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A GAUGE FORMULATION FOR RELATIVISTIC THEORIES OF PARTICLES AND STRINGS

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Relativistic theories of particles and strings are usually formulated either in a manifestly reparametrization invariant form or by using constraints with the corresponding Lagrangian multipliers. However, both approaches are not easy to adapt for describing several interacting relativistic particles or unusual strings. In the first approach one has to find an invariant Lagrangian (nonlinear and unrelated to the nonrelativistic one), and in the second approach some a priori constraints have to be chosen. In Ref. /1/ the relativistic theory of free particles was obtained from the nonrelativistic one by gauging the linear canonical symmetries of a rudimentary (formally nonrelativistic) bilinear Lagrangian. In this way a standard gauge theory emerges in which the canonical variables $p(t), q(t), \xi(t)$ (Greek characters are used for Grassmann variables) play the role of "matter fields" while the Lagrangian multipliers $\ell_i(t)$ act as components of a gauge potential $A(t)$. Here, we will show that this simple observation reveals the action of a rather general principle of gauging linear supercanonical symmetries. Using this principle allows one to construct, in addition to all known models of relativistic particles and strings, quite new theories (the first example is the theory of massless scalar particles suggested in Ref. /1/ ; the second, highly nontrivial application of the principle, is given at the end of this paper).

Following the ideas of Refs. /2/,/3/ consider the following rather general rudimentary Lagrangian that is bilinear in canonical variables

$$L_0 = g_{\mu\nu} p_i^\mu \dot{q}_i^\nu - \frac{i}{2} h_{\alpha\beta} \xi^\alpha \dot{\xi}^\beta - H_0(p, q, \xi). \qquad (1)$$

Here $\dot{q} \equiv dq/dt \equiv \partial_t q$; $\mu, \nu = 0, 1, \ldots, D-1$, the index i numbers the particles, say, i=1,..., N . The symmetrical matrices $g_{\mu\nu}$ and $h_{\alpha\beta}$ can be diagonalized by linear transformations of the canonical variables. Neglecting the new ones corresponding to zero eigenvalues we obtain $g_{\mu\nu} = (-1, -1, \ldots, -1, +1, \ldots, +1)$. It can be shown that the quantum theory of the gauge invariant Lagrangian for one scalar particle (see Ref. /1/) is consistent only for the Minkowski signature $g_{\mu\nu} = (-1, +1, \ldots, +1)$, otherwise the Hilbert space of the system has indefinite metrics. In this sense, the gauge principle implies relativistic invariance. In what follows we use the

Minkowski metrics $g_{\mu\nu}$ and suppress all contracted space-time indices μ, ν ; Lorentz invariance is trivially satisfied everywhere.

The Grassmann variables may be chosen, to some extent, arbitrarily, and this important observation can be used for describing spin and internal degrees of freedom. For simplicity, we consider a collection ξ_k^μ, $k=1,\dots,K$, where ξ_k^μ are Lorentz vectors, like p_i^μ, q_i^μ . Introducing the notation

$$\Psi^T \equiv (p_1 q_1 \dots p_N q_N \xi_1 \dots \xi_K),$$

we rewrite L_0 in the form

$$L_0 = \frac{1}{2} \Psi^T C(\partial_t - H_0)\Psi + \frac{1}{2}\partial_t(p_i q_i). \qquad (2)$$

Here H_0 is the supermatrix of the bilinear form $H_0(p,q,\xi)$, and C is easily derivable matrix $(C^2 = -\mathbb{1})$. The last term in Eq.(2) is important for deriving the boundary conditions for the local parameters $f(t)$, $\varphi(t)$ at the ends of the evolution interval, $0 \leq t \leq 1$ (they follow from the invariance of the action). The first term in the Lagrangian, depending on the derivative $\partial_t \Psi$ is invariant with respect to rigid transformations

$$\delta\Psi = F(f,\varphi)\Psi, \quad F^T C + CF = 0,$$

i.e. $F \in osp(2N|K)$ (remember that the transposed supermatrix F^T must be defined so as to satisfy the relation $(F\Psi)^T = \Psi^T F^T$). The full Lagrangian L_0 is invariant under the subalgebra of $osp(2N|K)$, satisfying the condition

$$[H_0, F] \equiv H_0 F - F H_0 = 0.$$

Now the gauged Lagrangian, that is invariant under the local transformations $F(f(t), \varphi(t))$, can easily be written

$$L = \frac{1}{2}\Psi^T C(\partial_t - A)\Psi. \qquad (3)$$

Here the matrix A is obtained from F simply by substituting

$$f(t) \to \ell(t), \quad \varphi(t) \to \lambda(t),$$

where we call $\ell(t), \lambda(t)$ the gauge potentials. The full gauge transformation is quite standard

$$\delta\Psi = F\Psi, \quad \delta A = \dot{F} + [F, A]. \qquad (4)$$

Remark that, due to the commutativity condition $[H_0, F] = 0$, the matrix H_0 can be included in A by a simple shift of the variables ℓ and λ . In the one-particle case (N=1) the Lagrangian (3) is, in the usual notation,

$$L = p\dot{q} - \frac{i}{2}\xi_k\dot{\xi}_k - \frac{1}{2}\ell p^2 - i\lambda_k(p\xi_k) - \frac{i}{2}\xi_i \ell_{ik}\xi_k, \quad (5)$$

where $\ell_{ik} = -\ell_{ki}$. This Lagrangian describes massless spinning particles, and for K=2 it has been derived in Ref./4/ by using much more complicated superfield construction (the general case has been treated in Ref./5/).

The problem of quantizing the theory (5) has been treated in Refs./4/,/5/ in the framework of the Dirac quantization scheme. This treatment is however incorrect as it uses the gauge choice $\ell \equiv 1$, $\lambda_k \equiv 0$ incompatible with the boundary conditions mentioned above. The consistent approach to quantizing such a system was first introduced by Fradkin and Vilkovisky/6/ (FV). In the FV approach the potentials ℓ, λ are promoted to the status of the other canonical variables by introducing conjugate momenta $k, \varkappa$ and adding to the Lagrangian the "kinetic" terms $k\dot{\ell}, i\varkappa\dot{\lambda}$. This is equivalent to the gauge fixing conditions $\dot{\ell}=0, \dot{\lambda}=0$ (more general gauge conditions can be introduced by adding "potential" terms, e.g. $c\,k^2$). To understand the necessity of the extended phase space , consider the gauge theory of the scalar particle. In this case $\delta\ell = \dot{f}(t)$ and the boundary conditions $f(0) = f(1) = 0$ require that

$$\delta\ell^{(0)} \equiv \delta\int_0^1 dt\, \ell(t) = \int_0^1 dt\, \dot{f}(t) = 0,$$

i.e. $\ell^{(0)}$ is gauge invariant. This excludes the "obvious" gauges like $\ell = 1$ while the FV procedure gives a correct gauge choice. Further extension of the phase space by introducing Faddeev-Popov ghosts allows one to construct a BRST-invariant Lagrangian. In such a "superextended" phase space the standard path integral quantization with the Liouville measure is possible. All unphysical degrees of freedom are automatically compensated in a relativistically invariant manner, and one finally arrives at the correct expressions for relativistic propagators (see e.g./7/). By introducing the fields depending on q^μ, ℓ, λ and ghost coordinates a gauge invariant formulation of the field theory can be found, such a program for scalar particles has recently been performed in/8/.

In the string case the index i is continuous, we denote it by s, $0 \leqslant s \leqslant 2\pi$. With the fundamental variables $p^{\mu}(t,s)$, $q^{\mu}(t,s)$, $\xi^{\mu}_{\alpha}(t,s)$, $\alpha = 1,2$, the simplest rudimentary Lagrangian obvilusly is

$$L_0 = \int_0^{2\pi} ds\, \mathcal{L}_0(t,s), \quad \mathcal{L}_0 = p\dot{q} - \frac{1}{2}(p^2 + q'^2) + \frac{i}{2}\xi_{\alpha}\dot{\xi}_{\alpha} - \frac{i}{2}\xi_{\alpha}\sigma_3^{\alpha\beta}\xi_{\beta}, \tag{6}$$

where $q' \equiv \partial_s q \equiv \partial q$, σ_3 is the Pauli matrix. Writing $\Psi^T = (p\ q\ \xi_1\ \xi_2)$ it is easy to represent L_0 in the form (2) and to find its rigid symmetries depending on the parameters $f(s)$, $\varphi(s)$

$$\delta\Psi = \mathcal{F}\Psi = D_+ F D_- \Psi, \quad D_{\pm} = \begin{pmatrix} \partial_{\pm} & 0 \\ 0 & \mathbb{1} \end{pmatrix}, \quad \partial_+ = \begin{pmatrix} \partial & 0 \\ 0 & 1 \end{pmatrix}, \quad \partial_- = \begin{pmatrix} 1 & 0 \\ 0 & \partial \end{pmatrix},$$

$$F = \begin{pmatrix} f & \varphi \\ \tilde{\varphi} & \tilde{f} \end{pmatrix}, \quad f = \begin{pmatrix} f_2 & f_1 \\ f_1 & f_2 \end{pmatrix}, \quad \tilde{f} = \begin{pmatrix} f_+\partial + \partial f_+ & 0 \\ 0 & f_-\partial + \partial f_- \end{pmatrix}, \quad \varphi = -i\begin{pmatrix} \varphi_1 & -\varphi_2 \\ \varphi_1 & \varphi_2 \end{pmatrix}, \quad \tilde{\varphi} = \begin{pmatrix} \varphi_1 & \varphi_1 \\ \varphi_2 & -\varphi_2 \end{pmatrix} \tag{7}$$

where $f_{\pm} = \frac{1}{2}(f_2 \pm f_1)$. The operator ∂ is acting on all functions to the right of it. The ordering of ∂ as well as the form of the matrix $\tilde{f}$ are defined by the closure requirement, $[\delta_1, \delta_2] \sim \delta_3$. Now, defining the gauge potential, $\mathcal{A} = D_+ A \cdot D_-$ by the usual substitution, $f_i \to \ell_i$, $\varphi_i \to \lambda_i$, we obtain the gauge invariant Lagrangian

$$\mathcal{L} = p\dot{q} - \frac{1}{2}\ell_1(p^2 + q'^2 + i\xi^T\sigma_3\xi) - \ell_2(pq' + \frac{i}{2}\xi^T\xi') - i\lambda^T\xi \cdot p - i\lambda^T\sigma_3\xi q'. \tag{8}$$

As usually, the gauge transformations are defined by Eq.(4) where $A \to \mathcal{A}$, $F \to \mathcal{F}$. The Lagrangian (8) for the fermionic string was originally proposed in Refs./9/ for description of the Neveu--Schwarz-Ramond superstring. The approach to quantizing the theory (8) may be developed in strict analogy to the particle case described above. The main difference and the principal difficulty is that the differential operator ∂ corresponds to infinity of generators (in a discrete representation it is represented by the set of matrices $ni\sigma_2$ where n=1,2,3,...). We hope that the transparent presentation of the gauge structure in string theories found here will allow one to search for some new and simpler approach to the difficult problem of constructing field theories of strings.

Finally, consider N particles bound by harmonic forces. The rudimentary Lagrangian is obviously

$$L_0 = p_i\dot{q}_i - \frac{1}{2}p_ip_i - \frac{1}{2}k_{ij}(q_i - q_j)^2, \quad i,j = 1,\dots,N. \tag{9}$$

By gauging this Lagrangian we come to the corresponding relativistic theory. Supposing $k_{ij} = k$ one obtains a theory with a rather rich gauge structure. We write the gauge invariant Lagrangian for the three-particle case, N=3:

$$\begin{aligned} L = {} & z\dot{y} + z_1\dot{y}_1 + z_2\dot{y}_2 - \frac{1}{2}\ell_1(z^2 + M^2) - \\ & - \frac{1}{2}\ell_2(z_1^2 + z_2^2 + y_1^2 + y_2^2 - z^2 - m^2) - \\ & - \frac{1}{2}\ell_3(z_1^2 + y_1^2 - z_2^2 - y_2^2) - \ell_4(z_1z_2 + y_1y_2) - \\ & - \ell_5(z_1y_2 - z_2y_1). \end{aligned} \tag{10}$$

Here y^{μ}, z^{μ} are the canonical center-of-mass coordinates, y_1^{μ}, z_1^{μ} describe the relative motion of the particles 1 and 2 while y_2^{μ}, z_2^{μ} are the canonical coordinates of the third particle in the center--of-mass system of the particles 1 and 2. The parameter k is absorbed in coordinates q_i with due rescaling of t. Considering the constraints appearing in Eq.(10) as generators of symmetry transformations, we see that the first constraint generates the usual translation group T_1, the second produces $U_1 \sim SO_2$ rotations while the rest generate the SO_3 -transformations. The full gauge group of the three-particle system is thus

$$T_1 \otimes U_1 \otimes SO_3\,.$$

This result can be generalized for arbitrary values of N. To obtain the two-particle Lagrangian one simply sets $y_2 = z_2 = 0$ in Eq.(10). Due to the abelian nature of the potentials ℓ_1 and ℓ_2, we are free to add arbitrary mass parameters M^2 and m^2. It has to be stressed that the individual masses of the particles cannot be defined in this gauge theory.

If the parameters k_{ij} are different (for example, we have particles of two sorts), the gauge symmetry SO_N is correspondingly broken. Our approach can also be used for constructing a gauge theory of the discrete string (the "linear" system of N particles, i.e. with such harmonic forces that $k_{ij} = k\,\delta_{|i-j|,1}$).

Using the standard quantization formalism described above will probably enable us to construct a relativistic quantum theory of interacting composite particles including the discrete strings).

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References

1. Filippov A.T. - JINR Rapid Communications, 1987, N3 (23), p.5.
2. Berezin F.A., Marinov M.S. - Pis'ma v Zs. Exptl. Teor. Fiz., 1975, 21, p.678.
3. Casalbuoni R. - Nuovo Cim., 1976, 33A, p.389.
4. Brink L., Di Vecchia P., Howe P. - Nucl. Phys., 1977, B118, p.76.
5. Gershun V.D., Tkach V.I. - Pis'ma v Zs. Exptl. Teor. Fiz., 1979, 29, p.320.
6. Fradkin E.S., VilkoviskyG.A. - Phys.Lett., 1975, 55B, p.224.
7. Henneaux M., Teitelboim C. - Ann. Phys., 1982, 143, p.127.
8. Neveu A., West P. Phys.Lett., 1986, 182B, p.393.
9. Deser S., Zumino B. Phys.Lett., 1976, 65B, p.369; Brink L., Di Vecchia P., Howe P. - ibid., p.471.

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О калибровочной формулировке теорий релятивистских частиц и струн

В работе показано, что применение принципа локализации линейных канонических симметрий позволяет конструировать релятивистские калибровочные теории частиц и струн, исходя из нерелятивистских. В качестве примеров получены известные лагранжианы для частиц с произвольным спином и для фермионной струны. Предложена также новая релятивистская калибровочная теория системы N частиц, связанных гармоническими силами, которую можно квантовать методами, обычно применяемыми для квантования калибровочных теорий поля.

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A Gauge Formulation for Relativistic Theories of Particles and Strings

Application of the principle of gauging the linear canonical symmetries is shown to produce relativistic gauge theories of particles and strings starting from the nonrelativistic ones. A relativistic gauge theory describing N particles bound by harmonic forces is obtained which can be quantized by the standard methods.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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