
(a) , H is self-adjoint,
(b) if the wavefunction has a support separated from the junctions, then $H$ describes the appropriate motion on the halfline or on the loop.

Such an operator can be constructed by taking first suitable pre-Hamiltonian $H_{0}$ which is non-selfadjoint, the branching points of the graph being removed from its domain. The admissible Hamiltonians are then obtained as self-adjoint extensions of $H_{0}$. The same method has been applied recently in different contexts $/ 1-4,1,3,16-22,28,31-36 /$.

The Hamiltonians describing a free motion on a branching graph have been studied in Refs. 20,21. In particular, we derived there boundary conditions which specify the self-adjoint extensions of the preHamiltonian $H_{0}$. They can be used obviousiy for an electron moving under influence of an external field.as long as the interaction remains bounded. This is true in our case : we shall show a little later that the starting operator $H_{O}$ may be chosen as

$$
\begin{equation*}
H_{0}=\stackrel{4}{\oplus}{ }_{j=1} H_{0, j} \tag{3.2a}
\end{equation*}
$$

with

$$
\begin{equation*}
H_{0, j}=\left(-\frac{k^{2}}{2 m^{*}} \frac{d^{2}}{d x_{j}^{2}}+V_{j}\left(x_{j}\right)\right)+c_{0}^{\infty}\left(\mathbb{M}_{j}\right) \tag{3.2b}
\end{equation*}
$$

where $M_{j}$ is the appropriate part of the configuration manifold, $m^{*}$ is the effective mass ; furthermore, $V_{j}$ is a bounded function on the loop, $j=2,3$, while $V_{j}=0$ on the leads, $j=1,4$.

In general, the deficiency indices of $H_{0}$ are $(18,18)$ so it has a vast family of self-adjoint extensions. We adopt the following restrictions on the Hamiltonian $H$ :
(c) $H$ is local in the sense that supp HuC supp $u$ for all $u \in D(H)$,
(d) $H$ is locally permutation-invariant at each junction : if the support of $u$ is sufficiently concentrated around one of the junctions, and $P_{j k}$ is the operator permuting the $j-$ th and $k$-th wire at the junction, then $P_{j k} u \in D(H)$ and $P_{j k} H u=H P_{j k} u$.
The last condition means that if the electric field is switched off, the electron whose motion is governed by $H$ does not distinguish the wires provided it is close enough to the juction.

Under the conditions (c) and (d), the self-adjoint extensions $H$ are characterized by two pairs of real parameters, each of them referring to one junction. Most of the extensions can be expressed by the following boundary conditions

$$
\begin{align*}
& u_{1}(0)=A_{1} u_{1}^{\prime}(0)+B_{1} u_{2}^{\prime}(0)+B_{1} u_{3}^{\prime}(0), \\
& u_{2}(0)=B_{1} u_{1}^{\prime}(0)+A_{1} u_{2}^{\prime}(0)+B_{1} u_{3}^{\prime}(0),  \tag{3.3a}\\
& u_{3}(0)=B_{1} u_{1}^{\prime}(0)+B_{1} u_{2}^{\prime}(0)+A_{1} u_{3}^{\prime}(0)
\end{align*}
$$

and

$$
\begin{align*}
& u_{2}\left(l_{2}\right)=A_{2} u_{2}^{\prime}\left(I_{2}\right)+B_{2} u_{3}^{\prime}\left(l_{3}\right)+B_{2} u_{4}^{\prime}(0), \\
& u_{3}\left(l_{3}\right)=B_{2} u_{2}^{\prime}\left(I_{2}\right)+A_{2} u_{3}^{\prime}\left(I_{3}\right)+B_{2}^{\prime} u_{4}^{\prime}(0),  \tag{3.3b}\\
& u_{4}(0)=B_{2} u_{2}^{\prime}\left(l_{2}\right)+B_{2} u_{3}^{\prime}\left(l_{3}\right)+A_{2} u_{4}^{\prime}(0),
\end{align*}
$$

where $u_{j}(0), u_{j}^{\prime}(\theta), u_{j}\left(I_{j}\right)$ and $u_{j}^{\prime}\left(I_{j}\right)$ are understood as the limits from the appropriate side. With the chosen orientation of the axes, the two junctions are the same if

$$
\begin{equation*}
A_{1}=-A_{2}, \quad B_{1}=-B_{2} \tag{3.4}
\end{equation*}
$$

The conditions (3.3) do not exhaust all the operators $H$ fulfilling the requirements (a)-(d). There are two additional one-parameter classes of boundary conditions at each junction that also lead to a selfadjoint $H-c f . R e f .21$. However, the considerations presented below are adapted easily to these exceptional cases, and we shall not treat them explicitly in the following.

- The last thing we must fix are the potentials $V_{j}$ appearing in (3.2b). A natural guess which can be supported by the standard quantization procedure $/ 14,37 /$ is

$$
\begin{equation*}
v_{j}\left(x_{j}\right)=-e \varepsilon_{y_{j}}\left(x_{j}\right) \tag{3.5}
\end{equation*}
$$

where $\gamma_{j}\left(x_{j}\right)$ marks the distance from a fixed equipotential line ; conventionally, we choose zero value of the potential on the leads. Unfortunately, the problem is more complicated. The graph lines in our model substitute thin stripes, and therefore the potentials should contain curvature-dependent terms $/ 8-10,25,26 /$. Simple estimates show, however, that for the ring structures mentioned in the introduction, the ansatz (3.5) represents a good approximation. We shall use it therefore in the present paper leaving the analysis of the curvature effects to a subsequent publication.

## 4. The transmission coefficient

Now we are ready to solve the scattering problem for the loop. We shall use the time-independent framework; then one has to find the function $u=\left(u_{1}, u_{2}, u_{3}, u_{4}\right)$ that belongs locally to the domain of the extension $H$ chosen to play the role of Hamiltonian, and fulfils

$$
\begin{align*}
& u_{1}\left(x_{1}\right)=e^{-i k x_{1}}+a e^{i k x_{1}}  \tag{4.1a}\\
& u_{2}\left(x_{2}\right)=c_{1} f_{1}\left(x_{2}\right)+c_{2} f_{2}\left(x_{2}\right)  \tag{4.1b}\\
& u_{3}\left(x_{3}\right)=d_{1} g_{1}\left(x_{3}\right)+d_{2} g_{2}\left(x_{3}\right)  \tag{4.1c}\\
& u_{4}\left(x_{4}\right)=b e^{-i k x_{4}} \tag{4.1d}
\end{align*}
$$

where $\mathrm{f}_{1}, \mathrm{f}_{2}$ are the solutions to the Schrodinger equation

$$
\begin{equation*}
-\frac{n^{2}}{2 m^{*}} f_{k}^{\prime \prime}\left(x_{2}\right)+v_{2}\left(x_{2}\right) f_{k}\left(x_{2}\right)=E f_{k}\left(x_{2}\right) \tag{4.2a}
\end{equation*}
$$

with $E=x^{2} k^{2} / 2 m$ fulfilling the boundary conditions

$$
f_{1}(0)=f_{2}^{\prime}(0)=1, \quad f_{1}^{\prime \prime}(\sigma)=f_{2}(0)=0
$$

and similarly, $g_{1}, g_{2}$ are the solutions of

$$
\begin{equation*}
-\frac{k^{2}}{2 m^{*}} g_{k}^{\prime \prime}\left(x_{3}\right)+V_{3}\left(x_{3}\right) g_{k}\left(x_{3}\right)=E g_{k}\left(x_{3}\right) \tag{4.2b}
\end{equation*}
$$

corresponding to the boundary conditions

$$
\begin{equation*}
g_{1}(0)=g_{2}^{\prime}(0)=1 \quad, \quad g_{1}^{\prime}(0)=g_{2}(0)=0 \tag{4.3b}
\end{equation*}
$$

the potentials $V_{2}, V_{3}$ are given by (3.5) as mentioned above. We shall need also the transfer matrices $\Pi_{j} \equiv \Pi_{j}\left(I_{j}\right), j=2,3$,

$$
\begin{equation*}
\binom{u_{j}\left(I_{j}\right)}{u_{j}^{\prime}\left(l_{j}\right)}=\Pi_{j}\binom{u_{j}(0)}{u_{j}^{\prime}(0)} \tag{4.4a}
\end{equation*}
$$

Using the above mentioned solutions to the eqs.(4.2), we may express them as

$$
\Pi_{2}=\left(\begin{array}{ll}
f_{1}\left(I_{2}\right) & f_{2}\left(I_{2}\right)  \tag{4.4b}\\
f_{1}^{\prime}\left(I_{2}\right) & f_{2}^{\prime}\left(I_{2}\right)
\end{array}\right), \quad \Pi_{3}=\left(\begin{array}{ll}
g_{1}\left(I_{3}\right) & g_{2}\left(I_{3}\right) \\
g_{1}^{\prime}\left(I_{3}\right) & g_{2}^{\prime}\left(I_{3}\right)
\end{array}\right)
$$

Our aim is to find the coefficient $b$ assuming that the functions (4.1) fulfil the boundary conditions (3.3). To solve this problem, we express first the coefficients $c_{1}, c_{2}$ with the help of $d_{1}, d_{2}$. The boundary conditions for the first junction yield the equations

$$
\begin{aligned}
1+a & =-i k A_{1}(1-a)+c_{2} B_{1}+d_{2} B_{1}, \\
c_{1} & =-i k B_{1}(1-a)+c_{2} A_{1}+d_{2} B_{1}, \\
d_{1} & =-i k B_{1}(1-a)+c_{2} B_{1}+d_{2} A_{1},
\end{aligned}
$$

Excluding a from here, we get a system of two linear equations for $c_{1}, c_{2}$ which is solved by

$$
\begin{equation*}
\binom{c_{1}}{c_{2}}=\binom{z_{1}(k)}{z_{2}(k)}+c_{1}(k)\binom{d_{1}}{d_{2}} \tag{4.5}
\end{equation*}
$$

where

$$
\begin{equation*}
z_{2}(k)=\left(A_{1}-B_{1}\right) z_{1}(k) \quad, \quad z_{1}(k)=\frac{2 i k}{1+i k\left(B_{1}-A_{1}\right)} \tag{4.6a}
\end{equation*}
$$

and

$$
\left.\begin{array}{l}
C_{1}(k)=\frac{1}{B_{1}\left[1+i k\left(B_{1}-A_{1}\right)\right]}
\end{array}\right)=\left(\begin{array}{cc}
A_{1}+i k\left(B_{1}^{2}-A_{1}^{2}\right) & \left(B_{1}-A_{1}\right)\left[\left(A_{1}+B_{1}\right)\left(1-i k A_{1}\right)+2 i k B_{1}^{2}\right] \\
1-i k A_{1} & -A_{1}-i k\left(B_{1}^{2}-A_{1}^{2}\right) \tag{4.6b}
\end{array}\right) .
$$

- The boundary conditions at the second junction yield the equations

$$
\begin{aligned}
& \tilde{c}_{1}=\tilde{c}_{2} A_{2}+\tilde{d}_{2} B_{2}-i k b B_{2}, \\
& \tilde{d}_{1}=\tilde{c}_{2} B_{2}+\tilde{d}_{2} A_{2}-i k b B_{2}, \\
& b=\tilde{c}_{2} B_{2}+\tilde{d}_{2} B_{2}-i k b A_{2},
\end{aligned}
$$

where we have introduced

$$
\begin{equation*}
\binom{\tilde{c}_{1}}{\tilde{c}_{2}}=\Pi_{2}\binom{c_{1}}{c_{2}}=\binom{u_{2}\left(l_{2}\right)}{u_{2}^{\prime}\left(1_{2}\right)} \tag{4.7a}
\end{equation*}
$$

and

$$
\begin{equation*}
\binom{\tilde{d}_{1}}{\tilde{d}_{2}}=\Pi_{3}\binom{d_{1}}{d_{2}}=\binom{u_{3}\left(1_{3}\right)}{u_{3}^{\prime}\left(1_{3}\right)} . \tag{4.7b}
\end{equation*}
$$

Excluding $b$ from the above system,

$$
\begin{equation*}
b=\frac{B_{2}}{1+i k A_{2}}\left(\tilde{c}_{2}+\tilde{d}_{2}\right) \tag{4.8}
\end{equation*}
$$

we arrive at equations for $\tilde{c}_{1}, \tilde{c}_{2}$. They are solved by

$$
\begin{equation*}
\binom{\tilde{c}_{1}}{\tilde{c}_{2}}=c_{2}(-k)\binom{\tilde{d}_{1}}{\tilde{\mathrm{a}}_{2}} \tag{4.9}
\end{equation*}
$$

where $C_{2}(k)$ is given by (4.6b) with $A_{1}, B_{1}$ replaced by:, $A_{2}, B_{2}$.
Now we are in position to find the coefficients in ( $4.1 \mathrm{~b}, \mathrm{c}$ ).
Notice that the matrices appearing in the above relations are non-singular ; one finds easily that $\operatorname{det} C_{j}(k)=-1$, and furthermore,
$\operatorname{det} \Pi_{j}$ is the Wronskian of the corresponding solutions, and therefore non-zero. The relations (4.5), (4.7) and (4.9) give

$$
\prod_{2}^{-1} c_{2}(-k) \Pi_{3}\binom{d_{1}}{d_{2}}=\binom{z_{1}(k)}{z_{2}(k)}+c_{1}(k)\binom{d_{1}}{d_{2}}
$$

From here the coefficients $d_{1}, d_{2}$ may be found ; the other three pairs of coefficients are then obtained from (4.7) and (4.9). In particular, we have

$$
\begin{align*}
& \binom{\tilde{c}_{1}}{\tilde{c}_{2}}=\left[\Pi_{2}^{-1}-c_{1}(k) \Pi_{3}^{-1} c_{2}(-k)^{-1}\right]^{-1}\binom{z_{1}(k)}{z_{2}(k)},  \tag{4.10a}\\
& \binom{\tilde{d}_{1}}{\tilde{d}_{2}}=\left[\eta_{2}^{-1} c_{2}(-k)-c_{1}(k) \Pi_{3}^{-1}\right]^{-1}\binom{z_{1}(k)}{z_{2}(k)}, \tag{4.10b}
\end{align*}
$$

provided the matrices in the square brackets are non-singular (it is sufficient that one of them is non-singular). Combining now the relations (4.8) and (4.10), we get the sought transmission coefficient

$$
\begin{equation*}
T(E)=|b|^{2}=\frac{B_{2}^{2}}{1+k^{2} A_{2}^{2}}\left|\tilde{c}_{2}+\tilde{d}_{2}\right|^{2} \tag{4.11}
\end{equation*}
$$

where $E=h^{2} k^{2} / 2 m^{*}$. In a similar way, the reflection coefficient at energy $E$ is given by

$$
\begin{equation*}
R(E)=|a|^{2}=\frac{\left|2-B_{1}\left(c_{2}+d_{2}\right)\right|^{2}}{1+k^{2} A_{1}^{2}} \tag{4.12}
\end{equation*}
$$

## 5. Semiclassical expressions for the transfer matrices

The relations (2.2) and (4.11) represent the solution to our problem. In order to calculate the conductivity, however, one must know the transfer matrices (4.4). They can be written down analytically for very few potentials, so one must look for another way. One possibility is to solve the eqs.(4.2) numerically ; it will be done in a separate paper $/ 15 /$.

Instead of that, we shall use here an analytically expressed but approximative solution. In fact, the WKB-approximation is applicable to nearly all situations in our model with the exception of those when the, energy is near the top of the potential barrier or a plateau in the "upper" branch of the loop. We restrict our attention, however, to the simplest situation represented by the weak-field case when no tunneling occurs (Fig.2). This is true if

$$
\begin{equation*}
\mathcal{L} \lesssim \frac{E}{2 a e} \tag{5.1}
\end{equation*}
$$

where a is acharacteristic size of the "upper" branch, say, the radius of the ring. Taking $a \approx$ $\approx 2 \cdot 10^{-5} \mathrm{~cm}$ and a typical Fermi energy for metals, we get $\mathcal{\varepsilon} \leqslant$ $\lesssim 10^{5} \mathrm{~V} \mathrm{~cm}^{-1}$. For semiconductors, this bound is correspondingly lower, $\varepsilon \lesssim 10^{3} \mathrm{~V} \mathrm{~cm}^{-\uparrow}$, but still - it is possible to obtain interference minima before the tunneling regime takes place.

The WKB-approximation may be applied if $\left|\left(K / p_{j}\left(x_{j}\right)\right)^{\prime}\right| \ll 1$, where

$$
\begin{equation*}
p_{j}\left(x_{j}\right)=\left(2 m^{*}\left(E-v_{j}\left(x_{j}\right)\right)\right)^{1 / 2} \tag{5.2}
\end{equation*}
$$

Since $p_{j}(x) \approx \sqrt{2 m^{*} E}$ in the weak-field case, it yields the condition

$$
\begin{equation*}
\varepsilon \ll \frac{\left(2 m^{*} E\right)^{3 / 2}}{m^{*} e \nmid} \tag{5.3}
\end{equation*}
$$

In metals, this leads to the requirement $\mathcal{E} \ll 10^{9} \mathrm{Vcm}^{-1}$. In semiconductors, we get $\mathcal{E} \ll 10^{5} \mathrm{~V} \mathrm{~cm}^{-1}$ which is still fine.

The general wKB-solution to eq.(4.2a) is well-known $/ 23 /$; it is only necessary to select the two solutions that fulfil the boundary conditions (4.3a). We obtain

$$
\begin{aligned}
& f_{1}\left(x_{2}\right)=\left(\frac{p_{2}(0)}{p_{2}\left(x_{2}\right)}\right)^{1 / 2} \cos \left(\frac{1}{h} \int_{0}^{x} p_{2}(y) d y\right)- \\
& -\frac{m^{*} K V_{2}^{\prime}(0)}{2 p_{2}(0)^{5 / 2} p_{2}\left(x_{2}\right)^{1 / 2}} \sin \left(\frac{1}{k} \int_{0}^{x_{2}} p_{2}(y) d y\right) \text {, } \\
& f_{1}^{\prime}\left(x_{2}\right)=m * \frac{v_{2}^{\prime}\left(x_{2}\right) p_{2}(0)^{3}-v_{2}^{\prime}(0) p_{2}\left(x_{2}\right)^{3}}{2\left(p_{2}(0) p_{2}\left(x_{2}\right)\right)^{5 / 2}} \cos \left(\frac{1}{h_{1}} \int_{0}^{x_{2}} p_{2}(y) d y\right)-
\end{aligned}
$$

$$
\begin{align*}
f_{2}\left(x_{2}\right)= & \frac{h}{\left(p_{2}(0) p_{2}\left(x_{2}\right)\right)^{1 / 2}} \sin \left(\frac{1}{h} \int_{0}^{x_{2}} p_{2}(y) d y\right),  \tag{5.4c}\\
f_{2}^{\prime}\left(x_{2}\right)= & -\left(\frac{p_{2}\left(x_{2}\right)}{p_{2}(0)}\right)^{1 / 2} \cos \left(\frac{1}{h} \int_{0}^{x_{2}} p_{2}(y) d y\right)+ \\
& +\frac{m^{*} h v_{2}^{\prime}\left(x_{2}\right)}{2 p_{2}(0)^{1 / 2} p_{2}\left(x_{2}\right)^{5 / 2}} \sin \left(\frac{1}{h} \int_{0}^{x_{2}} p_{2}(y) d y\right) ; \tag{5.4d}
\end{align*}
$$

substituting $x_{2}=l_{2}$ and using' (4.4b), we get the sought expression of $\Pi_{2}$. The transfer matrix $\Pi_{3}$ expresses in quite the same way by means of $p_{3}, V_{3}$ and $V_{3}^{\prime}$.

## 6. Conclusions

On the following pages, we present a few examples of conductivity plots calculated in the above described way. The shape of the loop is sketched on each graph ; the junctions are supposed to be the same and such that $A_{1}=B_{1}=-A_{2}=-B_{2}=1$. The first three plots are calculated for gold ( $E_{F}=5.49 \mathrm{eV}$ ), while the remaining ones refer to a GaAs-wire; in that case we choose $E=0.05 \mathrm{eV}$; in accord with the results of Ref. 24 . Variating the parameters that characterize the junctions, the loop shape and the energy $E$, we obtain other curves. The qualitative character does not change, however. The conductivity always exhibits large oscillations with well-distinguished minima at reasonably low field intensities.

Let us add a few comments. The results of our model for metals have only an academic meaning because the applied field is screened in this case. For a bulk-metal wire, the coreening would be complete. For ultrathin wires whose cross seotion contains $\sim 10^{2}$ atoms, however, at least the electrons near the surface feel the field and the effect survives, though the conductivity plot is expected to be smeared. In order to find it on the basis of the above considerations, one has to solve the corresponding electrostatic problem and to calculate the (appropriately weighted) average over the potentials $V_{j}$.

In semiconductors, a similar smearing effect might be caused by self-screening if sufficiently many electrons were injected into the structure. However, taking into account typical currents ( $\mu \mathrm{A}$ to nA region) and velocities ( $10^{6}-10^{7} \mathrm{~cm} / \mathrm{s}$ ) in these experiments, we see that typically a single conductivity electron is present on the loop.

Hence we can conclude that the prospect of constructing a new type of switching device (quantum interference transistors, if you like) based on the ultrathin semiconduetor wires is fully realistic. They would have two attractive features. For one, the values of field intensities corresponding to interference minima show that such a device could operate at very low switching voltages, since the electrodes generating the external field can be placed very close to the loop. Even more appealing is the possibility of tailoring the conductance plot by choosing an appropriate shape of the loop.






Fig.4. The conductivity
plofs for GaAs-loops


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Экснер П., Шеба П., Штовичек П. Квантовая интерферениия на графиках, управляемая внешним электрическим полем

Рассматривается движение заряженной квантовой частицы на петле с двумя внешними приөодами, помещенной в эпектростатическое поле. Гамильтониан петли выбирается простейшим образом; чтобы связать его с гамильтонианами приводов, используется метоД, основыяаюцийся на самосопряженных расширениях. Накладыөая условие симметрии, мы получаем полный гамильтониан, зависящий от четырех свободных параметров; каждый контакт характеризуется одной парой. Рассматриваемая система представпяет собой модень металлической или полупроводниковой структуры, которые могут производитьск при помощи доступных сегодня технологий. В предположении баллистического режима для электронов вычислено сопротив ление такой структуры в зависимости от интенсивности внешнего поля. Результа ты показывают возможность создания квантовых интерференционных транзисторов, у которых размеры и выключающсе напряжение могут быть намного меньше чем в применяемых сегодня микросхемах.

Работа выполнена в Лаборатории теоретической физики оияи.

Сообщение Объединенного института пдерных иселедований. Дубна 1987
Exner P., Šeba P., Štovíek P.
Quantum Interference on Graphs Controlled by an External
Electric Field

We consider motion of a charged quantum particle on a loop with two external leads which is placed into an electrostatic field. The loop Hamiltonian is chosen in the simplest possible way; in order to join it to the free Hamiltonians describing the leads, we employ a method based on self-adjoint extensions. Under a symmetry requirenient, the resulting full Hamiltonian contains four free parameters; each junction is characterized by a pair of them. The system under consideration represents a model of metalic or semiconductor structures that can be fabricated by presently available technologies. Assuming the ballistic regime for electrons in such a structure, we calculate the resistance dependence on intensity of the external field. The results suggest the possibility of constructing quantum interference transis tors whose size and switching voltage would be much smaller than in the presently used microchips.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

