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LOW-ENERGY THEOREMS FOR GRAVITINO-SCALAR PARTICLE SCATTERING

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At present much attention is paid to the construction of a unified theory of all interactions on the basis of supersymmetry. Meanwhile the "final" Lagrangian is not constructed, and we have many intermediate models. In such a situation the results, that are independent of the explicit form of Lagrangian or concrete models, but are found from basic principles of the theory - the conservation laws and general analyticity properties, are of great interest.

The considered process is related with supersymmetry theory predicting the existence of a fermionic partner of the graviton, gravitino, with spin 3/2. In some models to obtain a correct value of the cosmological density, the mass of gravitino is set to be zero (see, for example,/1/). Kinematically the simplest process with gravitino is its scattering on a scalar target.

Knowledge of the kinematic structure of helicity amplitudes of binary processes leads to interesting consequences. Kinematical constraints on helicity amplitudes determine the gravitino-graviton Born amplitudes in supergravity/2/. The spin kinematics allows one to obtain the low-energy theorems for photon-hadron processes/3,4/.

The process under consideration in the c.m. system of the s-channel is described by the amplitudes $f^{s}_{\frac{3}{2}0,\frac{3}{2}0}(s,t)$ and $f^{s}_{\frac{3}{2}0,-\frac{3}{2}0}(s,t)$. These amplitudes are decomposed into the Wigner rotation functions; as the latter are connected with the Jacoby polynomials $P^{mn}_{k}(\cos\theta)$/5/, they may be written in a general case for arbitrary binary processes as

$$f^{s}_{\lambda_3\lambda_4,\lambda_1\lambda_2}(s,t)=\left(\cos\frac{\theta_s}{2}\right)^{|\lambda+\mu|}\left(\sin\frac{\theta_s}{2}\right)^{|\lambda-\mu|}\sum_J(2J+1)f^{s,J}_{\lambda_3\lambda_4,\lambda_1\lambda_2}(s)P^{|\lambda-\mu|,|\lambda+\mu|}_{J-\max(|\lambda|,|\mu|)}(\cos\theta), \tag{1}$$

where λ_i are helicities of the corresponding particles; $\lambda=\lambda_1-\lambda_2$, $\mu=\lambda_3-\lambda_4$, θ_s is the scattering angle, $\cos\theta_s$ is linearly dependent on t. The reduced amplitudes $\hat{f}^{s}$ are related to the helicity amplitudes by

$$f^{s}_{\frac{3}{2}0,\frac{3}{2}0}(s,t)=\cos^3\frac{\theta_s}{2}\,\hat{f}^{s}_{\frac{3}{2}0,\frac{3}{2}0}(s,t),\quad f^{s}_{\frac{3}{2}0,-\frac{3}{2}0}(s,t)=\sin^3\frac{\theta_s}{2}\,\hat{f}^{s}_{\frac{3}{2}0,-\frac{3}{2}0}(s,t). \tag{2}$$

The reduced amplitudes are decomposed into polynomials in t and have no kinematic singularities in the variable. In (2) they are separated in the factors in front of the reduced amplitudes. The same factors automatically control the number of amplitudes which describe the process because for forward and backward scattering only one nonzero amplitude must remain, and the second amplitude is forbidden by the angular-momentum projection conservation law. The t-channel helicity and reduced amplitudes are related by

$$f^{t}_{\frac{3}{2}\frac{3}{2},00}(s,t)=\hat{f}^{t}_{\frac{3}{2}\frac{3}{2},00}(s,t),\quad f^{t}_{\frac{3}{2}-\frac{3}{2},00}(s,t)=\sin^3\frac{\theta_t}{2}\cos^3\frac{\theta_t}{2}\,\hat{f}^{t}_{\frac{3}{2}-\frac{3}{2},00}(s,t). \tag{3}$$

The reduced amplitudes in the annihilation channel are decomposed in series over polynomials of the variable s, and in this variable they are free from kinematic singularities. Here, both in the forward and backward scattering the second helicity amplitude is equal to zero and this is provided by the corresponding factors in (3).

In our case, when the scattered particle has a zero mass and the target spin is zero, the crossing-relations are very simple/6/:

$$f^{s}_{\frac{3}{2}0,\frac{3}{2}0}(s,t)=\alpha f^{t}_{\frac{3}{2},-\frac{3}{2},00}(s,t),\quad f^{s}_{\frac{3}{2}0,-\frac{3}{2}0}(s,t)=\beta f^{t}_{\frac{3}{2},-\frac{3}{2},00}(s,t). \tag{4}$$

(α and β are constant modulo unity). Crossing-relations for reduced helicity amplitudes have the following form

$$\hat{f}^{s}_{\lambda_3 0,\lambda_1 0}(s,t)=\left(\frac{s-m^2}{m^2}\right)^3\left(\frac{m}{\sqrt{s}}\right)^{|\lambda_1-\lambda_3|}F_{\lambda_1\lambda_3}(t)\hat{f}^{t}_{\lambda_3-\lambda_1,00}(s,t). \tag{5}$$

The function $F_{\lambda_3\lambda_1}$ does not depend on s. The expressions before the reduced amplitudes of the annihilation channel contain kinematic singularities in s of s-channel helicity amplitudes. So, we may define the dynamic amplitudes for the considered process which are free from kinematic singularities in both invariant variables and are connected with the helicity amplitudes in the following way:

$$f^{s}_{3/2\,0,\,3/2\,0}(s,t)=\left(\frac{s-m^2}{m^2}\right)^3\cos^3\frac{\theta_s}{2}\,\mathfrak{F}_{3/2\,0,\,3/2\,0}(s,t),$$

$$f^{s}_{3/2\,0,\,-3/2\,0}(s,t)=\left(\frac{s-m^2}{m^2}\right)^3\left(\frac{m}{\sqrt{s}}\right)^3\sin^3\frac{\theta_s}{2}\,\mathfrak{F}_{3/2\,0,\,-3/2\,0}(s,t). \qquad (6)$$

Dynamic amplitudes $\mathfrak{F}_h(s,t)$ $(h=\lambda_3\lambda_4,\lambda_1\lambda_2)$ possess singularities only by the unitarity conditions. They have a clear physical meaning. The physical observable quantities are simply expressed in terms of dynamic amplitudes. In such a parametrization the conservation rules are fulfilled automatically. In contrast with the regularized helicity amplitudes and invariant amplitudes, dynamic amplitudes have the same dimensions as those of the helicity amplitudes (for πN- and NN- scattering, dynamic amplitudes were considered in /7,8/).

Since the dynamic amplitudes are free from kinematic singularities, the dispersion relation may be writted for them

$$\mathfrak{F}_h(s,t)=\mathfrak{F}^B_h(s,t)+\frac{1}{\pi}\int_{t_0}^{\infty}\frac{dt'}{t'-t}[\mathfrak{F}_h(s,t')]^t+\frac{1}{\pi}\int_{u_0}^{\infty}\frac{du'}{u'-u}[\mathfrak{F}_h(s,u')]^u. \qquad (7)$$

$[\ldots]^{t(u)}$ denotes the absorptive part of the function in brackets in the $t(u)$ channel. $\mathfrak{F}^B_h(s,t)$ denotes the contributions of pole terms to dynamic amplitudes, corresponding to one-particle intermediate states in the unitarity condition.

The energy of gravitino E is related to s and t as follows:

$$E=\frac{s-m^2}{2\sqrt{s}},\qquad t=-2E^2\sin^2\frac{\theta_s}{2}, \qquad (8)$$

when $E\to 0$ (θ_s - fixed), $t\to 0$ and $s\to m^2$. We suppose that there is no degeneracy of masses: there are no fermions with the mass that exactly equals the target (pion) mass. Then the corresponding Born pole at $u=m^2$ is absent in dispersion relations. However, there exist t-channel pole terms with a massless photon and gravitino exchange. We neglect the graviton exchange contribution supposing that it is much smaller than the photon-exchange contribution because of the gravitational constant being small.

When an analogous process, photon Compton-effect, was considered, contributions from unitary diagrams with massless particles from two- and many-particle intermediate states were neglected because the electromagnetic constant is small. The results thus obtained were exact in strong interactions, but only in e^2 approximation in electromagnetic interactions. We shall consider that the constant (or constants) of gravitino interaction is small and neglect the contributions to the continuum from unitary diagrams which contain massless particles. From (8) we get the dispersion relations for helicity amplitudes /9/:

$$f^s_{3/2\,0,\,3/2\,0}(s,t)=\bar{f}^B_{3/2\,0,\,3/2\,0}(s,t)+\cos^3\frac{\theta_s}{2}\left(\frac{s-m^2}{m^2}\right)^3\Big\{\tilde{\mathfrak{F}}^B_{3/2\,0,\,3/2\,0}(s,t)+$$
$$+\frac{1}{\pi}\int_{t_0}^{\infty}\frac{dt'}{t'-t}\Big[\Big(\frac{m^2}{\sqrt{(s-m^2)^2+st'}}\Big)^3 f^s_{3/2\,0,\,3/2\,0}(s,t')\Big]^t+ \qquad (9)$$
$$+\frac{1}{\pi}\int_{u_0}^{\infty}\frac{du'}{u'-u}\Big[\Big(\frac{m^2}{\sqrt{m^4-su'}}\Big)^3 f^s_{3/2\,0,\,3/2\,0}(s,u')\Big]^u\Big\},$$

$$f^s_{3/2\,0,\,-3/2\,0}(s,t)=\bar{f}^B_{3/2\,0,\,-3/2\,0}(s,t)+\sin^3\frac{\theta_s}{2}\left(\frac{m}{\sqrt{s}}\right)^3\left(\frac{s-m^2}{m^2}\right)^3\times$$
$$\times\Big\{\tilde{\mathfrak{F}}^B_{3/2\,0,\,-3/2\,0}(s,t)+\frac{1}{\pi}\int_{t_0}^{\infty}\frac{dt'}{t'-t}\Big[\Big(\frac{m}{\sqrt{-t'}}\Big)^3 f^s_{3/2\,0,\,-3/2\,0}(s,t')\Big]^t+ \qquad (10)$$
$$+\frac{1}{\pi}\int_{u_0}^{\infty}\frac{du'}{u'-u}\Big[\Big(\frac{m}{\sqrt{s+u'-2m^2}}\Big)^3 f^s_{3/2\,0,\,-3/2\,0}(s,u')\Big]^u\Big\}.$$

The Born term with photon exchange is denoted by $\bar{f}^B_h(s,t)$. The contributions of all other possible Born terms to the dynamic amplitudes are denoted by $\tilde{\mathfrak{F}}^B_h(s,t)$. The expressions in the braces are regular functions at $s=m^2$ and $t=0$. In the low-energy limit unknown contributions of these functions are suppressed by the factor $(s-m^2)^3\sim E^3$. Therefore, at low energies both the helicity amplitudes up to $O(E^3)$ are determined by their t-channel Born terms with photon exchange:

$$f^{s}_{\lambda_3 0,\lambda_1 0}(s,t) = \bar{f}^{B}_{\lambda_3 0,\lambda_1 0}(s,t) + O(E^3). \qquad (11)$$

This is our result: the low-energy theorems in a general form have been found on the basis of a kinematical analysis. We use neither the Ward identities (as in [10,11] when the theorem for Compton effect was proved), nor the perturbative expansion (the consideration was carried out in the framework of the S-matrix approach). The dominant Born term may be calculated in any supersymmetric model. Analogous low-energy theorems for photon scattering determine the helicity amplitudes up to $O(E^2)$ [3,12].

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Низкоэнергетические теоремы
для рассеяния гравитино на скалярной
частице

Исследуется кинематическая структура спиральных амплитуд рассеяния безмассового гравитино на скалярной частице. Вводятся динамические амплитуды, которые имеют одинаковые размерности и свободны от кинематических особенностей. Получены дисперсионные соотношения и низкоэнергетические теоремы для спиральных амплитуд.

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Chavleishvili M.P. E2-87-69

Low-Energy Theorems for Gravitino-Scalar
Particle Scattering

The kinematic structure of the helicity amplitudes of massless gravitino scattering on scalar particle is investigated. Dynamic amplitudes which are of the same dimension and free from kinematic singularities are introduced. Dispersion relations and low-energy theorems for helicity amplitudes are obtained.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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