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E.Z.Avakyan*, S.L.Avakyan*, G.V.Efimov,
M.A.Ivanov

NONLEPTONIC DECAYS OF K-MESONS

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*Tashkent State University

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I. Introduction

The physics of kaons is extremely rich in interesting physical phenomena. Among them the nonleptonic decays of kaons are of particular interest. Investigation of these processes is needed for a deeper insight into the structure of weak interactions and relation of strong and weak quark interactions. Moreover, the empirical $\Delta I = 1/2$ rule has no full theoretical explanation, though there are many works devoted to this problem [1]. This rule implies that changing isospin I by 3/2 transitions are substantially suppressed in comparison with those with $\Delta I = 1/2$. There are two alternatives in modern physics of weak interactions for explanation of the $\Delta I = 1/2$ rule.

1. Being a production of quark currents, terms are introduced in the Lagrangian so that nonleptonic decays with $\Delta I = 3/2$ become suppressed, for example [2]. This approach is purely phenomenological and thus, cannot give useful information about weak and strong quark interactions.

2. Weak interactions of quarks are described by standard Weinberg-Salam model with QCD corrections. Consideration of strong quark interactions at small distances enhances amplitudes with $\Delta I = 1/2$, but only this enhancement is insufficient for the explanation of the $\Delta I = 1/2$ rule. An additional dynamical mechanism is searched for enhancement of $\Delta I = 1/2$ transitions. This mechanism is associated with evaluation of matrix elements of hadron-quark transitions at long distances (at the confinement distances).

We shall follow the second way. A weak quark-quark interaction is to be treated in more detail. Effective Hamiltonian of this interaction is obtained in Weinberg-Salam model with a glance to gluonic QCD corrections connected with a strong quark interaction at small distances. The Hamiltonian is usually written in the form [3]

$$H^{w}_{eff} = \frac{G_F}{2\sqrt{2}} \sin\theta_c \cos\theta_c \sum_{i=1}^{6} c_i O_i . \qquad (1.1)$$

Here O_i are the four-quark local operators, their explicit form is given by (4.1). Numerical coefficients c_i are defined by quarks behaviour at small distances and are calculated in QCD. Their explicit form is given in (B.1) from where on can see that the coefficients C_i depend on the normalization point μ, chromodynamic running constant α_s and masses of heavy quarks. Thus, they are not determined uniquelly.

Some sets of $C_1 - C_6$ proposed in literature are given in table 1. The coefficients c_i can be selected by fixing μ and α_s values from common considerations, as it has been done, for example, in /1,4/. The coefficient C_4 corresponding to O_4 -operator with $\Delta I = 3/2$ has to be noted 3-5 times smaller than C_1, i.e. strong interaction effects enhance Hamiltonian's part with $\Delta I = 1/2$ but this enhancement is insufficient to explain $\Delta I = 1/2$ rule. Set of coefficients /5/ has been chosen by fitting correctional coefficient ε. The coefficients used in /6/ have been obtained by artificial suppression of C_2, C_3, C_4 and artificial enhancement of C_5, C_6. We shall also start from Hamiltonian (1.1) where C_i are defined by (B.1) while the μ and α_s paramoters will be selected by experimental fit.

The main problem is evaluation of matrix elements of O_i -operators for which it is necessary to know the mechanism of hadronization and confinement. As these mechanism is unknown, the calculation of matrix elements needs different models to be used. Nonleptonic kaon decays were studied in different quark models /7,8,9/, in dispersion approaches /6,10,11/. It is impossible to calculate matrix elements of the two particle decay $K \to \pi\pi$ directly in most of the approaches; so, one has to resort to current algebra methods. Vacuum insertion method was used in some approaches /5,6/ for the calculation of matrix elements of four-quark operators. This method, however, was criticized in /12,13/ the last years.

Quark behaviour at long distances (or in the confinement region) causes the values of matrix elements of the $O_1 - O_6$ operators entering into the effective Hamiltonian (1.1). Only the O_4 operator from the set proposed in /3/ is responsible for $\Delta I = 3/2$ transitions. The O_5 operator is the subject of a separate discussion. This operator contributing to the $\Delta I = 1/2$ amplitudes contain "right" quark currents along with "left" ones unlike $O_1 - O_4$ consisting of "left" currents only. The role of this operator in the explanation of the $\Delta I = 1/2$ rule is a point of much controversy. There is an affirmation /3,6,7/ about the decisive role of O_5, O_6 operators in the enhancement of the corresponding amplitudes. On the other hand, it has been shown in /10,14/ O_5, O_6 enhancement is insufficient to ensure $\Delta I = 1/2$ rule.

Table 1

	C_1	C_2	C_3	C_4	C_5
QCM $\mu = 0.20$ GeV, $\alpha_s = 1.1$	-2.36	0.092	0.085	0.42	-0.059
[1] $\alpha_s = 1$, $m_t = 40$ GeV	-2.38	0.10	0.084	0.42	-0.047
[4] $\alpha_s(m_W) = 0.1$, $\mu = 3$ GeV	-3.04	0.32	0.22	1.08	-0.13
[5] $\alpha_s = 1$, $\Lambda = 0.1$ GeV, $\mu = m_\pi$, $\varepsilon = 1.1$	-5.11	0.03	0.04	0.2	-0.17
[6] $c_2, c_3, c_4 \to \frac{1}{4}(c_2, c_3, c_4)$, $c_5, c_6 \to 3(c_5, c_6)$	-2.538	0.0205	0.02	0.1	0.24

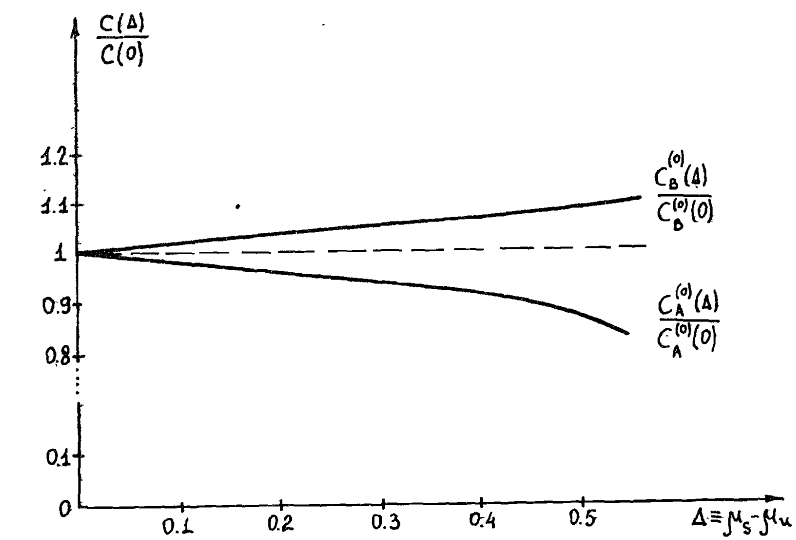


Fig.1

Table 2 summarizes decay widths $K \to \pi\pi$ obtained in different approaches. Enhancement of the $\Delta I = 1/2$ amplitudes in quark loop model /7/ is associated with a great contribution of diagrams with an intermediate ρ (770)-meson. The results obtained in /6,7/ depend on a great number of parameters and on a method of calculation. A sufficient agreement with the experimental data in /9,10/ has been achieved by means of phenomenological corrections to matrix elements. Agreement with experiment in /5/ is a result of fitting a correction parameter to the coefficients $C_1 - C_6$, a new neutral interaction is suggested in /15/ for description of nonleptonic decay amplitudes. The result depends on the choice of the coupling constant of this interaction (Ag_A = = 30).

It is to be stressed that the enhancement of the $\Delta I = 1/2$ transition amplitudes in /5,6,9,11,15/ is achieved by introducing new additional phenomenological parameters. So, in spite of good agreement of the results obtained in these approaches, the $\Delta I = 1/2$ problem cannot be regarded as a solved one.

The aim of this article is also studying of nonleptonic and electromagnetic K-meson decays. We shall start from the Hamiltonian of weak interactions (1.1) while the matrix elements will be evaluated in the framework of the Quark Confinement Model (QCM) /16/ . This model is based on two ansatzes. The first one is that quark confinement is realized as a definite approach of averaging the quark fields over gluonic vacuum. The second one is that quark hadronization is thought to be a transition to collective variables in the QCD Lagrangian. The quark masses and confinement region size are the parameters of the model.

QCM gives a sufficient description of low-energy hadronic physics of nonstrange quarks, as it was shown by calculations (see /17/). In this article we proceed with the study of physics of strange particles-kaons.

The work is organized in the following way:

Section 2 is devoted to the method of the calculation in QCM for processes with both strange and nonstrange quarks. Parameters of a strange quark are fixed from $K \to \mu\nu$, $K^* \to K\pi$, $K^* \to K\gamma$, $\varphi \to K^+K^-$, $\varphi \to e^+e^-$ decays. Section 3 deals with electromagnetic $K^\pm$ and K^0 radii $K \to \pi e \nu$ decay from factors and ratio $\gamma = f_A / f_V$ in $K \to e\nu\gamma$ decay. Calculation of these quantities is a good checking of any quark model.

Nonleptonic and electromagnetic K-meson decays are examined in section 4. Matrix elements of operators entering into the effective

Table 2

	$\Gamma(K_S^0 \to \pi^+\pi^-)$ 10^{-15} GeV	$\Gamma(K_S^0 \to \pi^0\pi^0)$ 10^{-15} GeV	$\Gamma(K^+ \to \pi^+\pi^0)$ 10^{-17} GeV	$\Gamma(K_L^0 \to \gamma\gamma)$ 10^{-21} GeV	$\Gamma(K_S^0 \to \gamma\gamma)$ 10^{-18} GeV
Experiment /24/	5.06±0.03	2.32±0.02	1.13±0.01	6.22±0.26	<2.95
Quark Loop Model /7/	5.94	2.98	1.8	6.7	1.75
MIT-bag model /9/	5.14	2.12	1.27	-	-
Harmonic oscillator model /9/	6.27	2.68	1.14	-	-
Vacuum insertion method /6/	4.98	2.71	0.21	-	-
Current algebra with cont.to phys.area /5/	5.11	2.54	1.12	-	-
1/N expansion /11/	5.34	2.13	1.89	6.26	-
Model with new neutral interaction /15/	5.58	2.18	2.19	-	-
Chiral Lagrangian method /2/	4.85	1.6	-	-	-
QCM	4.93	1.98	1.19	6.67	0.31

Table 3

$M(K_S^0 \to \pi^+\pi^-)$	$\frac{M_{O_1-O_4}(K_S^0 \to \pi^+\pi^-)}{M(K_S^0 \to \pi^+\pi^-)}$	$\frac{M_{O_5}(K_S^0 \to \pi^+\pi^-)}{M(K_S^0 \to \pi^+\pi^-)}$	$\frac{M^{\varepsilon}_{O_5}(K_S^0 \to \pi^+\pi^-)}{M(K_S^0 \to \pi^+\pi^-)}$
38.6 10^{-8} GeV	0.526	0.189	0.234

Hamiltonian of weak interaction are calculated. Matrix elements of the O_5 operator turn out to be enhanced but only this enhancement is insufficient for explanation of the $\Delta I = 1/2$ rule . Our result agrees with the point of view expressed in /14/ . Interaction with an intermediate ε (670)-meson, or the O_5 pole contribution, gives a considerable contribution to the $K_S^0 \to \pi^+\pi^-$ amplitude.

The coefficients c_i are μ and α_s dependent. Using analytical expression of this dependence we have fixed the c_i set by fitting μ and α_s by experimental values of $K_S^0 \to \pi^+\pi^-$, $K^\pm \to \pi^\pm\pi^0$, $K_L^0 \to \gamma\gamma$ widths. It turns out that a good agreement with experiment needs α_s to be 1,0-1,1 while μ is permitted to vary between 0.1-0.4 GeV. The set of coefficients we have obtained is given in table I. One can see that our values for the coefficient are close to those assumed in /1/ .

Table 2 represents $K_S^0 \to \pi^+\pi^-$, $K^\pm \to \pi^\pm\pi^0$, $K_S^0 \to \pi^0\pi^0$, $K_L^0 \to \gamma\gamma$, $K_S^0 \to \gamma\gamma$ widthes. The obtained results are in good agreement with the experimental data.

Table 3 shows that consideration of all operators including the diagrams with an intermediate ε - meson (the pole contribution of O_5) is important for solving $\Delta I = 1/2$ problem. The enhancement connected with the $O_1 - O_4$ operators is caused by relation of $c_1 - c_4$. A considerable contribution from contact diagrams with O_5 and those with an intermediate ε - meson is explained by great values of the matrix elements of O_5 .

2. Parameters of strange quarks in QCM and main kaonic decays

Let us state the main principles of QCM /16,17/ for mesonic interactions. Lagrangian of meson-quark interactions

$$\mathcal{L}_{Mq\bar{q}} = \sum_J \frac{g_J}{\sqrt{2}} M_J^a(x)\, \bar{q}(x)\, \Gamma_J \lambda_a\, q(x), \tag{2.1}$$

where

$$\Gamma_J = \begin{cases} I & \text{for scalar mesons} \\ i\gamma_5 & \text{for pseudoscalar mesons} \\ \gamma_\mu & \text{for vector mesons} \\ \gamma_\mu\gamma_5 & \text{for axial-vector mesons} \end{cases}$$

λ_a are the Gell-Mann matrices, g_J are the coupling constants. Weak and electromagnetic interactions are described by the standard Lagrangians $\mathcal{L}_{em}$ and $\mathcal{L}_w$.

Interactions between hadrons are described by the collision matrix:

$$S = \int d\sigma_{vac}\, T \exp\left\{ i \int dx \left[\mathcal{L}_{Mq\bar{q}} + \mathcal{L}_{em} + \mathcal{L}_w \right] \right\}. \tag{2.2}$$

Time ordering in (2.2) is thought to be a conventional Vick T-production for mesonic and quark fields with the following quarks field propagator:

$$S(x, x' | B_{vac}) = \overline{q(x)\,\bar{q}(x')} = (m_f - \hat{p} - \hat{B}_{vac})^{-1}\, \delta(x - x'). \tag{2.3}$$

Here m_f is quark's mass, $\hat{B}_{vac}$ is the vacuum gluonic field, $d\sigma_{vac}$ is indefinite measure of integration over vacuum field. Quark fields must be put equal to zero after transition to normal ordering in (2.2).

There are several assumptions concerning vacuum gluonic fields. S - matrix becomes a totality of exclusive colourless quarks cycles with attached mesonic fields after transition to normal production of hadronic fields in (2.2) with quark fields are put equal to zero.

The first ansatz of the model is that all these colourless cycles are averaged over measure $d\sigma_{vac}$ independently. This assumption means that quark loops at low energies may be connected by mesons only, while their connection through gluonic vacuum can be ignored.

The second ansatz is that averaging of a quark loop over measure $d\sigma_{vac}$ can be changed by one-dimension integral:

$$\int d\sigma_{vac}\, Sp\{ S(x_1, x_2 | B_{vac}) M(x_2) S(x_2, x_3 | B_{vac}) M(x_3) \ldots M(x_n) \Rightarrow$$
$$\Rightarrow \int d\sigma_\lambda\, Sp\{ S_\lambda(x_1 - x_2) M(x_2) S_\lambda(x_2 - x_3) M(x_3) \ldots M(x_n) \}, \tag{2.4}$$

where

$$S_\lambda(x_1 - x_2) = \int \left(\frac{dP}{2\pi} \right)^4 \frac{e^{-iP(x_1 - x_2)}}{\Lambda\lambda + m_f - \hat{P}},$$

m_f stands for the mass of a quark with flavour f, while the parameter Λ characterises the size of confinement.

Ansatz of confinement implies definition of the analytical structure of measure $d\sigma_{vac}$. Let us define $d\sigma_{vac}$ by the equality:

$$\int d\sigma_\lambda \frac{1}{\Lambda\lambda + m_f - \hat{p}} = \frac{1}{\Lambda_f} G\left(\frac{m_0 - \hat{p}}{\Lambda_f}\right) = \tag{2.5}$$

$$= \frac{1}{\Lambda_f}\left[a\left(\frac{p^2}{\Lambda_f^2}, \frac{m_f}{\Lambda_f}\right) + \frac{\hat{p}}{\Lambda_f} b\left(\frac{p^2}{\Lambda_f^2}, \frac{m_f}{\Lambda_f}\right)\right]$$

or

$$\int \frac{d\sigma_\lambda}{\lambda - z} = G(z).$$

Function $G(z)$ will be called the confinement function. One can easily see that the analytical structure will be expressed by the confinement function $G(z)$ or a and b in (2.5).

In the case of calculation of the loops of type (2.4) containing quarks with different flavours we shall use Λ, corresponding to the heaviest quark in the loop. Thus, parametres Λ_u and Λ_s are used for description of physics of nonstrange and strange mesons.

Let us assume $G(z)$ to satisfy the following conditions:

1. $G(z)$ is universal, i.e. it depends on neither colour nor flavour and is unique for all quark loops determining any mesonic interaction.

2. $G(z)$ is the analytical function in all complex z - plane with singularity permitted only at the infinitly distant point.

   This condition secures quark confinement and S-matrix unitarity in the space of physical states.

3. $G(z)$ decreases quicker than any polynomial in the Euclidean space, i.e.

$$\lim_{z^2 \to -\infty} (-z^2)^N |G(z)| = 0$$

for any $N > 0$. This condition provides convergency of any quark diagram.

4. The explicit form of $G(z)$ has not yet been found from any general investigations in QCD. Thus, the choice of the explicit form of the confinement function is also one of the model's assumptions. Our estimations have shown, however, that mainly integral characteristics of $G(z)$ are important for a sufficient description of low-energy mesonic processes.

The function:

$$G(z) = \exp\left[\left(\frac{3}{2} - z\right)^2\right] \tag{2.6}$$

has been used in the present work and in [17]. This functional form is taken from the virton-quark model.

One has to hold all the evaluations of the S - matrix elements in the Euclidean metric, with analytical continuation of them with respect to invariant momentum variables.

The S-matrix constructed under the rules mentioned above is a collision matrix of the theory with a nonlocal interaction (see [18]). The S-matrix is finite unitary and macrocausal on the space of physical mesonic states in every order of interaction lagrangian expansion.

The coupling constants g_J in (2.1) are determined by the binding condition implying (see for example [19]) the requirement for the wave functions of M_J meson renormalization constant to be equal to zero:

$$Z_{M_J} = 1 - g_J^2 \, \tilde{\Pi}_J'(m_J^2) = 0. \tag{2.7}$$

where $\tilde{\Pi}(p^2)$ is the mass operator, m_J is the mass of meson M_J.

The mass m_f differes for u and s quark. Thus, m_f-, Λ_u-, Λ_s- - quantities characterizing the size of confinement are the parametres of the model.

Let us illustrate our technique of calculation by example of a weak kaonic decay: $K \to \mu\nu$. The diagram determining the amplitudes of this process is represented in Table 4. The corresponding structure integral takes the form

$$\langle I^\mu_{K\to\mu\nu}\rangle = \left\langle \int \frac{d^4k}{i(2\pi)^4} Sp\left\{\gamma_5 \frac{1}{\Lambda_s\lambda + m_u - \hat{k}} O_\mu \frac{1}{\Lambda_s\lambda + m_s - (\hat{k}+\hat{p})}\right\}\right\rangle_{conf} =$$

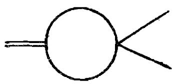
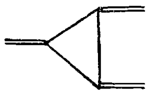
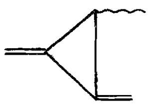
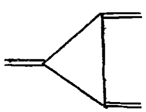
$$= \frac{\Lambda_s^2}{4} \int_0^1 d\alpha \int \frac{d^4k}{i(2\pi)^4} \left\langle \frac{}{[(\mu_u\alpha + \mu_s(1-\alpha))^2 - \Delta^2\alpha(1-\alpha) - \bar{k}^2 - \bar{p}^2\alpha(1-\alpha)]^2} \right\rangle_{conf}. \tag{2.8}$$

We have passed to dimensionless quantites μ_f, $\bar{k} = k/\Lambda_s$ in the latter expression. After standard transformations using formulae (2.4) and (2.5) one obtains:

$$\langle I^\mu_{K\to\mu\nu}\rangle = -\frac{\Lambda_s^2}{4} \bar{p}^\mu \int_0^1 d\alpha \int_0^\infty du \, a\left(u - \bar{p}^2\alpha(1-\alpha) + (\mu_s - \mu_u)^2\alpha(1-\alpha)\right). \tag{2.9}$$

Here $\mu_f = m_f/\Lambda_s$

Table 4

Process	Diagram	Decay constant	Experiment	QCM
$K\to\mu\nu$		$f_K = \frac{3\Lambda_s\sqrt{\lambda_K}}{\pi} C_A^{(0)}$	154.3±0.36 MeV	158.9 MeV
$K^{*+}\to K^0\pi^+$		$g_{K^*K\pi} = 48\pi\sqrt{\lambda_\rho\lambda_v\lambda_K}\,C_B^{(0)}$	4.47±0.04	3.9
$K^{*0}\to K^0\gamma$		$g_{K^*K\gamma} = \frac{32\sqrt{\lambda_K\lambda_v}}{\Lambda_s}a(0)$	1.02±0.24 GeV^{-1}	1.1 GeV^{-1}
$\varphi\to K^+K^-$		$g_{\varphi KK} = 48\pi\lambda_K\sqrt{\lambda_v}\,C_B^{(0)}$	4.44±0.12	3.53
$\varphi\to\gamma\to e^+e^-$		$\frac{1}{f_v} = \frac{2\sqrt{\lambda_v}}{3\pi}C_B^{(0)}$	0.076±0.002	0.080

We shall neglect squared mass in the case of light mesons. The parameters Λ_u and m_u were determined in /17/

$$\Lambda_u = 730\,\text{MeV}, \quad m_u = 220\,\text{MeV}. \tag{2.10}$$

Investigation of kaonic physics needs strange quark's parameter Λ_s to be fixed by fitting over main decays of strange mesons: $K\to\mu\nu$, $K^*\to K\pi$, $K^*\to K\gamma$, $\varphi\to K^+K^-$, $\varphi\to e^+e^-$.

Table 4 represents diagrams, invariant amplitudes, experimental data and theoretical values obtained in QCM for main decays of kaons. Best agreement with experiment (with an accuracy better than 20%) can be achieved at

$$m_s = 440\,\text{MeV}, \quad \Lambda_s = 1052\ \text{MeV}. \tag{2.11}$$

The following notation

$$C_A^{(0)}(\mu_u,\mu_s) = \int_0^1 d\alpha \int_0^\infty du\, a\big(u + \Delta^2\alpha(1-\alpha),\ \mu_u\alpha + \mu_s(1-\alpha)\big)$$
$$C_B^{(0)}(\mu_u,\mu_s) = \int_0^1 d\alpha \int_0^\infty du\, b\big(u + \Delta^2\alpha(1-\alpha),\ \mu_u\alpha + \mu_s(1-\alpha)\big) \tag{2.12}$$

is used in the table.
Here

$$\mu_u = m_u/\Lambda_s \qquad \mu_s = m_s/\Lambda_s$$
$$\mu_u = 0.21 \qquad \mu_s = 0.42\,.$$

The $C_A^{(0)}(\mu_u,\mu_s)/C_A^{(0)}(\mu_u)$ and $C_B^{(0)}(\mu_u,\mu_s)/C_B^{(0)}(\mu_u)$ dependence on $\Delta = \mu_s - \mu_u$ is plotted in fig.2. It turns our that at $\Delta = 0.25$ $C^{(0)}(\mu_u,\mu_s)$ differs from $C^{(0)}(\mu_u)$ less than by 10% . Therefore, we shall neglect strange and nonstrange quarks mass difference in further calculations. Thus, parameter Λ_s characterising the confinement region of a strange quark plays the key role in the description of kaonic physics. All one-loop diagrams are expressed through the structure integrals

$$C_A^{(n)}(\mu_u) = \frac{1}{n!}\int_0^\infty du\, u^n\, a(u,\mu_u)$$
$$C_B^{(n)}(\mu_u) = \frac{1}{n!}\int_0^\infty du\, u^n\, b(u,\mu_u). \tag{2.13}$$

3. Weak and electromagnetic characteristics of kaons

Let us consider weak and electromagnetic kaon characteristics: $K^\pm$ and K^0 electromagnetic radii, K_{e3} - decay form factors, and the ratio of the axial vector form factor to the vector one in the structure dependent part of the amplitude of $K\to e\nu\gamma$ decay. These quantities are more fine characteristics of processes than decay's widths are; so, the calculation of the former is a good checking of a quark model.

Electromagnetic radius of kaon

Electromagnetic radii of $K^{\pm}$ and K° mesons are determined by the diagram in table 5. An invariant matrix element has the form

$$M^{\mu}(K\to K\gamma)=e(p_1+p_2)^{\mu}F_K(t)\Big|_{t=(p_1-p_2)^2;\ p_1^2=p_2^2=m_K^2}\ , \qquad (3.1)$$

where

$$F_K(t)=F_{\Delta}(t)+\frac{g_{\rho KK}}{f_{\rho}}\frac{t}{m_{\rho}^2-t}+\frac{g_{\varphi KK}}{f_{\varphi}}\frac{t}{m_{\varphi}^2-t} \qquad (3.2)$$

with $F_{\Delta}(0)=1$ for $K^{\pm}$ mesons

$F_{\Delta}(0)=0$ for K° mesons .

It is to be noted that $F_{\Delta}(t)$ pays the main contribution to $F_K(t)$ in the case of $K^{\pm}\to K^{\pm}\gamma$ while a contribution of $F_{\Delta}(t)$ to $F_K(t)$ in the case of $K^{\circ}\to K^{\circ}\gamma$ is proportional to the non-strange and strange quark mass difference, and diagrams with intermediate vector mesons play the main role, the contribution of ρ- meson diagram being negative. The constants $g_{\rho KK}$ and $g_{\varphi KK}$ are the constants of $\rho\to KK, \varphi\to KK$ transitions, $\frac{1}{f_\rho}, \frac{1}{f_\varphi}$, are the constants of $\rho\to\gamma, \varphi\to\gamma$ transitions. Analytical expressions for $K^{\pm}, K^{\circ}$ electromagnetic radii are given in Appendix A.

The obtained numerical values for electromagnetic radii of kaons are represented in table 5.

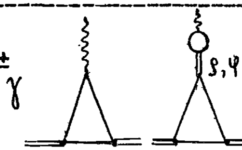
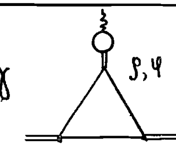
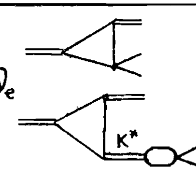
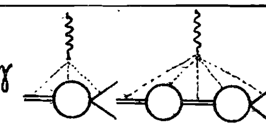
$K^{+}\to\pi^{\circ}e^{+}\nu_e$ Decay

The amplitude of the process considered is determined by diagrams represented in table 5. The matrix element is

$$M(K\to\pi e\nu)=-\frac{G_F}{\sqrt{2}}\sin\theta_c\,\ell_{\mu}^{w}\,T_{\mu}(p_1,p_2)\Big|_{p_1^2=m_K^2,\ p_2^2=m_{\pi}^2}\ , \qquad (3.3)$$

where ℓ_{μ}^{w} is the weak leptonic current, θ_c is the Cabibbo angle,

Table 5

Process	Diagram	Observable value	Experiment	QCM
$K^{\pm}\to K^{\pm}\gamma$		$\langle r^2\rangle_{K^{\pm}}$ Fm^2	0.26±0.07 [20]	0.31
$K^{\circ}\to K^{\circ}\gamma$		$\langle r^2\rangle_{K^{\circ}}$ Fm^2	−0.05±0.13 [21] −0.054±0.26 [22] 0.08± 0.05 [23]	−0.029
$K^{+}\to\pi^{\circ}e^{+}\nu_e$		$\xi(0)$ λ_+	−0.35± 0.14 [24] 0.026±0.008 [24]	−0.26 0.022
$K^{+}\to e^{+}\nu_e\gamma$		$\gamma=\frac{f_A(0)}{f_V(0)}$	0.05< γ < 0.6 [25]	0.21

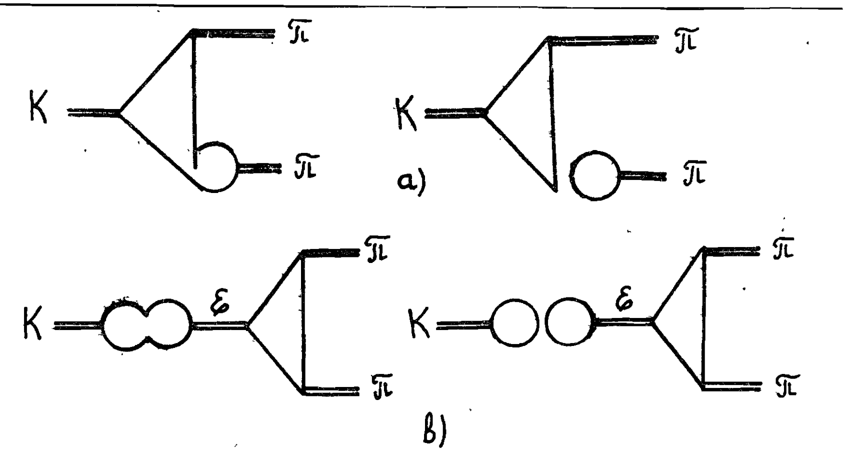


Fig.2.

$$T_\mu(P_1,P_2) = F_+(t)(P_1+P_2)_\mu + F_-(t)(P_1-P_2)_\mu\,,\quad t=(P_1-P_2)^2. \qquad (3.4)$$

One obtains λ_+ and $\xi(0)=F_-(0)/F_+(0)$ by using standard parametrization for the K_{e3} decay form factors

$$F_\pm(t) = F_\pm(0)\left[1 \pm \lambda_\pm \frac{t}{m_\pi^2}\right]. \qquad (3.5)$$

Numerical values for λ_+ and $\xi(0)$ are represented in table 5. Their analytical expressions are written in Appendix A.

$K \to e\nu_e\gamma$ Decay

The amplitude of this process is determined by diagrams represented in table 5. It may be written as

$$M(K\to e\nu\gamma) = M_{IB} + M_{SB}\,, \qquad (3.6)$$

where M_{SD} is the structure-dependent part of the amplitude, M_{SD} is interesting from the theoretical point of view because dependence on the inner structure of a kaon. Usually, M_{SD} is represented in the form

$$M_{SD}(K\to e\nu\gamma) = -e\frac{G_F}{\sqrt{2}}\sin\theta_c\, \ell^{w}_\mu\, \varepsilon^\nu(q_\gamma)\, T^{\mu\nu}_{SD}\,, \qquad (3.7)$$

where ℓ^w_μ is the weak leptonic current, $\varepsilon^\nu(q_\gamma)$ is the polarization vector of γ quantum and

$$T^{\mu\nu}_{SD} = f_A(t)\left[g^{\mu\nu}pq - p^\nu q^\mu\right] - i f_V(t)\varepsilon^{\mu\nu\alpha\beta}p_\alpha q_\beta\,,\quad t=(p-q)^2. \qquad (3.8)$$

Usually, the object of studying is

$$\gamma = \frac{f_A(0)}{f_V(0)} \qquad (3.9)$$

which is now known with a very great error

$$0.05 < \gamma < 0.6\,. \quad /25/$$

The inclusion of diagrams with an intermediate K_1 (1280)-meson affects the axial form factor and does not affect the vector one.

Thus,

$$\gamma = 1 - \frac{48\lambda_A}{\mu_A^2}\left(\frac{2C_A^{(0)}(\mu_u)\,C_B^{(0)}(\mu_u)}{a(0,\mu_u)} - C_B^{(1)}(\mu_u)\right) \qquad (3.10)$$

Using numerical values one obtains:

$$\gamma = 0.21$$

which is in agreement with the experimental estimation.

Table 5 shows that the calculated characteristics of electromagnetic, semileptonic and weak radiative decays are in good agreement with the experimental data. Thus, QCM describes physics of kaons correctly.

4. Nonleptonic decays of K-mesons

Let us study nonleptonic $K_S^0\to\pi^+\pi^-$, $K_S^0\to\pi^0\pi^0$, $K^+\to\pi^+\pi^0$ and electromagnetic $K_L^0\to\gamma\gamma$, $K_S^0\to\gamma\gamma$ decays of kaons with the effective Hamiltonian of weak interaction (1.1) being used, where O_i are the local four-quark operators defined as

$$O_1 = (\bar d\, O_L^\mu s)(\bar u\, O_L^\mu u) - (\bar d\, O_L^\mu u)(\bar u\, O_L^\mu s) \qquad (\Delta I = 1/2)$$

$$O_2 = (\bar d\, O_L^\mu u)(\bar u\, O_L^\mu s) + (\bar d\, O_L^\mu s)(\bar u\, O_L^\mu u) + 2(\bar d\, O_L^\mu s)(\bar d\, O_L^\mu d) + 2(\bar d\, O_L^\mu s)(\bar s\, O_L^\mu s) \qquad (\Delta I = 1/2)$$

$$O_3 = (\bar d\, O_L^\mu u)(\bar u\, O_L^\mu s) + (\bar d\, O_L^\mu s)(\bar u\, O_L^\mu u) + 2(\bar d\, O_L^\mu s)(\bar d\, O_L^\mu d) - (\bar d\, O_L^\mu s)(\bar s\, O_L^\mu s) \qquad (\Delta I = 1/2) \qquad (4.1)$$

$$O_4 = (\bar d\, O_L^\mu u)(\bar u\, O_L^\mu s) + (\bar d\, O_L^\mu s)(\bar u\, O_L^\mu u) - (\bar d\, O_L^\mu s)(\bar d\, O_L^\mu d) \qquad (\Delta I = 3/2)$$

$$O_5 = (\bar d\, O_L^\mu \lambda^A s)\sum_{q=u,d,s}(\bar q\, O_R^\mu \lambda^A q) \qquad (\Delta I = 1/2)$$

$$O_6 = (\bar d\, O_L^\mu s)\sum_{q=u,d,s}(\bar q\, O_R^\mu q). \qquad (\Delta I = 1/2)$$

Here

$$O^\mu_{L,R} = \gamma^\mu(1\mp\gamma^5),$$

λ^A is the Gell-Mann matrix acting in the colour space. The O_i operator properties with respect to the isotopic spin are indicated in brackets.

The coefficients C_i have been calculated in /3/ allowing for strong interactions at small distances. Expressions for $C_1 - C_6$ are given in Appendix B.

The coefficients $c_1 - c_6$ depend on W–boson and c -quark masses as well as on QCD parameters: values of the running constant $\alpha_s = g^2(\mu)/4\pi$ and the renormalization point μ.

$K \to \pi\pi$ Decays

Diagrams determining the amplitudes $K_S^0 \to \pi^+\pi^-$, $K_S^0 \to \pi^0\pi^0$, $K^+ \to \pi^+\pi^0$ decays are shown in fig.2. The matrix elements of nonleptonic decays of kaons take the form

$$M(K_S^0 \to \pi^+\pi^-) = \frac{G_F}{2\sqrt{2}} \sin\theta_c \cos\theta_c \{(c_1+2c_2+2c_3+2c_4)\, T^1_{K\pi^+\pi^-} + (c_5+\tfrac{3}{16}c_6)T^5_{K\pi^+\pi^-} + (c_5+\tfrac{3}{16}c_6)T^5_{K\varepsilon}\, \mathcal{D}_\varepsilon(m_K^2)\, M(\varepsilon \to \pi^+\pi^-)\} \qquad (4.2)$$

$$M(K_S^0 \to \pi^0\pi^0) = \frac{G_F}{2\sqrt{2}} \sin\theta_c \cos\theta_c \{(c_1+2c_2+2c_3-4c_4)\, T^1_{K\pi^0\pi^0} + (c_5+\tfrac{3}{16}c_6)T^5_{K\pi^0\pi^0} + (c_5+\tfrac{3}{16}c_6)T^5_{K\varepsilon}\, \mathcal{D}_\varepsilon(m_K^2)\, M(\varepsilon \to \pi^0\pi^0)\} \qquad (4.3)$$

$$M(K^+ \to \pi^+\pi^0) = \frac{G_F}{2\sqrt{2}} \sin\theta_c \cos\theta_c\, c_4\, T^4_{K^+\pi^+\pi^0} \qquad (4.4)$$

The following notaion is used:

$$T^{i*}_{K12} = \int dy\, dx_1 dx_2 dx_3\, e^{ip_1x_1+ip_2x_2+ip_3x_3} \langle 0|T\mathcal{L}_1(x_1)\mathcal{L}_2(x_2)\mathcal{L}_k(x_3) O^i(y)|0\rangle \qquad (4.5)$$

$$T^i_{K1} = \int dy\, dx_1 dx_2\, e^{ip_1x_1+ip_2x_2} \langle 0|T\mathcal{L}_1(x_1)\mathcal{L}_k(x_2) O^i(y)|0\rangle$$

$$M(\varepsilon \to \pi\pi) = -\lambda_p \sqrt{\lambda_s}\, \cos\delta_s\, \frac{32\pi}{\sqrt{2}}\, 3\Lambda_u C_A^{(0)}, \quad \delta_s = 23^\circ \qquad (4.6)$$

$\mathcal{D}_\varepsilon(m_K^2)$ is the ε - meson propagator.
The invariant amplitudes T^i calculated in QCM are represented in table 6.

The matrix elements of $\Delta I = 1/2$ decays $K_S^0 \to \pi^+\pi^-$ and $K_S^0 \to \pi^0\pi^0$ are determined either by the contact diagrams shown in fig.2a as well as the pole one with an intermediate ε - meson which is shown in fig.2b. It was claimed in /7/ that the contribution of this intermediate state plays the main role in the explanation of $\Delta I = 1/2$ rule.

Table 6

T^i_{KYX}	Analytical expression	Numerical value $(GeV)^4$ $(GeV)^3$
$T^1_{K_S^0\pi^+\pi^-}$	$\frac{24\sqrt{2}}{\pi}\lambda_p\sqrt{\lambda_K}\, m_K^2 \Lambda_s C_A^{(0)}\left(C_B^{(0)} + \frac{\mu_K^2 b(0)}{6}\right)$	0.06
$T^1_{K_S^0\pi^0\pi^0}$	$\frac{48\sqrt{2}}{\pi}\lambda_p\sqrt{\lambda_K}\, m_K^2 \Lambda_s C_A^{(0)}\left(C_B^{(0)} + \frac{\mu_K^2 b(0)}{6}\right)$	0.12
$T^4_{K^+\pi^+\pi^0}$	$\frac{18\sqrt{2}}{\pi}\lambda_p\sqrt{\lambda_K}\, m_K^2 \Lambda_s C_A^{(0)}\left(C_B^{(0)} + \frac{\mu_K^2 b(0)}{6}\right)$	0.05
$T^5_{K_S^0\pi^+\pi^-}$	$\frac{64\sqrt{2}}{\pi}\lambda_p\sqrt{\lambda_K}\, \Lambda_s^3 C_B^{(1)}\left(2C_A^{(0)} + \frac{\mu_K^2 a(0)}{2}\right)$	1.38
$T^5_{K_S^0\pi^0\pi^0}$	$\frac{128\sqrt{2}}{\pi}\lambda_p\sqrt{\lambda_K}\, \Lambda_s^3 C_B^{(1)}\left(2C_A^{(0)} + \frac{\mu_K^2 a(0)}{2}\right)$	2.76
$T^5_{K_L^0 A}$	$-\frac{32}{\pi^2}\sqrt{\lambda_K\lambda_A}\, \Lambda_s^2 m_K^2 \left(2C_A^{(1)}C_B^{(0)} + C_B^{(1)}C_A^{(0)}\right)$	- 0.06
$T^5_{K_S^0 S}$	$\frac{24}{\pi^2}\sqrt{\lambda_K\lambda_S}\, \Lambda_s^4 \left(\sqrt{3}\sin\theta_s - \cos\theta_s\right) C_B^{(1)} C_B^{(1)}$	0.19
$T^1_{K_L^0\gamma\gamma}$	$\frac{\alpha\sqrt{2}}{\pi^2}\sqrt{\lambda_K}\, \Lambda_s m_K^2 C_A^{(0)} b(0)$	$0.86\ 10^{-4}$
$T^5_{K_L^0\gamma\gamma}$	$\frac{-2\sqrt{2}\alpha}{3\pi^2}\sqrt{\lambda_K}\, \Lambda_s^3 \left(-\frac{8}{9}C_A^{(1)} b(0) + 8C_B^{(1)} a(0) + \frac{9}{2}\mu_K^2 C_A^{(0)} b(0)\right)$	$-0.35\ 10^{-2}$
$T^1_{K_L^0 P}$	$-\frac{6}{\pi^2}\sqrt{\lambda_K\lambda_P}\, \Lambda_s^2 m_K^2 C_A^{(0)} C_A^{(0)}$	$-0.47\ 10^{-2}$
$T^1_{K_L^0 A}$	$\frac{6}{\pi^2}\sqrt{\lambda_K\lambda_A}\, \Lambda_s^2 m_K^2 C_A^{(0)} C_B^{(1)}$	$0.81\ 10^{-2}$
$T^5_{K_L^0 P}$	$-\frac{32}{\pi^2}\sqrt{\lambda_K\lambda_P}\, \Lambda_s^2 m_K^2 \left(2C_A^{(0)}C_A^{(1)} - C_B^{(1)}C_B^{(1)}\right)$	0.21

We consider ε - meson as effective inclusion of s wave contribution to the $\pi-\pi$ interaction. In this case, its mass is a parameter determined from the $\pi-\pi$ scattering. The ε meson propagator is chosen as

$$\mathcal{D}_{\varepsilon}(p^2) = \frac{1}{m_{\varepsilon}^2 - p^2 - i m_{\varepsilon}\Gamma_{\varepsilon}} . \tag{4.7}$$

Formulae for the $\pi-\pi$ scattering lengths a_0^0 and a_0^2 are given in /26/. The a_0^0 and a_0^2 dependence on m_{ε} is plotted in fig.3. It turned out that for the best agreement with the $\pi-\pi$ scattering lengths m_{ε} is to be chosen

$$m_{\varepsilon} = 670 \text{ MeV},$$

Γ_{ε} is the total $\varepsilon \to \pi\pi$ decay width.
The QCM estimation gives $\Gamma_{\varepsilon} = 245$ MeV.

Electromagnetic Decays $K_L^0 \to \gamma\gamma$, $K_S^0 \to \gamma\gamma$

The amplitudes of electromagnetic decays of neutral kaons determined by the diagrams are shown in fig.4. It is to be noted that intermediate states play an important role in these decays. The amplitudes of these prcesses are

$$M(K_S^0 \to \gamma\gamma) = \frac{G_F}{2\sqrt{2}} \sin\theta_c \cos\theta_c \, c_5 \sum_{X=\varepsilon,\delta,S^*} T_{KX} \frac{M(X\to\gamma\gamma)}{m_X^2 - m_K^2}, \tag{4.8}$$

$$M(K_L^0 \to \gamma\gamma) = \frac{G_F}{2\sqrt{2}} \sin\theta_c \cos\theta_c \Big\{ (c_1 - 4c_2 + \tfrac{3}{2}c_3 + \tfrac{3}{2}c_4) T^1_{K\gamma\gamma} +$$

$$+ c_5 T^5_{K\gamma\gamma} + \sum_{X=\pi,a_1} \Big[(c_1 + 2c_2 + 2c_3 - 4c_4) T^5_{KX} \Big] \frac{M(X\to\gamma\gamma)}{m_X^2 - m_K^2} +$$

$$+ \sum_{X=\eta',D} \Big[(0{,}87c_1 + 1{,}74c_2 + 3{,}94c_3 - 3{,}92c_4) T^1_{KX} + 0{,}87 c_5 T^5_{KX} \Big] \frac{M(X\to\gamma\gamma)}{m_X^2 - m_K^2} + \tag{4.9}$$

$$+ \sum_{X=\eta,E} \Big[(0{,}76c_1 + 1{,}52c_2 - 9{,}8c_3 - 0{,}76c_4) T^1_{KX} + 0{,}76 c_5 T^5_{KX} \Big] \frac{M(X\to\gamma\gamma)}{m_X^2 - m_K^2} .$$

The matrix elements turn out to be equal to

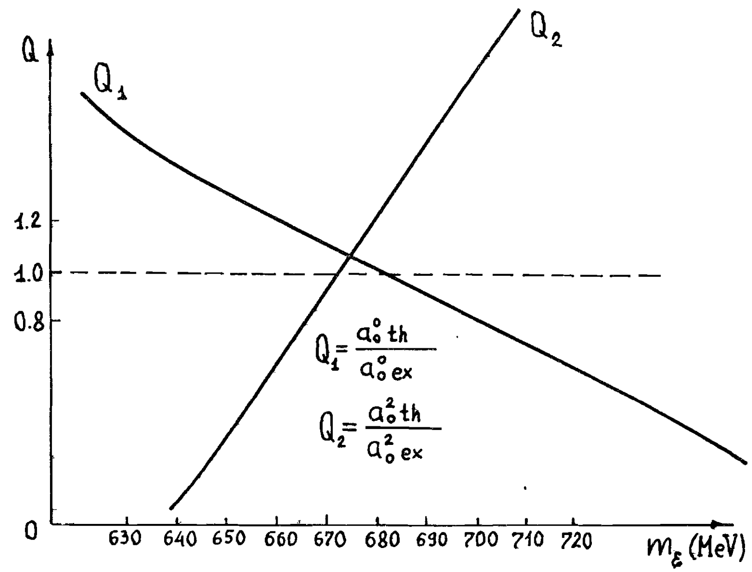


Fig.3.

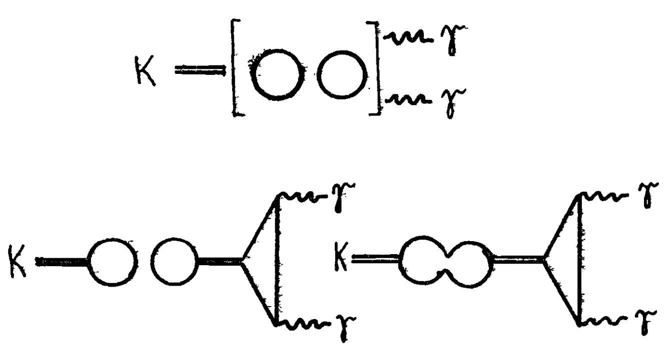


Fig.4.

$$M^{\mu\nu}(P\to\gamma\gamma)=\frac{24\alpha}{\sqrt{2}}\sqrt{\lambda_P}\,\frac{1}{\Lambda_u}C_M\,a(0,\mu_u)\,q_1^\sigma q_2^\rho\,\varepsilon^{\mu\nu\rho\sigma}$$

$$M^{\mu\nu}(A\to\gamma\gamma)=-\frac{9\alpha}{\sqrt{2}}\sqrt{\lambda_A}\,\frac{1}{\Lambda_u}C_M\,b(0,\mu_u)\,q_1^\sigma q_2^\rho\,\varepsilon^{\mu\nu\rho\sigma} \qquad (4.10)$$

$$M^{\mu\nu}(S\to\gamma\gamma)=-\frac{16\alpha}{\sqrt{2}}\sqrt{\lambda_S}\,\frac{1}{\Lambda_u}C_M\,a(0,\mu_u)\,q_1^\sigma q_2^\rho\,\varepsilon^{\mu\nu\rho\sigma}.$$

Here $\alpha=e^2/4\pi$

$$C_M=\begin{cases}1 & \text{for } \pi, a_1, \delta\\ (\cos\theta-2\sqrt{2}\sin\theta)/\sqrt{3} & \text{for } \eta, E, S^* \qquad \theta_P=-11^\circ\\ (2\sqrt{2}\cos\theta+\sin\theta)/\sqrt{3} & \text{for } \eta', D, \varepsilon \qquad \theta_S=58^\circ.\end{cases}$$

The calculated invariant amplitudes for a direct transition and for one-particle ones $K\to X$ are given in table 6. Intermediate pseudoscalar states turn out to play the crucial role in the $K_L^0\to\gamma\gamma$ decay.

It is very interesting to clarify the role of different operators in the $\Delta I=1/2$ amplitude enhancement. Table 3 represents relative contributions to $M(K_S^0\to\pi^+\pi^-)$ from different operators O_i taking account of the coefficients c_i : $M_{O_1-O_4}$ - the O_1-O_4 contribution, $M^c_{O_5}$ - contribution of the contact diagram with O_5, $M^{\varepsilon}_{O_5}$ - contribution of the pole diagrams with O_5 (i.e. diagrams with an intermediate ε - meson).

There are two opposite points of view concerning the role of the O_5 operator in the $\Delta I=1/2$ amplitudes enhancement . The contact diagrams with O_5 play an essential role in the explanation of the $\Delta I=1/2$ rule in /3,6/. It is claimed in /7/ that the $M_{O_1-O_4}$ and $M^c_{O_5}$ contributions are negligible in comparison with that from the pole diagram with intermediate ε - meson. On the other hand, it is shown in /10/ that the O_5 contribution with the terms included is less than 10% and the O_1-O_4 operators play the main role. Table 3 shows that in QCM all contributions to $M(K_S^0\to\pi^+\pi^-)$ have to be taken into account to achieve an agreement with the experimental data. The enhancement of the $\Delta I=1/2$ amplitude by O_1-O_4 is due to C_1-C_4 while the reason for a considerable contribution from O_5 is great values of the matrix elements of this operator.

The authors wish to thank G.Nardulli and M.K.Volkov for useful discussions.

APPENDIX A

Analytical expressions for electromagnetic radii of K-mesons

$$\frac{1}{6}\langle r^2\rangle_{K^+}=\frac{16\lambda_K}{3\Lambda_s^2}\,b(0)+\frac{g_{\rho K^+K^-}}{f_\rho m_\rho^2}+\frac{g_{\varphi K^+K^-}}{f_\varphi m_\varphi^2} \qquad (A.1)$$

$$\frac{1}{6}\langle r^2\rangle_{K^0}=\frac{g_{\rho K^0\bar K^0}}{f_\rho m_\rho^2}+\frac{g_{\varphi K^0\bar K^0}}{f_\varphi m_\varphi^2}.$$

$K\to\pi e\nu$ decays form factors

$$F_+(t)=8\sqrt{\lambda_K\lambda_P}\Big[C_B^{(0)}(\mu_u)+\frac{1}{6}b(0,\mu_u)(\mu_K^2-\mu_\pi^2)+\frac{b(0,\mu_u)}{6}+ \qquad (A.2)$$
$$+\frac{2\lambda_V C_B^{(0)}(\mu_u)}{\mu_{K^*}^2-t}\,t\,\Big(C_B^{(0)}+\frac{1}{6}b(0,\mu_u)(\mu_K^2-\mu_\pi^2)\Big)\Big]$$

$$F_-(t)=\frac{4\sqrt{\lambda_K\lambda_P}}{3}(\mu_K^2-\mu_\pi^2)\Big[-b(0,\mu_u)+\frac{1}{10}\frac{d}{dt}b(t,\mu_u)\Big|_{t=0}t-$$
$$-\frac{12\lambda_V C_B^{(0)}(\mu_u)}{\mu_{K^*}^2-t}\Big(C_B^{(0)}(\mu_u)+\frac{1}{6}b(0,\mu_u)(\mu_K^2-\mu_\pi^2)\Big)\Big]$$

$$\lambda_+=\mu_\pi^2\,\frac{\frac{1}{6}b(0,\mu_u)+\frac{2\lambda_V C_B^{(0)}(\mu_u)}{\mu_{K^*}^2}\Big(C_B^{(0)}(\mu_u)+\frac{b(0,\mu_u)}{6}(\mu_K^2-\mu_\pi^2)\Big)}{C_B^{(0)}(\mu_u)+\frac{1}{6}b(0,\mu_u)(\mu_K^2-\mu_\pi^2)} \qquad (A.3)$$

APPENDIX B

The coefficients c_i entering into the effective Hamiltonian of weak interactions are given by the formulae /3/

$$C_1=-\varkappa_1^{4/b}(0.98\varkappa_2^{0.42}+0.01\varkappa_2^{-0.8})+0.04\varkappa_1^{-2/b}(\varkappa_2^{0.42}-\varkappa_2^{-0.3})$$
$$C_2=0.2\varkappa_1^{-2/b}(0.96\varkappa_2^{-0.3}+0.03\varkappa_2^{-0.12})-0.02\varkappa_1^{4/b}(\varkappa_2^{0.42}-\varkappa_2^{-0.3})$$

$$c_3 = \frac{2}{15} \mathcal{H}_1^{-2/b} \mathcal{H}_2^{-2/9}$$

$$c_4 = \frac{2}{3} \mathcal{H}_1^{-2/b} \mathcal{H}_2^{-2/9}$$

$$c_5 = 10^{-2} \mathcal{H}_1^{4/b}(4.8\mathcal{H}_2^{0.42} - 0.6\mathcal{H}_2^{-0.3} - 2.9\mathcal{H}_2^{0.8} - 1.3\mathcal{H}_2^{-0.12}) + $$
$$+ 10^{-2}\mathcal{H}_1^{-2/b}(0.1\mathcal{H}_2^{0.42} + 2.9\mathcal{H}_2^{-0.3} - 1.4\mathcal{H}_2^{0.8} - 1.4\mathcal{H}_2^{-0.12}) \qquad (B.1)$$

$$c_6 = 10^{-2} \mathcal{H}_1^{4/b}(4.8\mathcal{H}_2^{0.42} - 0.6\mathcal{H}_2^{-0.3} - 2.9\mathcal{H}_2^{0.8} - 1.3\mathcal{H}_2^{-0.12}) +$$
$$+ 10^{-2}\mathcal{H}_1^{-2/b}(-0.2\mathcal{H}_2^{0.42} - 5.8\mathcal{H}_2^{-0.3} - 1.0\mathcal{H}_2^{0.8} + 7.0\mathcal{H}_2^{-0.12})$$

Here $b = 11 - \frac{2}{3} N_f$.

$$\mathcal{H}_1 = 1 + b\,\frac{\tilde{g}^2(m_c)}{16\pi^2} \ln \frac{m_W^2}{m_c^2} \qquad (B.2)$$

$$\mathcal{H}_2 = 1 + 9\,\frac{\tilde{g}^2(\mu)}{16\pi^2} \ln \frac{m_c^2}{\mu^2}.$$

Another set of operators suggested in [27] is used in [7,11] instead of the set of operators $O_1 - O_6$ given above

$$Q_1 = (\bar{s}_\alpha d_\alpha)_{V-A} (\bar{u}_\beta u_\beta)_{V-A}$$
$$Q_2 = (\bar{s}_\alpha d_\beta)_{V-A} (\bar{u}_\beta u_\alpha)_{V-A} \qquad (B.3)$$
$$Q_3 = (\bar{s}_\alpha d_\alpha)_{V-A} \sum_{q=u,d,s} (\bar{q}_\beta q_\beta)_{V-A}$$
$$Q_5 = (\bar{s}_\alpha d_\alpha)_{V-A} \sum_{q=u,d,s} (\bar{q}_\beta q_\beta)_{V+A}$$
$$Q_6 = (\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} (\bar{q}_\beta q_\alpha)_{V+A}.$$

The relation between $O_1 - O_6$ and $Q_1 - Q_6$ is [4]

$$Q_1 = -\frac{1}{2} O_1 + \frac{1}{10} O_2 + \frac{1}{15} O_3 + \frac{1}{3} O_4$$

$$Q_2 = \frac{1}{2} O_1 + \frac{1}{10} O_2 + \frac{1}{15} O_3 + \frac{1}{3} O_4$$

$$Q_3 = \frac{1}{2} (O_2 - O_1)$$

$$Q_5 = O_6$$

$$Q_6 = \frac{1}{2} O_5 + \frac{1}{3} O_6. \qquad (B.4)$$

The relation between the coefficients c_i entering into (1.1) and coefficients R_i entering into the effective Hamiltonian from [27] is given by [4]

$$c_1 = 2(R_2^* - R_1^* - R_3^*)$$
$$c_2 = \frac{2}{5}(R_2^* + R_1^*) + 2R_3^* \qquad (B.5)$$
$$c_3 = \frac{4}{15}(R_1^* + R_2^*)$$
$$c_4 = \frac{4}{3}(R_1^* + R_2^*)$$
$$c_5 = 8R_6^*$$
$$c_6 = \frac{4}{3}R_6^* + 4R_5^*.$$

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Авакян Е.З. и др. E2-87-630
Нелептонные распады К-мезонов

Лептонные, полулептонные, нелептонные и электромагнитные распады К-мезонов рассматриваются в модели конфайнмированных кварков. Для описания нелептонных распадов каонов используется эффективный гамильтониан слабого взаимодействия. Вычислены ширины нелептонных и электромагнитных распадов каонов, параметры наклона для полулептонных распадов, электромагнитные радиусы каонов.

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Nonleptonic Decays of K-mesons

Leptonic, semileptonic, nonleptonic and electromagnetic decays of K-mesons are treated in the framework of quark confinement model. Effective Hamiltonian of weak interactions is used for description of nonleptonic decays of kaons. Widths of nonleptonic and electromagnetic kaonic decays, slope parameters for semileptonic decays and electromagnetic radii of kaons are calculated.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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