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SOME INTEGRALS FOR ANALYTIC BREMSSTRAHLUNG CALCULATION.

Photon and Z^0-Boson Exchange in the Ultrarelativistic Limit

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1. INTRODUCTION

This communication is a continuation of paper [1] where we have presented integrals used for analytic bremsstrahlung calculations exact in fermion masses in QED [2]. Here, we extend our results [1] to processes with an intermediate massive vector boson, but take the limit extreme relativistic in fermion masses (m_e^2, $m_f^2 \ll s$). Additional difficulties are related with the appearance of a new complex parameter

$$M^2 = M_V^2 - iM_V\Gamma_V, \tag{1}$$

real and imaginary parts of which are in general not small, $\sim S$.

We dealt with the completely inclusive situation in the photon variables [2-6] for the processes:

$$e^+e^- \to \gamma,\ Z^\circ \to \bar{f}f(\gamma), \tag{2a}$$

$$V \to \bar{f}_1 f_2(\gamma), \quad V = W^\pm,\ Z^\circ, \tag{2a}$$

i.e. we calculated the observables integrated over photon degrees of freedom

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi^2}{4S}\int_0^1 x\,dx \frac{1-x}{1-x+m_f^2/s}\ \frac{1}{2}\int_{-1}^{+1} d\cos\theta_R\ \frac{1}{2\pi}\int_0^{2\pi} d\phi_R |M_{br}|^2, \tag{3}$$

$$\sigma_{tot} = \int_{-1}^{+1} d\cos\theta\ \frac{d\sigma}{d\cos\theta}, \tag{4a}$$

$$A_{FB} = \frac{1}{\sigma_{tot}}\left[\int_0^1 d\cos\theta\ \frac{d\sigma}{d\cos\theta} - \int_{-1}^{1} d\cos\theta \frac{d\sigma}{d\cos\theta}\right], \tag{5}$$

$$\Gamma_V = \frac{\pi^2}{4M_V^2}\int_0^1 x\,dx \frac{1-x}{1-x+m_2^2/M_V^2}\ \frac{1}{2}\int_{-1}^{+1} d\cos\theta_R\ \frac{1}{2\pi}\int_0^{2\pi} d\phi_R\ |M_{br}|^2. \tag{4b}$$

The isolation of the infrared singularity and calculation of the soft-photon contribution have been described in detail in [4] following the method developed in [7]. Hard bremsstrahlung is treated by SCHOONSCHIP [8] in the following way (for more details see [9]): with the aid of algebraic identities between the kinematical variables one produces an expression which contains the so-called canonical set of R_γ-integrals of Table I, see below. After applying them and additional algebra, Table II may be used, and upon a similar third stage based on Table III one gets (4,5). The differential cross section (3) may be derived with Tables I and III only, but now in Table III one has to deal with integrands yet being dependent on the scattering angle.

All details concerning the underlying physics and the meaning of abbreviations may be found in [2-6] and refs. cited therein. We only again remark that the tabulations being presented here are useful in similar contents, too.

Table I

$$[A] = \frac{S(1-x)}{4\pi\tau}\int_{-1}^{+1} d\cos\theta_R \int_0^{2\pi} d\phi_R A(\theta_R,\phi_R), \tag{6}$$

where (θ_R, ϕ_R) are the photon angles in the R_γ system defined by $\vec{p}(f)+\vec{p}(\gamma) = 0$.

1. $[1] = \frac{S(1-x)}{\tau}$,

2. $[\frac{1}{Z_\mp}] = \frac{1}{S(1-xc_\mp)}\cdot\{L_e + L_f - L_\tau + 2\ln(1-xc_\mp)\}$

3. $[Z_\mp] = \frac{S^2}{2\tau}(1-xc_\mp)(1-x)[1]$

4. $[\frac{1}{Z_- Z_+}] = \frac{2}{S^2(1-x)}L_e$

5. $[\frac{1}{X_m}] = \frac{1}{Sx}\{2L_f - L_\tau + 2\ln x\}$

6. $[X_m] = \frac{S}{2}(1-x)(\frac{S}{\tau}-1)[1]$

7. $[\frac{1}{m^2}] = \frac{1}{Sx}\{L_\tau + 2\ln x\}$

8. $[\frac{1}{Z_\mp^2}] = \frac{1}{m_e^2\cdot S(1-x)}$

9. $[\frac{1}{X_m^2}] = \frac{1}{m_f^2\, S(1-x)}$

10. $[\frac{1}{S_m Z_\mp}] = \frac{1}{S^2(1-x)}L_e$

11. $[\frac{Z_\mp}{X_m}] = S(1-x)c_\mp[\frac{1}{X_m}] + \frac{e_\mp}{x}[1]$

12. $[\frac{1}{X_m Z_\mp}] = \frac{1}{S^2x(1-x)c_\mp}\{L_e + L_f + 2\ln c_\mp + 2\ln x\}$

13. $[\frac{X_m}{Z_\mp}] = \frac{Sx(1-x)c_\mp}{1-xc_\mp}[\frac{1}{Z_\mp}] + \frac{xe_\mp}{(1-xc_\mp)^2}[1]$

14. $[\frac{1}{m^2 Z_\mp}] = \frac{1}{S^2xc_\pm}\{L_e + L_f + 2\ln c_\pm + 2\ln x\}$

15. $[\frac{Z_\mp}{m^2}] = Sc_\pm[\frac{1}{m^2}] + \frac{1-x}{x}\{-\frac{S}{\tau} + c_\mp(2+\frac{Sx}{\tau})\}$

16. $[\frac{Z_\mp^2}{X_m}] = S^2c_\mp^2(1-x)^2[\frac{1}{X_m}] + \frac{S(1-x)}{x}c_\mp(c_\pm - 2(1-x)c_\mp) + \frac{S^3(1-x)^2}{2x\tau^2}(1-xc_\mp)^2$

17. $[\frac{1}{m^2 Z_\mp^2}] = \frac{1}{m_e^2 S^2}\cdot\frac{1-xc_\mp}{c_\pm x(1-x)}$

18. $[\frac{Z_\mp^2}{X_m^2}] = \frac{1}{m_f^2}\cdot S(1-x)c_\mp^2$

19. $[\frac{Z_\mp}{X_m^2}] = \frac{1}{m_f^2}\cdot c_\mp$

20. $[\frac{1}{(M^2-m^2)Z_\mp^2}] = \frac{1}{m_e^2}\cdot\frac{1}{S^2(R-1)}\cdot\{\frac{1}{1-x} + \frac{1}{d_\mp}\}$

21. $[\frac{Z_\mp}{M^2-m^2}] = \frac{e_\mp}{x}\{[1] - R\cdot S[\frac{1}{M^2-m^2}]\} + Sc_\pm[\frac{1}{M^2-m^2}]$

22. $[\frac{1}{(M^2-m^2)Z_\mp}] = -\frac{1}{S^2d_\mp}\cdot\ln\frac{Sd_\mp^2}{m_e^2\tau_Z}$

23. $[\frac{1}{M^2-m^2}] = \frac{1}{Sx}\cdot\ln\frac{\tau R - m_f^2}{\tau(R-x)}$

Integrals with m^{-4} can be traced back to pure QED and may be taken from [1]; those with $|M^2-m^2|^{-2}$ may be linearized using

$$\frac{1}{|M^2 - m^2|^2} = \frac{1}{2i F_V M_V} \cdot \left\{ \frac{1}{(M^2 - m^2)} - \frac{1}{(M^2 - m^2)^*} \right\}. \qquad (7)$$

In Table I and below we use the extreme relativistic approximations: $m_e^2,\ m_f^2 \ll S,\ M_V^2$. The following abbreviations are used:

$$x = \frac{X}{S} \qquad Z_- = Z \qquad b_\mp = c_\pm + c_\mp R$$

$$\tau = S - X + m_f^2 \qquad Z_+ = \bar{Z} \qquad e_\mp = c_\pm - c_\mp (1 - x)$$

$$c_\mp = \frac{1}{2}(1 \mp c) \qquad S_m = S - m^2 \qquad R = \frac{M^2}{S}$$

$$c = \cos\theta \qquad X_m = X - m^2 \qquad \epsilon_f = \frac{m_f^2}{S}$$

$$\tau_Z = R(R-x)(1-x) + \epsilon_f (1-R)^2, \quad d_\mp = x b_\mp - R, \quad m^2 = -(p_f + p_{\bar f})^2$$

$$L_e = \ln\frac{S}{m_e^2}, \quad L_f = \ln\frac{S}{m_f^2}, \quad L_\tau = \ln\frac{\tau}{m_f^2}. \qquad (8)$$

Dealing with complex logarithms one has to carefully handle with the rule of decomposition:

$$\ln(a \cdot b) = \ln a + \ln b + 2\pi i\, E(a,b)$$

$$E(a,b) = \begin{cases} \mp 1 & \text{if sign Im } a = \text{sign Im } b = -\text{sign Im } ab = \mp 1 \\ 0 & \text{otherwise} \end{cases} \qquad (9)$$

Table II

$$[A] = \int_0^{1-2\epsilon} dc\, A(c), \quad \epsilon = \frac{m_e^2}{S}. \qquad (10)$$

If the integrand is not singular at the end point, the limit of vanishing electron mass has been taken. The physical origin of the mass singularity is explained, e.g. in /3/ . The $Li_k(z)$ are Euler polylogarithms, $Li_2(1) = \pi^2/6$.

1. $[c^k] = \frac{1}{k+1}, \quad k = 0, 1, \ldots$

2. $[\ln c_\mp] = \mp \ln 2 - 1$

3. $[c \ln c_\mp] = -\frac{1}{2}\ln 2 + \frac{1}{4}\begin{pmatrix} -3 \\ 1 \end{pmatrix}$

4. $[c^2 \ln c_\mp] = \mp \frac{1}{3}\ln 2 - \frac{1}{18}\begin{pmatrix} 11 \\ 5 \end{pmatrix}$

5. $[c^3 \ln c_\mp] = -\frac{1}{4}\ln 2 + \frac{1}{48}\begin{pmatrix} -25 \\ 7 \end{pmatrix}$

6. $[c^4 \ln c_\mp] = \mp \frac{1}{5}\ln 2 - \frac{1}{300}\begin{pmatrix} 137 \\ 47 \end{pmatrix}$

7. $[\frac{1}{c_\pm} \ln c_\mp] = -Li_2(1) \mp \ln^2 2$

8. $[\frac{1}{c_\pm^2} \ln c_\mp] = (1 + L_e) \cdot \begin{pmatrix} 0 \\ -2 \end{pmatrix} \mp 4 \ln 2$

9. $[\frac{1}{c_\pm^3} \ln c_\mp] = \begin{pmatrix} -1 \\ 3/2 \end{pmatrix} - (L_e + \frac{2}{\epsilon})\begin{pmatrix} 0 \\ 1 \end{pmatrix} \mp 4 \ln 2$

10. $[\frac{1}{c_\pm^4} \ln c_\mp] = \begin{pmatrix} -5/3 \\ 22/9 \end{pmatrix} \mp \frac{16}{3}\ln 2 - \begin{pmatrix} 0 \\ 1 \end{pmatrix}\left\{ \frac{2}{3} L_e + \frac{1}{\epsilon} + \frac{1}{\epsilon^2} \right\}$

11. $[\frac{1}{c_\mp} \ln c_\mp] = \begin{pmatrix} -1 \\ 0 \end{pmatrix} L_e^2 \pm \ln^2 2$

12. $[\ln^2 c_\mp] = \pm \ln^2 2 \pm 2 \ln 2 + 2$

13. $[c \ln^2 c_\mp] = \begin{pmatrix} 7/4 \\ -5/4 \end{pmatrix} + \frac{1}{2}\ln^2 2 + \frac{3}{2}\ln 2$

14. $[c^2 \ln^2 c_\mp] = \frac{1}{54}\begin{pmatrix} 85 \\ 55 \end{pmatrix} \pm \frac{1}{3}\ln^2 2 \pm \frac{11}{9}\ln 2$

15. $[c^3 \ln^2 c_\mp] = \frac{1}{288}\begin{pmatrix} 415 \\ -241 \end{pmatrix} + \frac{25}{24}\ln 2 + \frac{1}{4}\ln^2 2$

16. $[c^4 \ln^2 c_\mp] = \frac{1}{9000}\begin{pmatrix} 12019 \\ 6589 \end{pmatrix} \pm \frac{137}{150}\ln 2 \pm \frac{1}{5}\ln^2 2$

17. $\left[\frac{1}{c_{\pm}}\ln^2 c_{\mp}\right] = \mp 4S_{1,2}(\tfrac{1}{2}) + 4\binom{1}{0} S_{1,2}(1)$

$$S_{1,2}(1) = \zeta(3), \quad S_{1,2}(\tfrac{1}{2}) = \frac{1}{8}\zeta(3) - \frac{1}{6}\ln^3 2$$

18. $\left[\frac{1}{c_{\pm}^2}\ln^2 c_{\mp}\right] = 2\,\mathrm{Li}_2(1) \pm 4\ln^2 2$

19. $\left[\frac{1}{c_{\pm}^3}\ln^2 c_{\mp}\right] = \mathrm{Li}_2(1) + \binom{0}{1}\{3 + 2L_e\} \pm 4(\ln 2 + \ln^2 2)$

20. $\left[\frac{1}{c_{\pm}^4}\ln^2 c_{\mp}\right] = \frac{2}{3}\mathrm{Li}_2(1) \pm \frac{16}{3}(\ln 2 + \ln^2 2) + \binom{2/3}{1} + \binom{0}{2}\{L_e + \frac{1}{\epsilon}\}$

21. $[\ln c_- \ln c_+] = -\mathrm{Li}_2(1) + 2$

22. $\{c \ln c_- \ln c_+\} = \frac{1}{2}\ln^2 2 - \frac{1}{2}\ln 2 + \frac{1}{4}$

23. $[c^2 \ln c_- \ln c_+] = -\frac{1}{3}\mathrm{Li}_2(1) + \frac{17}{27}$

24. $\left[\frac{1}{c_{\pm}}\right] = \binom{0}{2} L_e \pm 2\ln 2$

25. $\left[\frac{1}{c_{\pm}^2}\right] = \binom{0}{2}\frac{1}{\epsilon} + \binom{2}{-4}$

26. $\left[\frac{1}{c_{\pm}^3}\right] = \binom{0}{1}\frac{1}{\epsilon^2} + \binom{3}{-4}$

27. $[\mathrm{Li}_2(c_{\mp})] = \frac{1}{2}\binom{1}{3}\mathrm{Li}_2(1) \mp \frac{1}{2}\ln^2 2 \pm \ln 2 - 1$

28. $[c\,\mathrm{Li}_2(c_{\mp})] = \frac{1}{4}\{\mathrm{Li}_2(1) - \ln^2 2 + \ln 2\} + \frac{1}{8}\binom{-3}{1}$

29. $[c^2\mathrm{Li}_2(c_{\mp})] = \frac{1}{6}\binom{1}{3}\mathrm{Li}_2(1) \mp \frac{1}{6}\ln^2 2 \pm \frac{5}{18}\ln 2 - \frac{1}{108}\binom{37}{31}$

30. $[c^3\mathrm{Li}_2(c_{\mp})] = \frac{1}{8}\{\mathrm{Li}_2(1) - \ln^2 2 + \frac{7}{6}\ln 2\} + \frac{1}{576}\binom{-127}{49}$

31. $[c^4\mathrm{Li}_2(c_{\mp})] = \frac{1}{10}\binom{1}{3}\mathrm{Li}_2(1) \mp \frac{1}{10}\ln^2 2 \pm \frac{47}{300}\ln 2 - \frac{1}{18000}\binom{3739}{2869}$

32. $\left[\frac{1}{c_{\mp}}\mathrm{Li}_2(c_{\mp})\right] = \pm 2\mathrm{Li}_3(\tfrac{1}{2}) + 2\binom{0}{1}\mathrm{Li}_3(1)$

$$\mathrm{Li}_3(\tfrac{1}{2}) = \frac{7}{8}\zeta(3) - \frac{1}{2}\ln 2\,\zeta(2) + \frac{1}{6}\ln^3 2, \qquad \mathrm{Li}_3(1) = \zeta(3)$$

33. $\left[\frac{1}{c_{\mp}^2}\mathrm{Li}_2(c_{\mp})\right] = \binom{2}{0}L_e - \binom{2}{0}\mathrm{Li}_2(1) \pm 2\ln^2 2 \mp 4\ln 2 + \binom{4}{0}$

34. $\left\{\frac{1}{c_{\mp}^3}\mathrm{Li}_2(c_{\mp})\right\} = \binom{1}{0}\{\frac{1}{2}L_e - \frac{1}{2} + \frac{2}{\epsilon}\} \pm 2\ln^2 2 \mp 2\ln 2 + \binom{-2}{1}\mathrm{Li}_2(1) + \frac{1}{2}\binom{0}{1}$

35. $\left[\frac{1}{c_{\mp}^4}\mathrm{Li}_2(c_{\mp})\right] = \binom{1}{0}\{\frac{2}{9}L_e + \frac{1}{2\epsilon} + \frac{1}{\epsilon^2}\} \pm \frac{8}{3}\ln^2 2 \mp \frac{16}{9}\ln 2 + 2\binom{-4/3}{1}\mathrm{Li}_2(1) + \frac{5}{9}\binom{-4/3}{1}$

36. $\left[\mathrm{Li}_2(1 - \frac{c_{\mp}}{R})\right] = \binom{2R}{0}\mathrm{Li}_2(1) - 2\binom{0}{R-1}\mathrm{Li}_2(1 - \frac{1}{R}) \mp (2R-1)\mathrm{Li}_2(1 - \frac{1}{2R}) + L_V - 1 \mp \ln 2$

37. $\left[c\,\mathrm{Li}_2(1 - \frac{c_{\mp}}{R})\right] = -2R(R-1)\binom{1}{0}\mathrm{Li}_2(1) - 2R(R-1)\binom{0}{1}\mathrm{Li}_2(1 - \frac{1}{R}) + \frac{1}{2}(1-2R)^2 \times$

$\times\,\mathrm{Li}_2(1 - \frac{1}{2R}) + (\frac{3}{4} - R)\binom{1}{0}L_V + (-\frac{1}{4} + R)\binom{0}{1}L_V \pm R + \frac{1}{8}\binom{-7}{5} + (R - \frac{3}{4})\ln 2$

38. $\left[c^2\mathrm{Li}_2(1 - \frac{c_{\mp}}{R})\right] = 2(R - 2R^2 + \frac{4}{3}R^3)\binom{1}{0}\mathrm{Li}_2(1) + 2(\frac{1}{3} - R + 2R^2 - \frac{4}{3}R^3)\binom{0}{1}\mathrm{Li}_2(1 -$

$-\frac{1}{R}) \mp \frac{8}{3}(R - \frac{1}{2})^3\,\mathrm{Li}_2(1 - \frac{1}{2R}) \mp \frac{1}{6}(\frac{11}{3} - 10R + 8R^2)\ln 2 + \frac{1}{3}(\frac{11}{6} - 5R + 4R^2) \times$

$\times\binom{1}{0}L_V + \frac{1}{3}(\frac{5}{6} - 3R + 4R^2)\binom{0}{1}L_V - \frac{4}{3}R^2 + \frac{R}{2}\binom{11/3}{3} - \frac{1}{108}\binom{85}{55}$

39. $\left[\frac{1}{1 - x c_{\mp}}\right] = \mp\frac{2}{x}\ln(1 - \frac{x}{2}) - \frac{2}{x}\binom{0}{1}\ln(1 - x)$

40. $[\ln(1 - x c_{\mp})] = \pm(1 - \frac{2}{x})\ln(1 - \frac{x}{2}) - 1 + 2(1 - \frac{1}{x})\binom{0}{1}\ln(1 - x)$

41. $[c\ln(1 - x c_{\mp})] = \pm\frac{1}{x} + \frac{2}{x^2}(x-1)\binom{0}{1}\ln(1-x) + \frac{(2-x)^2}{2x^2}\ln(1 - \frac{x}{2}) + \frac{1}{4}\binom{-3}{1}$

42. $[\frac{\ln(1-xc_{\pm})}{1-xc_{\mp}}] = \mp\frac{1}{x}\ln^2(1-\frac{x}{2}) - \frac{1}{x}\binom{0}{1}\ln^2(1-x)$

43. $[\frac{1}{d_{\mp}}] = \frac{2}{x(R-1)}\{\pm\ln d_R - \binom{1}{0}\ln(1-\frac{x}{R}) + \binom{0}{1}\ln(1-x)\}$

44. $[\frac{\ln(d_{\pm}^2/\tau_Z)}{d_{\mp}}] = \pm\frac{2}{x(R-1)}\{\ln(1-x)\ln(1-\frac{x}{R}) + \ln^2 d_R - \ln d_R\cdot L_B\}$

45. $[\ln\frac{d_{\pm}^2}{\tau_Z}] = -2-\ln(1-x)+\ln(1-\frac{x}{R})\pm 2\{\ln(1-\frac{x}{R}) - \ln d_R\} + \frac{4R}{R-1}\cdot\frac{1-x}{x}\times$

$\times\{\binom{1}{0}[\ln(1-\frac{x}{R})-\ln d_R] - \binom{0}{1}[\ln(1-x)-\ln d_R]\}$

46. $[c\ln\frac{d_{\mp}^2}{\tau_Z}] = \frac{1}{2}\binom{-3}{1} \pm \frac{2(R-x)}{x(R-1)} + \ln d_R - \frac{1}{2}L_B +$

$+\frac{4R(1-x)(R-x)}{x^2(R-1)^2}[\ln d_R - \binom{1}{0}\ln(1-\frac{x}{R}) - \binom{0}{1}\ln(1-x)]$

47. $[c^2(\ln\frac{d_-^2}{\tau_Z} - \ln\frac{d_+^2}{\tau_Z})] = \frac{4}{3}\{\ln(1-\frac{x}{R}) - \ln d_R\} + S_1$

48. $[c^2(\ln\frac{d_-^2}{\tau_Z} - \ln\frac{d_+^2}{\tau_Z})\frac{1}{1-x}] = \frac{1}{1-x}\{\frac{4}{3}\ln 2 - \frac{4}{3}\ln(1+R\frac{1-x}{R-x}) + S_1\}$

49. $[c(\ln\frac{d_-^2}{\tau_Z}+\ln\frac{d_+^2}{\tau_Z})\cdot\frac{1}{1-x}] = \frac{1}{1-x}\{-2\ln 2+2\ln(1+R\frac{1-x}{R-x}) + L_f -$

$-L_W - 1 + 4\frac{R(R-x)(1-x)}{x^2(R-1)^2}(2\ln d_R - L_B)\}$

50. $[(\ln\frac{d_-^2}{\tau_Z} - \ln\frac{d_+^2}{\tau_Z})\frac{1}{1-x}] = \frac{4}{1-x}\{\ln 2 - \ln(1+R\frac{1-x}{R-x}) - \frac{R(1-x)}{x(R-x)}(2\ln d_R - L_B)\}$

The additional notation is as follows

$$S_1 = \frac{2}{3} + \frac{4}{3}\cdot\frac{R(1-x)}{x(R-1)} - \frac{2}{3}\{-1+(1+\frac{2R(1-x)}{x(R-1)})^3\}\cdot(2\ln d_R - L_B) \qquad (11)$$

$$L_B = \ln(1-x) + \ln(1-\frac{x}{R})$$

$$d_R = 1 - x\frac{1+R}{2R}$$

$$L_V = \ln\frac{S}{M_V^2}$$

$$L_W = \ln\frac{R(1-x)+\epsilon_f(R-1)}{\epsilon_f(R-x)}.$$

Table III

$$[A] = \int_0^{1-2\epsilon_f\bar{\omega}} dx\, A(x) \qquad (12)$$

All but one integrals are regular in the limit $\bar{\omega}\to 0$, its singularity is connected with the handling of the IR-divergency.

1. $[x^k] = \frac{1}{k+1}$

2. $[\frac{1}{1-x}] = -\ln\frac{2\bar{\omega}}{m_f} + L_f$

3. $[\frac{1}{R-x}] = -L_R$

4. $[\frac{1}{\tau}] = \frac{1}{S}L_f$

5. $[\frac{1}{\tau^2}] = \frac{1}{Sm_f^2}$

6. $[x^k\ln x] = -\frac{1}{(k+1)^2}$

7. $[L_\tau] = L_f - 1$

8. $[xL_\tau] = \frac{1}{2}L_f - \frac{3}{4}$

9. $[\frac{1}{1-x}L_\tau] = \frac{1}{2}L_f^2 + Li_2(1)$

10. $[\frac{1}{R-x}L_r] = -L_f L_R - \frac{1}{2}L_R^2 - Li_2(\frac{1}{R})$

11. $[\frac{1}{1-xc_\mp}L_r] = \frac{-1}{c_\mp}\{L_f\cdot\ln c_\pm + \frac{1}{2}\ln^2 c_\pm + Li_2(c_\mp)\}$

12. $[\ln(1-\frac{x}{R})] = (1-R)L_R - 1$

13. $[x\ln(1-\frac{x}{R})] = \frac{1}{2}(1-R^2)L_R - \frac{1}{4} - \frac{R}{2}$

14. $[x^2\ln(1-\frac{x}{R})] = \frac{1}{3}(1-R^3)L_R - \frac{1}{9} - \frac{R}{6} - \frac{R^2}{3}$

15. $[\frac{1}{x}\ln(1-\frac{x}{R})] = -Li_2(\frac{1}{R})$

16. $[\frac{1}{R-x}\ln(1-\frac{x}{R})] = -\frac{1}{2}L_R^2$

17. $[\frac{1}{d_R}\ln(1-\frac{x}{R})] = \frac{2R}{1+R}\cdot\mathcal{D}$

18. $[\frac{1}{d_\mp}\ln(1-\frac{x}{R})] = \frac{1}{b_\mp}\{\frac{1}{2}(\ln c_\pm + L_R)^2 + \mathcal{F}_\mp\}$

19. $[\frac{1}{R-x}\ln(1-x)] = -Li_2(\frac{1}{R}) - \frac{1}{2}L_R^2$

20. $[\frac{1}{d_R}\ln(1-x)] = \frac{2R}{1+R}Li_2(\frac{1+R}{1-R})$

21. $[\frac{1}{d_\mp}\ln(1-x)] = \frac{1}{b_\mp}\{\frac{1}{2}(\ln c_\pm + L_R)^2 + Li_2(\frac{b_\mp}{R})\}$

22. $[\ln\frac{d_\mp}{c_\pm}] = -1 + \ln(1-R) - \frac{R}{b_\mp}\cdot(\ln c_\pm + L_R)$

23. $[x\ln\frac{d_\mp}{c_\pm}] = \frac{R}{2b_\mp}[\ln\frac{d_\mp}{c_\pm}] + \frac{1}{2}(1-\frac{R}{b_\mp})\ln(1-R) - \frac{1}{4}$

24. $[x^2\ln\frac{d_\mp}{c_\pm}] = \frac{1}{3}\cdot\frac{R^2}{b_\mp^2}[\ln\frac{d_\mp}{c_\pm}] + \frac{1}{3}(1-\frac{R^2}{b_\mp^2})\ln(1-R) - \frac{R}{6b_\mp} - \frac{1}{9}$

25. $[\frac{1}{R-x}\ln\frac{d_\mp}{c_\pm}] = -\{\ln c_\mp - \ln c_\pm + \ln(-R) + \frac{1}{2}L_R\}L_R + \mathcal{K}_\mp$

26. $[\frac{1}{d_\mp}\ln\frac{d_\mp}{c_\pm}] = \frac{1}{2b_\mp}\{L_R^2 - \ln^2 c_\pm + 2(\ln c_\pm + L_R)\ln(-R)\}$

27. $[\frac{1}{d_\mp}] = \frac{1}{b_\mp}(\ln c_\pm + L_R)$

28. $[\frac{1}{1-xc_\mp}] = -\frac{1}{c_\mp}\ln c_\pm$

29. $[\ln\frac{1-xc_\mp}{c_\pm}] = -1 - \frac{1}{c_\mp}\ln c_\pm$

30. $[x\ln\frac{1-xc_\mp}{c_\pm}] = -\frac{1}{4} + \frac{1}{2c_\mp}[\ln\frac{1-xc_\mp}{c_\pm}]$

31. $[\frac{1}{1-xc_\mp}\ln\frac{1-xc_\mp}{c_\pm}] = \frac{1}{2c_\mp}\ln^2 c_\pm$

32. $[\frac{1}{1-x}\ln\frac{1-xc_\mp}{c_\pm}] = Li_2(c_\mp) + \frac{1}{2}\ln^2(c_\pm)$

33. $[\frac{1}{1-x}L_W] = Li_2(1) - Li_2(\frac{1}{R}) + \frac{1}{2}L_f^2 - L_f L_R$

34. $[\frac{1}{1-x}\ln\frac{d_\mp^2 m_f^2}{S r_Z c_\pm^2}] = -Li_2(1) - Li_2(\frac{1}{R}) - \frac{1}{2}L_f^2 + L_f L_R - L_R^2 - 2Li_2[\frac{b_\mp}{c_\pm(1-R)}]$

The new notation implies:

$$\mathcal{D} = Li_2(-\frac{1}{R}) + Li_2(\frac{R+1}{R-1}) - Li_2(1) + \ln\frac{R+1}{R-1}\ln(-\frac{1}{R})$$

$$L_R = \ln(1-\frac{1}{R}) \qquad (13)$$

$$\mathcal{F}_\mp = Li_2(-R\frac{c_\mp}{c_\pm}) - Li_2(c_\mp(1-R)) - \ln b_\mp(\ln c_\pm + L_R)$$

$$\mathcal{K}_\mp = Li_2(-c_\mp\frac{1-R}{b_\mp}) - Li_2(c_\mp\frac{R}{b_\mp}) - L_R\ln\frac{b_\mp}{c_\mp}$$

35. $[\ln d_R] = -1 + \frac{1-R}{1+R}(L_R - \ln 2)$

36. $[x \ln d_R] = -\frac{1}{4} - \frac{R}{1+R} + \frac{(1-R)(1+3R)}{2(1+R)^2}(L_R - \ln 2)$

37. $[\frac{1}{R-x} \ln d_R] = -L_R(L_R - \ln 2) - \mathfrak{D}$

38. $[\ln^2 d_R] = \frac{1-R}{1+R} \cdot (L_R - \ln 2)^2 - 2 \cdot [\ln d_R]$

39. $[x \ln^2 d_R] = \frac{(1-R)(1+3R)}{2(1+R)^2} \cdot (L_R - \ln 2)^2 - [x \ln d_R] - \frac{2R}{1+R}[\ln d_R]$

40. $[\frac{1}{1-x} \ln(1 + R\frac{1-x}{R-x})] = -Li_2(\frac{1}{R}) - \frac{1}{2}L_R^2 - Li_2(\frac{1+R}{1-R})$

41. $[\ln(1-\frac{x}{2})] = -1 + \ln 2$

42. $[x \ln(1-\frac{x}{2})] = -\frac{5}{4} + \frac{3}{2}\ln 2$

43. $[\frac{1}{1-x}\ln(1-\frac{x}{2})] = \frac{1}{2}Li_2(1) - [\frac{1}{1-x}]\ln 2$

44. $[\ln^2(1-\frac{x}{2})] = 2 - 2\ln 2 - \ln^2 2$

45. $[\ln(1-x)\ln(1-\frac{x}{2})] = 2 - \ln 2 - \frac{1}{2}Li_2(1)$

46. $[\ln(1-x)\ln(1-\frac{x}{R})] = Li_2(\frac{1}{R}) + \frac{1}{2}L_R^2 - [\ln(1-\frac{x}{R})] + 1 + R[\frac{\ln(1-x)}{R-x}]$

47. $[x\ln(1-x)\ln(1-\frac{x}{R})] = \frac{1}{2}\{Li_2(\frac{1}{R}) + \frac{1}{2}L_R^2\} - \frac{1}{2}[x\ln(1-\frac{x}{R})] - \frac{1}{2}[\ln(1-$

$-\frac{x}{R})] + \frac{R^2}{2}[\frac{\ln(1-x)}{R-x}] + \frac{R}{2} + \frac{3}{8}$

48. $[\ln(1-x)\ln d_R] = -Li_2(\frac{1+R}{1-R}) + 1 - [\ln d_R] + [\frac{1}{d_R}\ln(1-x)]$

49. $[x\ln(1-x)\ln d_R] = -\frac{1}{2}Li_2(\frac{1+R}{1-R}) - \frac{1}{2}[x\ln d_R] - \frac{1}{2}[\ln d_R] +$

$+ \frac{R}{1+R}[\frac{1}{d_R}\ln(1-x)] + \frac{3}{8} + \frac{R}{1+R}$

50. $[\ln(1-\frac{x}{R})\ln d_R] = (L_R - \ln 2)L_R - [\ln(1-\frac{x}{R})] - [\ln d_R] + R[\frac{\ln d_R}{R-x}] + [\frac{\ln(1-\frac{x}{R})}{d_R}]$

51. $[x\ln(1-\frac{x}{R})\ln d_R] = \frac{1}{2}(L_R - \ln 2)L_R - \frac{1}{2}[x\ln d_R] - \frac{R}{2}[\ln d_R] + \frac{R^2}{2} \times$

$\times [\frac{\ln d_R}{R-x}] - \frac{1}{2}[x\ln(1-\frac{x}{R})] - \frac{R}{1+R}[\ln(1-\frac{x}{R})] + \frac{R}{1+R}[\frac{1}{d_R}\ln(1-\frac{x}{R})]$

APPENDIX

Many of the integrals of the Tables are trivial, a lot of them may be got from integral tables of general use. Particularly useful are tables[10]. Some of the integrals require definite efforts to be calculated. In the Appendix we list some auxiliary formulae often used.

$$\int_0^1 dx \ln(1-ax)\ln(1-bx) = -\int_0^1 dx \ln(1-bx) - \int_0^1 dx \ln(1-ax) + \int_0^1 dx \frac{\ln(1-bx)}{1-ax} + R_0(a,b),$$

$$2\int_0^1 dx\, x\ln(1-ax)\ln(1-bx) = -\int_0^1 dx(x+\frac{1}{a})\ln(1-bx) - \int_0^1 dx(x+\frac{1}{b}) \times \ln(1-ax) + \frac{1}{a}\int_0^1 dx \frac{\ln(1-bx)}{1-ax} + \frac{1}{b}\int_0^1 dx \frac{\ln(1-ax)}{1-bx} + R_1(a,b), \tag{A.1}$$

$$R_i(a,b) = \ln(1-a)\ln(1-b) + \frac{1}{b^i}\int_0^1 dx \frac{\ln(1-ax)}{1-bx},$$

$$R_i(a,1) = -Li_2(\frac{a}{a-1}).$$

$$\int_0^1 \frac{\ln x\, dx}{x+C} = Li_2(-\frac{1}{C}) \quad \text{where C is complex.} \tag{A.2}$$

The Euler dilogarithm,

$$Li_2(x) = -\int_0^1 \frac{dt}{t} \ln(1 - xt), \tag{A.3}$$

has the following Taylor expansions:

$$Li_2(1-\epsilon) = Li_2(1) + \sum_{k=1}^{\infty} \frac{\epsilon^k}{k}\left(-\frac{1}{k} + \ln\epsilon\right), \quad 0 < \epsilon < 1, \tag{A.4}$$

$$Li_2(x+\epsilon) = \sum_{k=0}^{\infty} \left(\frac{\epsilon}{x}\right)^k \cdot \phi_k, \quad |x| < 1,\ |x+\epsilon| \le 1, \tag{A.5}$$

$$\phi_k = \frac{1}{k}\left\{(-)^k \ln(1-x) + \sum_{\ell=0}^{k-2} (-)^\ell \binom{k-1}{\ell} \cdot \frac{1}{k-\ell-1} \cdot \left[-1 + \frac{1}{(1-x)^{k-\ell-1}}\right]\right\}.$$

The first terms of the expansion (A.5) are

$$\phi_0 = Li_2(x),$$

$$\phi_1 = -\ln(1-x),$$

$$\phi_2 = \frac{1}{2}\left\{\ln(1-x) + \frac{x}{1-x}\right\}, \tag{A.6}$$

$$\phi_3 = \frac{1}{3}\left\{-\ln(1-x) + \frac{x(3x-2)}{2(1-x)^2}\right\}.$$

On the unit circle the real part of the Euler dilogarithm is a simple expression:

$$\mathrm{Re}\,Li_2(e^{i\phi}) = -\frac{1}{2} Li_2(1) + \frac{1}{4}(\pi - |\phi|)^2. \tag{A.7}$$

The following general integrals have been quite useful:

$$\int_a^b dx \frac{Li_2(x)}{x^2} = \frac{1}{a} Li_2(a) - \frac{1}{b} Li_2(b) + \ln b - \ln a + \frac{1-b}{b}\ln(1-b) - \frac{1-a}{a}\ln(1-a),$$

$$\int_0^1 dx\, Li_2(a+bx) = \frac{a+b}{b} Li_2(a+b) - \frac{a}{b} Li_2(a) + \frac{a+b-1}{b}\ln(1-a-b) + $$

$$+ \frac{1-a}{b}\ln(1-a) - 1, \tag{A.8}$$

$$\int_0^1 dx\, x\, Li_2(a+bx) = \frac{b^2-a^2}{2b^2} Li_2(a+b) + \frac{a^2}{2b^2} Li_2(a) + (3a-b-1)\frac{1-a-b}{4b^2}\ln(1-$$

$$-a-b) - (3a-1)\frac{1-a}{4b^2}\ln(1-a) + \frac{1}{8b}(6a-b-2),$$

$$\int_0^1 dx\, x^2 Li_2(a+bx) = \frac{b^3+a^3}{3b^3} Li_2(a+b) - \frac{a^3}{3b^3} Li_2(a) + (-2+7a-11a^2-2b-5ab-$$

$$-2b^2)\times\frac{1-a-b}{18b^3}\ln(1-a-b) - (-2+7a-11a^2)\frac{1-a}{18b^3}\ln(1-a) +$$

$$+ \frac{1}{108b^2}(-4b^2 + 15ab - 66a^2 - 6b + 42a - 12).$$

At the end of this Appendix we would like to correct some misprints of [1]; the corresponding equations should read:

1. $t' = (k_1 - p_1)^2$ – definition of t', p.1.
2. $[\frac{1}{Z}] = [\overline{\frac{1}{Z}}]$ – first line, p.4.
3. $[\frac{1}{Z X_M}] = \frac{1}{S\, X\sqrt{\lambda_t}} L_t$ – eighth line, p.4.
4. $\lambda_{X_t} = X_t^2 - 4m^2\mu^2$ – last line, p.5.
5. $d = \mu^2 S^2 - m^2 X^2$ – definition of d, p.7.
6. $\lambda_S^F = \lambda(S, \mu^2, \mu^2) = S^2 - 4\mu^2 S$ – definition of λ_S^F, p.8.

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Интегралы, встречающиеся при вычислении тормозного излучения в процессах с обменом фотоном и Z^0-бозоном в ультрарелятивистском приближении

Представлен исчерпывающий набор интегралов, которые совместно с системой аналитических вычислений SCHOONSCHIP были использованы для получения интегрированных по фотонным степеням свободы наблюдаемых для процессов $e^+e^- \to \gamma, Z^0 \to \bar{f}f(\gamma)$, $V \to \bar{f}_1 f_2(\gamma)$, $V = W^{\pm}, Z^0$. При вычислениях используется ультрарелятивистское приближение по массам фермионов ($m_f^2 \ll s$). Приводимые таблицы представляют достаточно общий интерес для вычисления вклада тормозного излучения в КЭД, КХД и теории электрослабых взаимодействий.

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Some Integrals for Analytic Bremsstrahlung Calculation. Photon and Z^0-Boson Exchange in the Ultrarelativistic Limit

An exhaustive list of integrals is presented which has been used together with the system of analytic calculations SCHOONSCHIP to obtain observables inclusive in the photon degrees of freedom for the processes $e^+e^- \to \gamma, Z^0 \to \bar{f}f(\gamma)$, $V \to \bar{f}_1 f_2(\gamma)$, $V = W^{\pm}, Z^0$. The extreme relativistic approximation in fermion masses is used. The Tables presented are of general interest for calculations of bremsstrahling contributions in QED, QCD and electroweak processes.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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