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THE ELECTROMAGNETIC α^3 CONTRIBUTIONS TO e^+e^--ANNIHILATION INTO FERMIONS IN THE ELECTROWEAK THEORY.

Total Cross Section σ_T and Integrated Asymmetry A_{FB}

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1. Introduction

This article contains some new analytic results on hard photon bremsstrahlung in the reaction

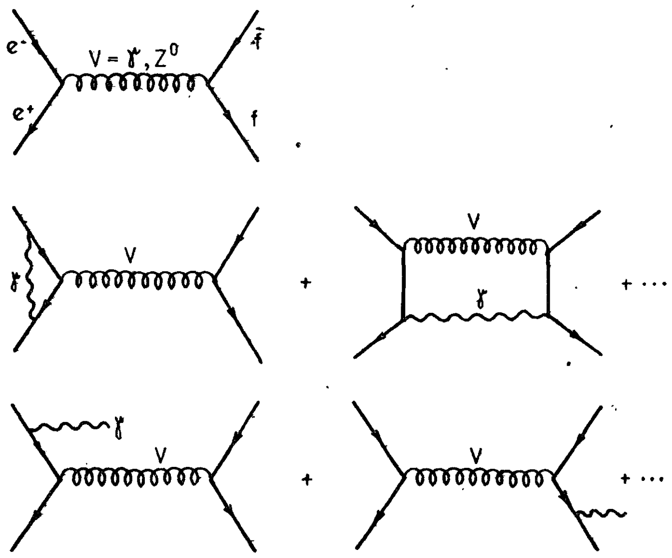
$$e^+e^- \to (\gamma, Z^0) \to \bar{f}f(\gamma) \qquad (1)$$

for ultra-relativistic energies including the resonance region $s \sim M_Z^2$. These QED radiative corrections are less interesting than the genuine electroweak one-loop insertions but numerically quite large so that a careful study of them is necessary for precision experiments as being planned at the new e^+e^--colliders TRISTAN, SLC, LEP. To obtain analytic results for hard bremsstrahlung in the presence of a heavy neutral vector boson Z^0 interfering with the photon as is predicted by the standard electroweak theory[1] has been proved technically involved[2,3,4,5], much more involved than in pure QED[6,7]. This fact and unavoidable inclusion of realistic experimental cuts promoted the development of very effective Monte-Carlo algorithms; here we only mention the classic results described in[7,8] (see also[9,10]).

With extensive use of the system of analytical calculations on computers SCHOONSCHIP[11] and of phase space integration methods developed at Dubna[12-15] we have determined the total cross section σ_T and the integrated forward-backward asymmetry A_{FB} for reaction (1)*). We assumed the photon registration conditions to be totally inclusive and used extreme relativistic approximation for fermion masses $m_f^2 \ll s$. In parallel, the angular distribution (differential cross section $d\sigma/d\cos\theta$) of the fermion has been obtained. Some preliminary results may be found in[14,15]. Concerning the technique used to carry through the three- and fourfold integrations, we only remark that it has been quite important to tackle the phase space integrals successively as complex functions of the

*) Including control calculations, the SCHOONSCHIP programmes developed consist of about 5000 commands. They are available at Dubna.

parameter $M^2 = M_Z^2 - iM_Z\Gamma_Z$, where Γ_Z is the total width of the Z^0 -boson[16]. There was no additional difficulty to avoid the assumption of a small width Γ_Z once we decided to assume the fermion masses to be small: m_e, $m_f \ll M_Z$, E; $E = \sqrt{s}/2$ is the beam energy. The underlying method is general enough to be useful for other reactions, too, as long as one does not apply complicated cuts. The results presented here may be generalized straightforwardly to the case of more than one heavy neutral boson or to similar reactions, e.g. $e^+e^- \to e^+e^-(\gamma)$. In the following, we present the contributions from the diagrams of the Figure to σ_T and A_{FB}. These diagrams, together with the photon self-energy diagram, not discussed here, are the QED radiative correction to the reaction (1).



The QED α^3 radiative corrections to the e^+e^- annihilation into a fermion pair considered in this article.

2. Definitions

We have calculated for the process (1) the differential with respect to $c = \cos\theta$ cross section $d\sigma/dc$, the total cross section σ_T and the integrated forward-backward asymmetry A_{FB} up to α^3 terms:

$$\sigma_T = \int_{-1}^{+1} dc \, \frac{d\sigma}{dc}, \tag{2}$$

$$A_{FB} = \sigma_T^{-1}\left[\int_0^1 dc\,\frac{d\sigma}{dc} - \int_{-1}^0 dc\,\frac{d\sigma}{dc}\right]. \tag{3}$$

The angle θ is defined as the cms scattering angle of $\bar{f}$ with respect to e^+. The following parametrisations are used:

$$\begin{aligned}
\frac{d\sigma}{dc} = \frac{\pi\alpha^2}{2s}\Big\{ & Q_f^2\left[1+c^2+\frac{\alpha}{\pi}\left(F_0+Q_fF_1+Q_f^2F_2\right)\right]+ \\
& +2|Q_f|v_ev_f\,\mathrm{Re}\left[\chi(1+c^2)+\frac{\alpha}{\pi}\chi\left(G_0+Q_fG_1+Q_f^2G_2\right)\right]+ \\
& +2|Q_f|a_ea_f\,\mathrm{Re}\left[\chi\,2c+\frac{\alpha}{\pi}\chi\left(G_3+Q_fG_4+Q_f^2G_5\right)\right]+ \\
& +(v_e^2+a_e^2)(v_f^2+a_f^2)|\chi|^2\left[1+c^2+\frac{\alpha}{\pi}\mathrm{Re}\left(H_0+Q_fH_1+Q_f^2H_2\right)\right]+ \\
& +4v_ea_ev_fa_f|\chi|^2\left[2c+\frac{\alpha}{\pi}\mathrm{Re}\left(H_3+Q_fH_4+Q_f^2H_5\right)\right]\Big\},
\end{aligned} \tag{4}$$

$$\begin{aligned}
\sigma_T = \sigma_0\Big\{ & Q_f^2\left[1+\frac{\alpha}{\pi}\left(F_0^T+Q_f^2F_2^T\right)\right]+2|Q_f|v_ev_f\,\mathrm{Re}\left[\chi+\frac{\alpha}{\pi}\chi\left(G_0^T+Q_f^2G_2^T\right)\right]+ \\
& +2|Q_f|a_ea_f\frac{\alpha}{\pi}Q_f\,\mathrm{Re}\left(\chi G_4^T\right)+ \\
& +(v_e^2+a_e^2)(v_f^2+a_f^2)|\chi|^2\left[1+\frac{\alpha}{\pi}\mathrm{Re}\left(H_0^T+Q_f^2H_2^T\right)\right]+4v_ea_ev_fa_f|\chi|^2\frac{\alpha}{\pi}Q_f\,\mathrm{Re}\left(H_4^T\right)\Big\},
\end{aligned} \tag{5}$$

$$A_{FB}=\frac{\sigma_0}{\sigma_T}\left\{\frac{\alpha}{\pi}Q_f^3F_1^T+2|Q_f|v_e v_f\frac{\alpha}{\pi}Q_f\,Re(\chi G_1^T)+\right.$$

$$+2|Q_f|a_e a_f\,Re\left[\frac{3}{4}\chi+\frac{\alpha}{\pi}\chi\left(G_3^T+Q_f^2G_5^T\right)\right]+$$

$$+(v_e^2+a_e^2)(v_f^2+a_f^2)|\chi|^2\frac{\alpha}{\pi}Q_f\,Re\left(H_1^T\right)+$$

$$\left.+4v_e a_e v_f a_f|\chi|^2\left[\frac{3}{4}+\frac{\alpha}{\pi}Re\left(H_3^T+Q_f^2H_5^T\right)\right]\right\}. \qquad (6)$$

The QED radiative corrections are contained in the functions F, G, H. These functions depend only on particle masses, the Z^0 -width and on the beam energy. In Eqs. (4)-(6) Q_f is the charge of the produced light fermion, $Q_\mu=-1$. The v, a are arbitrary vector and axial vector couplings to the massive neutral vector boson which in the standard electroweak theory become

$$a_f=1,$$
$$v_f=1-4s_w^2|Q_f|. \qquad (7)$$

The quantity χ may be obtained from two factors of different origin. The real constant k measures the relative strength of the photon and weak neutral boson couplings, in the standard theory:

$$k=\frac{g^2}{16c_w^2e^2}, \qquad (8)$$

where we use the on-mass-shell renormalization scheme:

$$c_w^2=1-s_w^2=M_W^2/M_Z^2, \qquad (9)$$

$$g=e/s_w. \qquad (10)$$

The complex kinematic variable $\varkappa$ relates the corresponding propagators:

$$\varkappa=\frac{s}{s-M^2}, \qquad (11)$$

$$M^2=M_Z^2-iM_Z\Gamma_Z, \qquad (12)$$

$$s=4E^2, \qquad (13)$$

where M_Z, M_W are the weak neutral and charged gauge boson masses.

There are two common definitions of χ:

$$\chi_I=K\varkappa\,(1-\delta r)^{-1}=\frac{G_\mu}{\sqrt{2}}\,\frac{M_Z^2}{8\pi\alpha}\,\frac{s}{s-M^2}, \qquad (14)$$

$$\chi_{II}=K\varkappa\qquad=\frac{1}{16s_w^2c_w^2}\,\frac{s}{s-M^2}. \qquad (15)$$

Here $\delta r=(\alpha/4\pi)X$ is the radiative correction to the muon decay constant G_μ /17,18/. Following the recomendations of the study group of electroweak radiative corrections at LEP/10/, we will understand the parameter χ in (4-6) as defined in (14) with

$$K=K_I=0.38894\,(M_Z/93)^2. \qquad (16)$$

The point-like QED cross section σ_0 is

$$\sigma_0=\frac{4\pi\alpha^2}{3s}. \qquad (17)$$

The QED-correction presented here may be used also for the integrated left-right asymmetry,

$$A_{LR}=\sigma_T^{-1}\left[\int dc\,\frac{d\sigma(\lambda)}{dc}-\int dc\,\frac{d\sigma(-\lambda)}{dc}\right], \qquad (18)$$

where λ is a longitudinal polarization of the electron and the angular integration corresponds either to (2) or (3). Here, the electron couplings in (4-6) must be changed as follows:

$$v_e v_f \longrightarrow (v_e - \lambda a_e) v_f,$$
$$a_e a_f \longrightarrow (a_e - \lambda v_e) a_f,$$
$$(v_e^2 + a_e^2)(v_f^2 + a_f^2) \longrightarrow (v_e^2 + a_e^2 - 2\lambda v_e a_e)(v_f^2 + a_f^2),$$
$$4 v_e a_e v_f a_f \longrightarrow 2\left[2 v_e a_e - \lambda (v_e^2 + a_e^2)\right] v_f a_f. \qquad (19)$$

In the same way the polarization of a created fermion may be taken into account. This simple procedure of inclusion of the longitudinal polarization is no longer true in the presence of genuine weak loop corrections because they destroy this factorization property of the coupling, valid here. But even for weak corrections we obtained some substitutions which are only slightly more complicated than (19), see /19/. In this first part of the article we present only complete expressions for the totally integrated quantities (5), (6) together with relevant numerical results. The expressions for the differential spectrum (4) and further numerical results, including those for A_{LR} and A_{pol}, will be presented in subsequent publications.

3. The QED radiative corrections

The radiative corrections F^T, G^T, H^T may formally be obtained from the corresponding functions in the angular distributions as follows:

$$\left\{F_i^T, G_i^T, H_i^T\right\} = \frac{3}{4}\int_0^1 dc \left\{F_i, G_i, H_i\right\}, \qquad (20)$$

where for the initial state radiation (i=0,3) the correct upper bound is $1 - 2m_e^2/s$ with m_e being the electron mass. Without going into details, here we present the final results in the following form ($R^T = F^T, G^T, H^T$):

$$R_i^T = R_{i,v}^T + R_{i,b}^T \quad (i=0,2,3,5), \quad R_i^T = R_{i,B}^T + R_{i,b}^T \quad (i=1,4). \qquad (21)$$

The R_V (R_B) are contributions from vertex (box) diagrams, and R_b is the complete bremsstrahlung, i.e. the sum of soft and hard

bremsstrahlung contributions. The R_b does not depend on any cut-off parameter since the photons have been taken into account totally inclusive.

3.1. The C-even QED radiative corrections:

$$F_0^T = A + B\left(L_f - \frac{7}{6}\right), \qquad (22)$$

$$G_0^T = A + B\left[R + \frac{1}{2} + (1+R^2)L_R\right], \qquad (23)$$

$$H_0^T = A + B\left[2R + \frac{1}{2} - |R|^2 + \frac{i}{g}(1-R^*)R(1+R^2)L_R\right], \qquad (24)$$

with
$$A = \frac{\pi^2}{3} - \frac{1}{2}, \quad B = L_e - 1;$$

$$F_2^T = G_2^T = H_2^T = 3/4. \qquad (25)$$

It is quite simple to single out, from (22-25), the corresponding vertex and bremsstrahlung terms /7,12,20/. For the interference functions we quote the box and bremsstrahlung terms separately:

$$\begin{pmatrix} G_4^T \\ H_4^T \end{pmatrix}_b = 3\left\{2\bar{P}_{IR} - 3 + \begin{pmatrix} 1/2 \\ 1 \end{pmatrix} R\left[1 + (1+R)L_R\right]\right\}, \qquad (26)$$

$$\begin{pmatrix} G_4^T \\ H_4^T \end{pmatrix}_B = 3\left\{-2\bar{P}_{IR} + 3\begin{pmatrix} 3/4 \\ 1 \end{pmatrix} - \begin{pmatrix} 1/2 \\ 1 \end{pmatrix}\left(R\left[1 + (1+R)\right] + L_Z\right)\right\}. \qquad (27)$$

In sum, (26, 27) give the remarkably simple expressions for the contributions to the total cross section σ_T from the interference of initial and final state radiations which have no analogue in QED:

$$\begin{pmatrix} G_4^T \\ \frac{1}{2} H_4^T \end{pmatrix} = -\frac{9}{4}\begin{pmatrix} 1 \\ 0 \end{pmatrix} - \frac{3}{2}L_Z. \qquad (28)$$

In eqs. (22)-(28) and below the following abbreviations are used:

$$L_a = \ln\frac{s}{m_a^2}, \quad a = e, f, \qquad (29)$$

$$L_Z = \ln\frac{s}{M^2}, \qquad (30)$$

$$L_R \doteq \ln\left(1-\frac{1}{R}\right), \tag{31}$$

$$R=\frac{M^2}{S}\ ,\quad R^*=\frac{M_Z^2+iM_Z\Gamma_Z}{S}\ , \tag{32}$$

$$g=\frac{\Gamma_Z M_Z}{S}\ , \tag{33}$$

$$\bar{P}_{IR}=P_{IR}+\frac{1}{2}\ln\frac{S}{M_W^2}=\ln\frac{2E}{\lambda}\ , \tag{34}$$

$$P_{IR}=\frac{1}{n-4}+\frac{1}{2}\gamma_E-\ln 2\sqrt{\pi}=\ln\frac{M_W}{\lambda}\ , \tag{35}$$

where λ is a small regularizing photon mass used by some authors.

The expressions F_0^T, F_2^T are known from /6/. The three final state functions are trivial. The G_0^T, H_0^T may be read off from the program MUSTRAAL /8/. All the initial state radiation functions show the well-known from QED mass singularity due to the electron mass. The singularity in F_0^T from the photon propagator kinematics due to the final fermion mass has been naturally replaced in the interference (G) and weak (H) functions by logarithms of the Z^0-boson mass. The function H_0^T gives rise at and beyond the resonance pole the radiative tail as may be seen from (24) - the logarithm L_R has a large imaginary part for $S>M_Z^2$ that is closely connected to the usually defined phase shift δ_Z /7/ of the resonance*):

$$L_R=\ln(R-1)-\ln R\ ,$$
$$\delta_Z=-\arg[\ln(R-1)]\ . \tag{36}$$

This, together with the factor t/Γ_Z in (24) produces the radiative tail.

*) The complex logarithm has a cut along the negative real axis.

Despite their simple structure, the interference functions G_4^T, H_4^T of (28) seem to be so far lacking in the literature as are the C-odd contributions to A_{FB}.

3.2. The C-odd radiative corrections :

$$F_1^T=-\frac{3}{4}Li_2(1)+\frac{3}{2}-\frac{15}{2}\ln 2+\frac{3}{4}\ln^2 2=-4.572\ , \tag{37}$$

$$\begin{aligned}\begin{pmatrix}G_1\\ \frac{1}{2}H_1\end{pmatrix}_B^T=&\frac{1}{2}\begin{pmatrix}2\\1\end{pmatrix}(1+8\ln 2)\bar{P}_{IR}-\frac{1}{8}\left[5R-\begin{pmatrix}6\\3\end{pmatrix}\right]+\\ &+\frac{1}{4}(1-R)\left[5-R+5R^2\right]Li_2\left(\frac{1}{R}\right)-\frac{3}{8}\left[\begin{pmatrix}2\\1\end{pmatrix}+R-2R^2\right]Li_2(1)+\\ &+\frac{1}{2(1+R)}\left[-12\begin{pmatrix}2\\1\end{pmatrix}-3\begin{pmatrix}5\\1\end{pmatrix}R+8R^2+5R^3\right]\ln 2\div\\ &-\frac{R}{2(1+R)}\left(1-4R+R^2\right)L_R+\frac{1}{4}R\left(6-3R+5R^2\right)D_2+\\ &+\frac{1}{4}\left(5-3R+6R^2\right)D_1\ ,\end{aligned} \tag{38}$$

$$\begin{aligned}\begin{pmatrix}G_1\\ \frac{1}{2}H_1\end{pmatrix}_B^T=&-\frac{1}{2}\begin{pmatrix}2\\1\end{pmatrix}(1+8\ln 2)\bar{P}_{IR}+\frac{3}{8}\begin{pmatrix}3\\2\end{pmatrix}-\frac{R}{2}+\frac{1}{8}\begin{pmatrix}11\\8\end{pmatrix}\ln^2 2+\\ &+2\left[\frac{9}{8}\begin{pmatrix}3\\2\end{pmatrix}-R(1+R)\right]\ln 2+\left(R-\frac{5}{4}\right)L_Z+\\ &+\frac{1}{2}\left[2-\frac{9}{2}R+\frac{3}{2}R^2+\left(-5+3R-6R^2\right)\ln 2\right]L_R+\\ &+\frac{1}{2}\left(1-3R+6R^2-8R^3\right)\left[Li_2\left(1-\frac{1}{2R}\right)-Li_2\left(1-\frac{1}{R}\right)\right]+\\ &+2R^3\left[Li_2(1)-Li_2\left(1-\frac{1}{R}\right)\right],\end{aligned} \tag{39}$$

$$\frac{1}{3}G_3^T = -\frac{1}{8} + \frac{1-R}{1+R}(1-\ln 2) + (1-R)\ln^2 2 + \frac{1}{4}(1+2R)Li_2(1) +$$

$$+\frac{1}{2}\frac{1-R}{1+R}\left[-\frac{1+3R}{1+R}\ln 2 + \frac{7-R}{4(1-R)}\right](L_e-1) + R\frac{1+R^2}{(1+R)^2}D_3 , \qquad (40)$$

$$\frac{1}{3}H_3^T = -\frac{2R}{|1+R|^2} - \left|\frac{1-R}{1+R}\right|^2\ln 2 + |1-R|^2\ln^2 2 + \frac{1}{4}(1+4R-2|R|^2)Li_2(1) +$$

$$+\left(\frac{7}{8} - 2\frac{R}{|1+R|^2}\right)L_e + \left(-\frac{5}{2} + 2\frac{6R-1}{|1+R|^2} + 4\frac{1-R^2}{|1+R|^4}\right)(L_e-1)\ln 2 +$$

$$+ R^2\frac{1+R^2}{(1+R)^2}\left[2+\frac{i}{g}(1-R)\right]D_3 , \qquad (41)$$

$$G_5^T = H_5^T = 0. \qquad (42)$$

Here

$$Li_2(z) = -\int_0^1 \frac{dt}{t}\ln(1-zt) , \qquad (43)$$

$$D_{\binom{0}{1}} = Li_2\left(\mp\frac{1+R}{1-R}\right) + \frac{1}{2}L_R^2 , \qquad (44)$$

$$D_2 = D_0 + Li_2\left(-\frac{1}{R}\right) - Li_2(1) + \ln\left(\frac{R+1}{R-1}\right)\ln\left(-\frac{1}{R}\right), \qquad (45)$$

$$D_3 = D_1 + D_2 + \ln^2 2 + (L_e - 1 - 2\ln 2)L_R . \qquad (46)$$

The integrated QED -asymmetry F_1^T may be found together with some technical details in /20/. The other C-odd corrections are much more involved than those to the total cross section σ_T . Additionally to the products of two logarithms occurring there, here one meets several nontrivial combinations of the complex Euler Dilogarithm function (41). In analogue to the radiative tail of σ_T

arising from H_0^T it is developed here by the initial state radiation function H_3^T (39), where the complex function D_3 together with the imaginary factor proportional to $1/\Gamma_Z$ gives rise to an additional contribution beyond the resonance, again essentially due to L_R .

That G_5 and H_5 are vanishing may be most easily seen from the impossibility to add an axial vector term to the photon or Z -boson self-energies whose imaginary parts the G_5 and H_5 are related with, as are the vector-like terms F_2, G_2, H_2.

4. Numerical results

Instead of a summary, we would like to show some numerical results which have been obtained, with the Fortran code BREMU. This program has been developed along the lines described here and calculates, besides σ_T and A_{FB} , the angular distribution. The corresponding numerical integration of the latter has to coincide with σ_T or A_{FB}, a powerful internal consistency check.

In Tables 1 and 2 the individual contributions to the total cross section and to the integrated forward-backward asymmetry are shown. Instead of the complete complex box contributions we included in the Tables their resonance parts (App.A). This makes our results directly comparable to those obtained with the Monte--Carlo program MUSTAAL /8/ which are shown, too. For this purpose one has, of course, to run the MC-program without photon cuts. Further, we redefined the Born expressions in MUSTRAAL corresponding to our eqs. (4-6, 16). As input parameters for this model calculations we used

$$M_Z = 93\,GeV,\ \Gamma_Z = 2.5\,GeV,\ s_W^2 = 0.23. \qquad (47)$$

As may be seen from the Tables, in σ_T the most important contributions at large energies besides the Born cross section are from initial state radiation, especially from the Z^0 -exchange. In A_{FB}, the interference terms dominate both in the Born approximation (I) and in the bremsstrahlung (G_3 , the initial state photon-interference).

To the MC-results in the Tables we have assigned the statistical errors as follows:

$$\Delta\sigma_T = \sigma_T \frac{1}{\sqrt{N_{ev}}} , \quad (48)$$

$$\Delta A_{FB} = 2 \frac{\Delta\sigma_T}{\sigma_T} \sqrt{1+A_{FB}^2} , \quad (49)$$

where N_{ev} is the number of events generated by the MC-program. For N_{ev} = 750 000, the Tables show both for σ_T and A_{FB} a good coincidence of the MC-generated quantities with our analytic results within 1.2 standard deviation.

Table 1. Individual contributions to σ_T as defined in (5) in units of σ_0 (17) as functions of $\sqrt{s} = 2E$. The A, I, Z are the corresponding Born values due to photon exchange (A), Z-boson exchange) (Z) and their interference (I). Box diagram contributions are taken from the appendix. The parameters are M_Z = 93 GeV, Γ_Z = 2.5 GeV, S_W^2 = 0.23.

$\sqrt{s}$, GeV	60	82	92.5	93.0	93.5	100
σ_0, nb	0.02413	0.01292	0.01015	0.01004	0.00994	0.00869
A	1	1	1	1	1	1
I	-0.00354	-0.01714	-0.06305	0	0.06467	0.03579
Z	0.07772	1.84241	179.00599	212.01602	186.59627	8.15261
F_0	0.60431	0.65436	0.67415	0.67504	0.67593	0.68710
F_2	0.00174	0.00174	0.00174	0.00174	0.00174	0.00174
G_0	0.00011	0.00188	0.00039	-0.02916	-0.05094	-0.00530
G_2	-0.00001	-0.00003	-0.00011	0	0.00011	0.00006
G_4	-0.00830	-0.02703	-0.08279	-0.03223	0.02162	0.01445
H_0	-0.00824	-0.36007	-63.06491	-66.01508	-41.76859	10.45730
H_2	0.00014	0.00321	0.31185	0.36936	0.32507	0.01420
H_4	0.00004	0.00045	0.00660	0.00458	0.00124	-0.00077
σ_T/σ_0	1.66397	3.09978	117.78985	147.99026	146.86712	20.35718
MC N_{ev}= 750000	1.665 $\pm$0.002	3.102 $\pm$0.004	117.899 $\pm$0.136	148.139 $\pm$0.171	147.006 $\pm$0.170	20.360 $\pm$0.024

Table 2. Individual contributions to A_{FB} as defined in (6). specifications are those of Table 1.

$\sqrt{s}$, GeV	60	82	92.5	93.0	93.5	100
σ_c, nb	0.02413	0.01292	0.01015	0.01004	0.00994	0.00869
A	0	0	0	0	0	0
I	-24.84049	-64.48871	-6.27289	0	5.16006	20.58367
Z	0.08817	1.12140	2.88083	2.71579	2.40830	0.75849
F_1	0.63553	0.34098	0.00902	0.00718	0.00723	0.05212
G_1	-0.00332	-0.00562	-0.00011	0.00018	0.00038	0.00052
G_3	1.07974	7.41222	0.05347	-2.29709	-4.05205	-2.94119
H_1	0.09744	0.61216	0.23961	0.13301	0.03737	-0.15693
H_3	-0.00959	-0.22010	-1.01508	-0.84575	-0.53926	0.96719
A_{FB},%	-22.95251	-55.22766	-4.10515	-0.28668	3.02203	19.26388
MC N_{ev}= 750000	-22.973 $\pm$0.237	-55.399 $\pm$0.264	-3.822 $\pm$0.231	-0.156 $\pm$0.231	2.774 $\pm$0.231	19.408 $\pm$ 0.235

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Appendix

Some groups take their Monte-Carlo results for hard photon bremsstrahlung together with approximate expressions for the γZ -box diagrams, the so-called resonance part/7,8/. To get one-to-one correspondence, one has to replace the box formulae of (27), (39) by the following expressions which are integrals of their corresponding distributions:

$$g_{1,B}^{T} = \frac{1}{2}\left(F_{1,B}^{T} + h_{1,B}^{T}\right) + \frac{i\pi}{4}\left(2 - 5\ln 2\right), \qquad (A.1)$$

$$g_{4,B}^{T} = -3\,\overline{P}_{IR} + \frac{9}{4} + \frac{1}{2}h_{4,B}^{T}, \qquad (A.2)$$

$$F_{1,B}^{T} = \frac{3}{4}\left(1 + 6\ln 2 + \ln^2 2\right), \qquad (A.3)$$

$$h_{1,B}^{T} = -(1+8\ln 2)\left[\overline{P}_{IR} + \ln(R-1) + \frac{1}{2}L_Z\right] + \frac{1}{2} + (1+4\ln 2)\ln 2 + 2\,Li_2(1), \qquad (A.4)$$

$$h_{4,B}^{T} = -6\left[\overline{P}_{IR} + \ln(R-1) + \frac{1}{2}L_Z - 1\right]. \qquad (A.5)$$

Concerning these expressions, one should note that compared to the above definition of g_1 and g_4, in /7/ the small contribution of $\arg(L_Z)$ has been neglected; in their notation

$$\delta_Z = -\arg\left[\ln(R-1)\right]. \qquad (A.6)$$

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Бардин Д.Ю., Федоренко О.М., Риман Т. E2-87-663
Электромагнитные поправки порядка α^3 к e^+e^--аннигиляции в пару фермионов в теории электрослабого взаимодействия. Полное сечение σ_T и интегральная асимметрия A_{FB}

Получены аналитические выражения для полностью проинтегрированных КЭД-вкладов порядка α^3 в полное сечение σ_T и интегральную асимметрию вперед-назад A_{FB} для процесса $e^+e^- \to \bar{f}f(\gamma)$. Предполагается, что фотоны ненаблюдаемы. Расчет выполнен в ультрарелятивистском приближении по массам фермионов, а масса M_Z и ширина Γ_Z нейтрального векторного промежуточного бозона не считаются малыми параметрами.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна 1987

Bardin D.Yu., Fedorenko O.M., Riemann T. E2-87-663
The Electromagnetic α^3 Contributions to e^+e^--Annihilation into Fermions in the Electroweak Theory. Total Cross Section σ_T and Integrated Asymmetry A_{FB}

Analytic expressions are obtained for the completely integrated QED α^3 contributions to the total cross section σ_T and the integrated forward-backward asymmetry A_{FB} in the process $e^+e^- \to \bar{f}f(\gamma)$. The photons are assumed not to be observable. The mass M_Z and width Γ_Z of the neutral weak gauge boson are treated with no approximation.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1987