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PARISI - SOURLAS TYPE SYMMETRY AND HAMILTONIAN BRST FORMALISM

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1. BRST symmetry has been discovered [1] in connection with the quantization of non-abelian gauge theories. At present time it received much attention in connection with string theory. It plays a central role in the attempts to construct a covariant string field theory [2,3].

Fradkin and Vilkovisky proposed the hamiltonian BRST formalism working in an extended phase space for treating systems with first class constraints [4,5]. Extensions of the BRST symmetry in the lagrangian formalism have been discussed in a number of papers [6]. Siegel and Zwiebach [3] used the BRST symmetry extended to the group OSP(1,1 2) in order to reconstruct a gauge-covariant string field theory from the gauge-fixed light-cone formalism. The OSP(1,1|2) supersymmetry, rather in the euclidean form OSP(2|2), was encountered already by Parisi and Sourlas [7] in analyzing the mechanism of dimensional reduction in the critical behaviour of systems in a stochastic background. This reduction mechanism was used in stochastic quantization [8]. Relating different approaches to string field theory Neveu and West [9] pointed out the connection of the BRST formalism to the Parisi - Sourlas reduction; see also [10].

The OSP(1,1|2) extension of the hamiltonian BRST approach has been applied to the relativistic point particle [9,11,12]. General systems with first class constraints have been considered under this aspect by Aratyn, Ingermanson and Niemi [12]. They showed that for a system with an abelian constraint algebra the BRST hamiltonian can always be chosen in such a form, that the symmetry is extended to OSP(1,1|2). Then the Parisi - Sourlas reduction provides a clear understanding of how the extended phase space formalism works. The generalization to the case of a non-commuting set of first class constraints requires a construction which relates these constraints to an abelian constraint algebra.

In discussing the relation of the BRST formalism to the Parisi-Sourlas reduction the emphasis in the mentioned papers was put on the possibility of extending the BRST symmetry. Especially in [12] the hamiltonian formalism has been presented in a form to exhibit this relation. In the present note we consider more closely the other side of this important relation. We point out that it is not necessary to require the full OSP(1,1|2) symmetry for the Parisi - Sourlas reduction. A subgroup directly related to the BRST symmetry without extension is enough. In this way we are able to understand on the basis of the Parisi - Sourlas reduction how the extended phase space formalism works also in the general case, if the BRST symmetry is not extended. Starting from the reduction based on the subgroup only it becomes simpler, compared to ref. /12/, to treat the case of non-commuting constraints. Only the BRST and the fermionic ghost charges have to be constructed from the subgroup generators. It is not necessary to construct a relation to an abelian constraint algebra.

2. Parisi and Sourlas /7/ discovered that the mechanism of dimensional reduction by 2 units in the critical behaviour of systems in a stochastic background is based on the supersymmetry OSP(2|2) rotating two bosonic x_1, x_2 and two fermionic $\theta, \bar\theta$ coordinates. The essential point is that the integral of a function of $x_1, x_2, \theta, \bar\theta$ depending on these variables only via the combination $x_1^2+x_2^2+\theta\bar\theta$ (and tending to zero for $x_1^2+x_2^2 \to \infty$) reduces to the value of the function at argument zero. There is a cancellation in the sum over all configurations such that only the one with $x_1 = x_2 = 0$, $\theta = \bar\theta = 0$ remains. The same holds if we choose in the bosonic part of the quadratic form the pseudoeuclidean metric, i.e. require the function to depend only on the combination $x^+x^- + \theta\bar\theta$, $x^\pm = x_1 \pm x_2$.

Our point is that in both cases the reduction works for functions depending on $x^+, x^-, \theta, \bar\theta$ in a more general way. Consider a function depending on these variables via $x^+x^- + \theta\bar\theta$ and x^+ : $f(x^+, x^+x^- + \theta\bar\theta)$. We assume that the dependence on the second argument can be represented by a Fourier integral which converges uniformly

$$f(x^+, y) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(x^+, s)\, e^{-isy}\, ds . \tag{1}$$

Calculate the integral

$$\begin{aligned} &\int dx^+ dx^- d\theta\, d\bar\theta\ \ f(x^+, x^+x^- + \theta\bar\theta) = \\ &\frac{1}{2\pi}\int_{-\infty}^{\infty} ds \int dx^+\ F(x^+, s)\int_{-\infty}^{\infty} dx^- \int d\theta\, d\bar\theta\ e^{-is(x^+x^- + \theta\bar\theta)} . \end{aligned} \tag{2}$$

A non-vanishing contribution to the integral over θ , $\bar\theta$ is obtained only from the term in the decomposition proportional to $\theta\bar\theta$. The integral over x^- yields the δ-function. We have

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} ds \int dx^+\ is\, 2\pi\, \delta(sx^+)\ \ F(x^+, s) =$$

$$i\int_{-\infty}^{\infty} ds\ F(0, s) \ = 2\pi i\, f(0, 0) . \tag{3}$$

The integration interval over x^+ should include $x^+ = 0$, but it needs not to cover the whole axis.

Thus we have shown that also with less restrictions on the function of $x^+, x^-, \theta, \bar\theta$ the sum over all configurations reduces to the contribution at $x^+ = x^- = 0$, $\theta = \bar\theta = 0$. A further generalization is possible, allowing for an additional dependence on either θ or $\bar\theta$.

3. The restriction on the function to depend on $x^+, x^-, \theta, \bar\theta$ only via $x^+x^- + \theta\bar\theta$ can be expressed by the condition of invariance with respect to OSP(1,1|2), acting on these variables as follows.

$$\begin{aligned}
&\delta^{+-}x^+ = x^+ , \quad \delta^{+-}x^- = -x^- , \quad \delta^{+-}\theta = \delta^{+-}\bar\theta = 0 , \\
&\delta^{\bar\theta\theta}\theta = \theta , \quad \delta^{\bar\theta\theta}\bar\theta = -\bar\theta , \quad \delta^{\bar\theta\theta}x^\pm = 0 , \\
&\delta^{\pm\theta}x^\mp = -\theta , \quad \delta^{\pm\theta}\bar\theta = x^\pm , \quad \delta^{\pm\theta}x^\pm = 0 , \quad \delta^{\pm\theta}\theta = 0 , \\
&\delta^{\pm\bar\theta}x^\mp = \bar\theta , \quad \delta^{\pm\bar\theta}\theta = x^\pm , \quad \delta^{\pm\bar\theta}x^\pm = 0 , \quad \delta^{\pm\bar\theta}\bar\theta = 0 , \\
&\delta^{\bar\theta\bar\theta}\theta = \bar\theta , \quad \delta^{\bar\theta\bar\theta}\theta = \delta^{\bar\theta\bar\theta}x^\pm = 0 , \\
&\delta^{\theta\theta}\bar\theta = \theta , \quad \delta^{\theta\theta}\bar\theta = \delta^{\theta\theta}x^\pm = 0 .
\end{aligned} \tag{4}$$

δ^{+-} and $\delta^{\bar\theta\theta}$ generate $U(1)$ subgroups and define bosonic and fermionic charges (θ , $\bar\theta$, $x^\pm$ have fermionic charges $1, -1, 0$, respectively). The remaining generators are nilpotent.

Consider the subgroup generated by $\delta^{+\theta}$ and $\delta^{\bar\theta\theta}$. It leaves x^+ and $x^+x^- + \theta\bar\theta$ invariant. There are more invariant combinations, but they reduce to functions of these two. Therefore the condition on a function to have the form $f(x^+, x^+x^- + \theta\bar\theta)$ can be expressed by the condition of invariance with respect to this subgroup.

If we would demand invariance with respect to $\delta^{+\theta}$ only but admit the fermionic charge $\delta^{\bar\theta\theta}$ to have non-negative values, we would allow for an additional dependence on θ .

The subgroup generated by $\delta^{+\bar\theta}$ and $\delta^{\bar\theta\theta}$ works equally well. The subgroups ($\delta^{-\theta}$, $\delta^{\bar\theta\theta}$) or ($\delta^{-\bar\theta}$, $\delta^{\bar\theta\theta}$) play an analogous role for x^+ replaced by x^- .

In the framework of hamiltonian formalism we introduce variables $p^\pm, \mathcal{P}, \bar{\mathcal{P}}$ conjugate to $x^\mp, \bar{\theta}, \theta$ with the Poisson brackets

$$\{p^+, x^-\} = \{p^-, x^+\} = 1,$$
$$\{\bar{\mathcal{P}}, \theta\} = \{\mathcal{P}, \bar{\theta}\} = 1, \tag{5}$$

and represent the action of the generators δ by Poisson brackets with operators R.

$$\delta^{AB} x^C = \{R^{AB}, x^C\}, \quad (A, B, C \hat{=} +, -, \theta, \bar{\theta}), \tag{6}$$

where

$$R^{+-} = x^+ p^- - x^- p^+, \quad R^{\bar{\theta}\theta} = \theta\bar{\mathcal{P}} - \bar{\theta}\mathcal{P},$$
$$R^{\bar{\theta}\bar{\theta}} = \bar{\theta}\bar{\mathcal{P}}, \quad R^{\theta\theta} = \theta\mathcal{P}, \tag{7}$$
$$R^{\pm\theta} = x^\pm \mathcal{P} - \theta p^\pm, \quad R^{\pm\bar{\theta}} = x^\pm \bar{\mathcal{P}} + \bar{\theta} p^\pm.$$

4. Consider a hamiltonian system with first class constraints Φ_i, $i = 1,\dots,n$. The action including the constraints via Lagrange multiplyers λ_i is given by

$$S = \int_0^T dt\, (p_a \dot{q}_a - \mathcal{H}(p_b, q_b) - \lambda_i \Phi_i(p_b, q_b)), \tag{8}$$
$$a, b = 1, \dots, f > n.$$

In general the constraints do not commute strongly with each other or with the hamiltonian

$$\{\Phi_i, \Phi_j\} = V_{ij}^k \Phi_k, \quad \{\Phi_i, \mathcal{H}\} = U_i^j \Phi_j. \tag{9}$$

We restrict ourselves to V_{ij}^k, U_i^j independent of Φ_ℓ. The treatment of the general case is known [4,5] and there is nothing to be added from the viewpoint of Parisi - Sourlas reduction.

Following Fradkin and Vilkovisky we introduce for each i the momentum $\pi_i(t)$ conjugate to $\lambda_i(t)$ and anticommuting variables $\theta_i(t)$, $\bar{\theta}_i(t)$ with their conjugate momenta $\bar{\mathcal{P}}_i(t)$ and $\mathcal{P}_i(t)$. For each i and time t the constraint and the additional variables $\Phi_i(t)$, $\lambda_i(t)$, $\theta_i(t)$, $\bar{\theta}_i(t)$ and their conjugates shall be related to the above variables x^+, x^-, θ, $\bar{\theta}$ and their conjugates, respectively. The BRST operator Q restricted to the subspace of variables of a given i shall act just as $\delta^{+\theta}$, whereas the fermionic charge F is counted by $\delta^{\bar{\theta}\theta}$.

In the case of commuting constraints the corresponding operators are represented by the sums [12]

$$Q^{(0)} = \sum_{i=1}^n R_i^{+\theta}, \quad F = \sum_{i=1}^n R_i^{\bar{\theta}\theta}, \tag{10}$$

where the R_i^{AB} are obtained from R^{AB} in (7) by replacing $x^+, x^-, \theta, \bar{\theta}$ and their conjugates by $\Phi_i(t)$, $\lambda_i(t)$, $\theta_i(t)$, $\bar{\theta}_i(t)$ and their conjugates. In the general case $Q^{(0)}$ has to be modified since

$$\{Q^{(0)}, Q^{(0)}\} = \sum_{i,j,k} \mathcal{P}_i \mathcal{P}_j V_{ij}^k \Phi_k \neq 0 \tag{11}$$

and therefore it does not generate a supersymmetry transformation. The proper BRST operator is

$$Q = Q^{(0)} - \frac{1}{2} \mathcal{P}_i \mathcal{P}_j V_{ij}^k \bar{\theta}_k. \tag{12}$$

The extended action invariant under Q and F is written in the usual way [4,5].

$$S_{ext} = \int_0^T dt\, (p_a \dot{q}_a + \pi_i \dot{\lambda}_i + \bar{\mathcal{P}}_i \dot{\theta}_i + \mathcal{P}_i \dot{\bar{\theta}}_i - \mathcal{H}(p_b, q_b) - U_i^j \mathcal{P}_i \bar{\theta}_j + \{Q, \Psi\}) \tag{13}$$

Ψ is an arbitrary function of the variables with fermionic charge -1.

Consider a transition amplitude calculated with this action between states invariant with respect to Q and F. We insert a complete set of intermediate states at time t, $0 < t < T$, characterized by the values of a complete set of commuting variables including $\bar{\theta}_i(t)$, $\theta_i(t)$, $\lambda_i(t)$ ($i = 1, \dots, n$) and $\Phi_1(t)$. We show that in the sum over all intermediate states only the ones obeying the constraint $\Phi_1(t) = 0$ contribute. Consider the integral over all variables but θ_1, $\bar{\theta}_1$, λ_1. $Q^{(0)}$ and F act on the latter as $\delta^{+\theta}$ and $\delta^{\bar{\theta}\theta}$ on $\theta, \bar{\theta}, x^-$ in (4). The action of Q on these variables deviates from that of $Q^{(0)}$ only by terms involving $\bar{\theta}_j$. Especially, the invariants $\{Q, \theta_1\}$ and $\{Q, \lambda_1 \theta_1\}$ coincide with Φ_1 and $\Phi_1\lambda_1 + \theta_1\bar{\theta}_1$ up to terms involving $\bar{\theta}_j$. The additional $\bar{\theta}_j$ terms are projected out in the integral over all θ_i, $\bar{\theta}_i$. Because of the condition $F = 0$ all θ_j come in pairs with some $\bar{\theta}_k$. Performing the integral over λ_1, θ_1, $\bar{\theta}_1$ we obtain the δ-function of Φ_1. We are left with a set of intermediate states with $\lambda_1 = \Phi_1 = 0$, $\theta_1 = \bar{\theta}_1 = 0$. This set can be parametrized by the values of commuting variables including $\bar{\theta}_j$, θ_j, λ_j ($j = 2, \dots, n$) and Φ_2. The above argument can be repeated.

In this way we understand on the basis of the Parisi - Sourlas reduction caused by a subgroup of OSP(1,1|2) generated by the same algebra as Q and $\mathcal{F}$ that in the extended phase space formalism invariance with respect to Q and $\mathcal{F}$ ensures unitarity.

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Киршнер Р. E2-87-676
Симметрия типа Паризи и Сургласа
и гамильтонов формализм БРСТ

Симметрия БРСТ в формализме расширенного фазового пространства Фрадкина и Вилковиского связана с подгруппой группы OSP (1, 1/2). Этот формализм можно понимать на основе редукции Паризи и Сургласа, не расширяя алгебру БРСТ. Оказывается, что инвариантность относительно соответствующей подгруппы достаточна для редукции, которая обеспечивает взаимное сокращение всех вкладов, кроме удовлетворяющих условиям связи.

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Kirschner R. E2-87-676
Parisi - Sourlas Type Symmetry
and Hamiltonian BRST Formalism

The BRST symmetry in the extended phase space formalism of Fradkin and Vilkovisky is related to a subgroup of OSP (1, 1/2). Without extending the BRST algebra we understand the formalism on the basis of Parisi - Sourlas reduction. This is because invariance under the corresponding subgroup is sufficient for reduction, which guarantees cancellation of all contributions besides of those satisfiying the constraints.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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