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MINIMAL QUANTIZATION OF TWO-DIMENSIONAL MODELS WITH CHIRAL ANOMALIES

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Introduction

In the common opinion, anomalous theories have been considered for a time as mathematically interesting but physically unacceptable constructions. So, cancellation of anomalies in the local symmetries has been used as a kind of "selection" rule or a criterion for the physical consistency of different models. This principle was plausible in the GWS model and in D = 10 supergravity without matter fields. Nowadays, it is widely used for fixing the gauge group in superstring theories.

The problem with anomalous theories is in their consistent quantization. Since not all classical symmetries are respected on the quantum level, one finds that some of the first-class constraints have become second-class ones, which is crucial for the quantization procedure itself. Some progress in our understanding of anomalies has been achieved with the observation that a scalar field, added by hand to the action through an additional Wess-Zumino term, preserves the constraints to be of the first class after quantization. This maintains gauge invariance on the quantum level [1]. Later it has been shown [2,3] that the W-Z term is an indispensable ingredient of the theory whose presence is simply explained by the impossibility of neglecting the gauge-group volume in the Faddeev-Popov integral in the case of anomalous theories contrary to the case of anomaly-free ones. In any way, it becomes clear that quantum and classical theories may have a different number of degrees of freedom, since some gauge transformations acquire the status of a physical field when gauge-noninvariant interactions are present.

These new developments revived interest in two-dimensional gauge models as a playground where the ideas and mechanisms of gauge-symmetry restoration, anomaly cancellation and consistency of the solution can be verified. For example, the chiral Schwinger model was largely discussed in different approaches because of the contradictory results [4,5]. Recently, both its gauge invariance and non-anomalous nature were argued [6].

In the present paper an approach to the quantization of anomalous two-dimensional gauge models is developed. It is based on the explicit solution of the constraint equations and controls the gauge invariance at each stage. The paper is organized as follows: Section 1 presents a brief review of the minimal quantization method of gauge theories. In sections 2,3 the "left-handed" QED_{1+1} and the chiral Schwinger model are considered. A table of the anomalous commutators in D = 2 is given in the Appendix.

1. The minimal quantization method

The minimal quantization method /7/ is based on the gauge-invariance principle used for the construction of physical variables and in the choice of the energy-momentum tensor. The method is self-consistent in the sense that ensures the same transformation properties under Lorentz transformations for classical and quantum variables. The gauge freedom is reduced to an algebraic one connected with the choice of the time axis of quantization.

Let us illustrate the method by the example of the Schwinger model-massless two-dimensional QED:

$$\mathcal{L} = -\frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} + i\bar{\Psi}\gamma^{\mu}(\partial_{\mu} - ieA_{\mu})\Psi \tag{1}$$

$$\mathcal{F}_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}, \quad \mu,\nu = 0,1.$$

According to the minimal quantization method, we exclude the nondynamical field A_0 from (1) through the constraint

$$\frac{\delta S}{\delta A_0} = 0 \quad \Rightarrow \quad A_0 = \frac{1}{\partial_1^2}\left(\partial_1\partial_0 A_1 + e j_0\right). \tag{2}$$

Then, a transverse projection is done with the operator $v(A_1)$, $v = \exp\{-ie\partial_1^{-1}A_1\}$, following from (2)

$$\begin{aligned} A_1^T &= v\left(A_1 + \frac{i}{e}\partial_1\right)v^{-1} \\ \Psi^T &= v\Psi, \qquad \bar{\Psi}^T = \bar{\Psi}v^{-1} \end{aligned} \tag{3}$$

and the effective Lagrangian is obtained

$$\mathcal{L}^T_{eff} = \frac{e^2}{2}\left(\partial_1^{-1}j_0\right)^2 + i\bar{\Psi}^T\gamma^{\mu}\partial_{\mu}\Psi^T. \tag{4}$$

We would like to emphasize the main feature of the transverse variables (3) – their invariance under gauge transformations of the initial fields A_{μ}, Ψ /8/

$$\left.\begin{aligned} A_1^g &= g\left(A_1 + \frac{i}{e}\partial_1\right)g^{-1} \\ \Psi^g &= g\Psi \\ \bar{\Psi}^g &= \bar{\Psi}g^{-1} \\ v^g &= v(A_1^g) = vg^{-1} \end{aligned}\right\} \Rightarrow \begin{aligned} A_1^{T,g} &= v^g\left(A_1^g + \frac{i}{e}\partial_1\right)(v^g)^{-1} = A_1^T \\ \Psi^{T,g} &= v^g\Psi^g = \Psi^T \\ \bar{\Psi}^{T,g} &= \bar{\Psi}^g(v^g)^{-1} = \bar{\Psi}^T. \end{aligned}$$

The field A_{μ} disappeared from $\mathcal{L}$, thus reflecting the absence of transverse degrees of freedom in the two-dimensional world. In the general case we are left with the fields Ψ^T, $\bar{\Psi}^T$, A_i^T and quantize just them and not the initial ones. It is important to note that vacuum has to be defined so as to ensure the positive definiteness of the free fermion Hamiltonian. This task is achieved by filling in the negative energy states in the Dirac sea, i.e. by introducing the Dirac vacuum. The price is the anomalous term in the current commutator though Ψ's themselves canonically anticommute /9/:

$$\{\Psi(x), \Psi^+(y)\} = \delta(x-y) \quad ; \quad j_{\mu} = \bar{\Psi}(x)\gamma_{\mu}\Psi(x) \tag{5}$$

$$[j_1(x), j_0(y)] = \frac{1}{\pi i}\partial_y\delta(x-y).$$

Such a commutator structure allows an equivalent formulation of the model in terms of a massive scalar field $\varphi(x)$ through the correspondence relation

$$j_{5\mu}(x) = \frac{1}{\sqrt{\pi}}\partial_{\mu}\varphi(x) \quad ; \quad j_{5\mu}(x) = \bar{\Psi}(x)\gamma_5\gamma_{\mu}\Psi(x).$$

So, the mass spectrum is easily found with the help of the Heisenberg equation of motion for $j_{50}(x)$:

$$\partial_0 j_{50}(x) = i\int dy\,[\mathcal{H}^T(y), j_{50}(x)] = \partial_1 j_{51} - \frac{e^2}{\pi}\partial_1^{-1}j_{51}$$

($\mathcal{H}^T$ being the Hamiltonian obtained from eq. (4)) that gives us

$$\partial^{\mu}j_{5\mu}(x) = -\frac{e^2}{\pi}\partial_1^{-1}j_{51}(x) \text{ or } (\Box + m^2)\varphi(x) = 0, \quad m^2 = \frac{e^2}{\pi}. \tag{6}$$

In the same manner the conservation of the vector current j^{μ} may be proved

$$\partial^0 j_0(x) = i[H^T, j_0(x)] = \partial_1 j_1 \quad \Rightarrow \quad \partial^{\mu}j_{\mu} = 0. \tag{7}$$

Note, that in this scheme the role of the Dirac sea is evident since the filling of the negative-energy states is the reason for the appearance of the Schwinger term in (5) and further polarization of this vacuum by the gauge (Coulomb) field leads to the anomaly (6).

The minimal-quantization method is based on the excluding of nondynamical degrees of freedom through their equations of motion (in fact, constraints). So, the choice of the time axis of quantization becomes an essential step that has to be physically motivated. This is the only arbitrariness of our method and it is to be considered separately in each case.

This operator method can be translated into a functional integral language. Let us write the Green-function generating functional

$$Z[\bar{\eta},\eta] = \int \mathcal{D}\bar{\Psi}\mathcal{D}\Psi\mathcal{D}A_\mu \exp\{i\int d^2x(\mathcal{L} + \bar{\Psi}\eta + \bar{\eta}\Psi)\}$$

with $\mathcal{L}$ given by eq. (1). Transition to transverse variables (3) does not cause any change in the fermionic measure since vector gauge invariance is present. However, as $A_1^T = 0$, we have

$$\mathcal{D}A_\mu \to \mathcal{D}A_0^T.$$

Solving the constraint for A_0 is equivalent to integrating over $\mathcal{D}A_0^T$ in the sense of extremals. So, we are led to the following $Z^T[\bar{\eta}^T,\eta^T]$:

$$Z[\bar{\eta}^T,\eta^T] = \int \mathcal{D}\bar{\Psi}^T\mathcal{D}\Psi^T \exp\{i\int d^2x(\mathcal{L}^T_{eff} + \bar{\eta}^T\Psi^T + \bar{\Psi}^T\eta^T)\}$$

with

$$\eta^T = v\eta$$
$$\bar{\eta}^T = \bar{\eta}v^{-1}$$

and $\mathcal{L}_{eff}$ given by eq. (4). Now, the bosonization is performed[10]

$$\Psi^T(x) = :e^{i\sqrt{\pi}\gamma_5(\phi(x)+\Sigma(x))}:\chi_0(x) \quad , \quad i\not{\partial}\chi_0(x) = 0,$$

and we finally obtain

$$Z[\bar{\eta}^T,\eta^T] = \int \mathcal{D}\bar{\chi}_0\mathcal{D}\chi_0\mathcal{D}\phi\mathcal{D}\Sigma \exp\{i\int d^2x[\tfrac{1}{2}\partial_\mu\phi\partial^\mu\phi -$$
$$-\tfrac{1}{2}m^2\phi^2 - \tfrac{1}{2}\partial_\mu\Sigma\partial^\mu\Sigma + i\bar{\chi}_0\gamma^\mu\partial_\mu\chi_0 + \mathcal{L}_{(s)}]\} \qquad (8)$$
$$\mathcal{L}_{(s)} = \bar{\eta}^T :e^{i\sqrt{\pi}\gamma_5(\phi+\Sigma)}:\chi_0 + \bar{\chi}_0 :e^{i\sqrt{\pi}\gamma_5(\phi+\Sigma)}:\eta.$$

Here ϕ is a massive scalar field and Σ is a massless one, quantized with an indefinite metrics. Its introduction is connected with the consistence of bosonization procedure on the Green-function level in the case of the free fermion theory.

Generating functional (8) leads to the correct two-point fermion Green function in the model[10]

$$G(x-y) = e^{-i\pi[\Delta_m(x-y) - \Delta_0(x-y)]}G_0(x-y),$$

where Δ_m, Δ_0 and G_0 are the free Green functions of a massive and massless scalar field and massless spinor field respectively.

Note, that the fermionic measure in (8) is not changed again since the transverse spinors Ψ^T are invariant under arbitrary gauge transformation. So, the change of the action contains the whole physically significant information.

However this is not the case in an anomalous theory. Due to the anomaly, transition to transverse variables in Z is accompanied by a Jacobian factor apart of that, comming from the bosonization. Thus, the operator formulation is more convenient for consideration of anomalous theories.

2. Two-dimensional U(1) theory with left-handed fermions

In the recent discussion of the chiral Schwinger model two different (though similar) theories have been considered: a model with only left-handed fermions and a model with left-current coupling to the gauge field. We shall treat them separately.

The "left-handed" QED_{1+1} is determined by *)

$$\mathcal{L} = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\Psi}_L\gamma^\mu(\partial_\mu - ieA_\mu)\Psi_L =$$
$$= \tfrac{1}{2}F_{01}^2 + i\Psi_L^+\partial_+\Psi_L + e\Psi_L^+\Psi_L A_+. \qquad (9)$$

So, we have two possible candidates for a time axis: x^0 and $x^+ = x^0 + x^1$. As has been mentioned above, this is the only ambiguity in our quantization procedure.

The choice $t = x^0$ leads to the constraint

*) Throughout this paper the following matrix convention is adopted

$$\gamma_0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \quad \gamma_1 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}; \quad \gamma_5 = \gamma_0\gamma_1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}; \quad \Gamma_\pm = \tfrac{1}{2}(1 \pm \gamma_5)$$

and

$$g_{00} = -g_{11} = 1 \quad ; \quad \Psi_{L(R)} = \Gamma_{-(+)}\Psi.$$

$$\frac{\delta S}{\delta A_0} = 0 \quad \Rightarrow \quad A_0 = \frac{1}{\partial_1^2}\left(\partial_1\partial_0 A_1 + e\mathcal{J}_0^+\right), \tag{10}$$

where

$$\mathcal{J}_\mu^+ = \bar{\Psi}\Gamma_+\gamma_\mu\Psi = \frac{1}{2}\left(j_\mu + j_{5\mu}\right) \quad ; \left(\mathcal{J}_0^+ = -\mathcal{J}_1^+ = \Psi_L^+\Psi_L = \Psi^+\Gamma_-\Psi\right)$$

is just the Noether current in the theory. Lagrangian (9) on the solution of (10) becomes

$$\mathcal{L} = i\bar{\Psi}\Gamma_+\gamma^\mu\partial_\mu\Psi + \frac{e^2}{2}\mathcal{J}_0^+\frac{1}{\partial_1^2}\mathcal{J}_0^+ + e\mathcal{J}_0^+\partial_1^{-1}\partial_0 A_1 - e\mathcal{J}_1^+ A_1$$

and for the Hamiltonian we find

$$\mathcal{H} = i\bar{\Psi}\Gamma_+\gamma_1\partial_1\Psi - \frac{e^2}{2}\mathcal{J}_0^+\frac{1}{\partial_1^2}\mathcal{J}_0^+ + e\mathcal{J}_1^+ A_1 .$$

It is easily seen that neither a vector nor a (left) chiral gauge invariance takes place here

$$\begin{aligned}\partial_0 j_0(x) &= i\left[H, j_0(x)\right] = \partial_1\mathcal{J}_1^+ + \frac{e^2}{2\pi}\partial_1^{-1}\mathcal{J}_0^+ \\ &\Rightarrow \partial^\mu j_\mu = -\partial_1\mathcal{J}_1^- + \frac{e^2}{2\pi}\partial_1^{-1}\mathcal{J}_0^+\end{aligned} \tag{11a}$$

and

$$\partial_0\mathcal{J}_0^+(x) = i\left[H, \mathcal{J}_0^+(x)\right] = \partial_1\mathcal{J}_1^+(x) + \frac{e^2}{2\pi}\partial_1^{-1}\mathcal{J}_0^+$$

$$\Rightarrow \partial^\mu\mathcal{J}_\mu^+ = \frac{e^2}{2\pi}\partial_1^{-1}\mathcal{J}_0^+ . \tag{11b}$$

In these calculations the anomalous commutators have been taken into account (see Appendix)

$$\left[\mathcal{J}_0^+(x), \mathcal{J}_0^+(y)\right] = \left[\mathcal{J}_1^+(x), \mathcal{J}_1^+(y)\right] = \frac{1}{2\pi i}\partial_x\delta(x-y) \tag{12a}$$

$$\left[\mathcal{J}_0^+(x), \mathcal{J}_1^+(y)\right] = \frac{1}{2\pi i}\partial_y\delta(x-y) \tag{12b}$$

$$\left[H_0, \mathcal{J}_0^+(x)\right] = -\left[H_0, \mathcal{J}_1^+(x)\right] = \frac{ie}{2\pi}\partial_1 A_1(x). \tag{12c}$$

There are different ways to compute them. We have followed the BJL--method (see, for example,/11/).

Since just the current that is coupled to the gauge field, is

anomalous, one cannot expect that transition to transverse variables will make the model content more transparent on the Hamiltonian level. Indeed, one finds

$$\mathcal{H}^T = i\bar{\Psi}^T\Gamma_+\gamma_1\partial_1\Psi^T - \frac{e^2}{2}\mathcal{J}_0^+\frac{1}{\partial_1^2}\mathcal{J}_0^+ \tag{13}$$

with the same expression for the chiral-current divergence (11b).

Relations (13),(11b) are very similar to (4), (6). Nevertheless, an analogous interpretation in terms of a bose field is impossible due to eq. (11a). It destroys the canonical structure of the pair $\{\varphi, \pi = \partial_0\varphi\}$, defined through the relation

$$\mathcal{J}_\mu^+(x) = \frac{1}{\sqrt{2\pi}}\partial_\mu\varphi(x)$$

since commutators $[\varphi(x), \varphi(y)]$ and $[\pi(x), \pi(y)]$ do not vanish.

The other possibility for t is $t = x^+$. The Lagrangian takes the form

$$\mathcal{L} = \frac{1}{8}\left(\partial_- A_+ - \partial_+ A_-\right) + i\Psi_L^+\partial_+\Psi_L + e\Psi_L^+\Psi_L A_+ . \tag{14}$$

Instead of (10) we have the constraint

$$\frac{\delta S}{\delta A_+} = 0 \quad \Rightarrow \quad A_+ = \frac{1}{\partial_-^2}\left(\partial_-\partial_+ A_- + 4e\mathcal{J}_+\right) \tag{15}$$

and the Lagrangian on its solution becomes

$$\mathcal{L} = i\Psi_L^+\partial_+\Psi_L + 2e^2\mathcal{J}_+\frac{1}{\partial_-^2}\mathcal{J}_+ + e\mathcal{J}_+\partial_-^{-1}\partial_+ A_- . \tag{16a}$$

The corresponding Hamiltonian is

$$\mathcal{H} = -2e^2\mathcal{J}_+\frac{1}{\partial_-^2}\mathcal{J}_+ . \tag{16b}$$

Since the only anomalous commutator in the cone frame is (see Appendix)

$$\left[\mathcal{J}_+(x^-), \mathcal{J}_+(y^-)\right] = -\frac{1}{2\pi i}\partial_-\delta(x^- - y^-)$$

the Heisenberg equation with x^+ as a physical time and Hamiltonian (16b) determine the following time evolution of $\mathcal{J}_+$:

$$\partial_+\mathcal{J}_+ = \frac{i}{2}\left[H, \mathcal{J}_+(x^-)\right] = -\frac{e^2}{\pi}\partial_-^{-1}\mathcal{J}_+(x^-) .$$

In other words, $\mathcal{J}_+$ satisfies the Klein-Gordon equation

$$(\Box + m^2/4)\,\mathcal{J}_+ = 0$$

$$\Box = \partial_+\partial_- \, , \qquad m^2 = \frac{4e^2}{\pi} \, .$$

Recalling the one-parameter formula for the mass spectrum of the model[12]

$$m^2 = \frac{e^2}{\pi}\cdot\frac{a^2}{a-1} \tag{17}$$

one easily sees that we have just singled out the value $a = 2$.

Fig. 1 gives some evidence for the physical motivation of the choice of x^+ as a time axis. The only physical field Ψ_L propagates just along this axis and the only interaction that is present in the model has a Coulomb structure

a) with respect to the frame (x^0, x^1) when $t = x^0$,

b) with respect to the cone frame when $t = x^+$.

So, it is not surprising that the choice of case (b) ensures internal consistency of our further considerations.

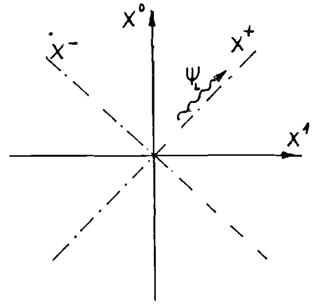

Fig. 1

In such a way, the minimal quantization method leads to a consistent unitary theory with mass generation. The massive mode represents a fermion bound state and the value of the mass is fixed uniquely.

In the functional integral language the situation looks out much more complicated. The transition into the transverse space is connected with a nontrivial Jacobian factor. Here we mean consideration of quantum fermions in an external field which has to be consistently quantized. If we start from a classical system and try to quantize it as a whole, these troublesome transformations can be performed on a classical level without any influence on the fermionic measure, which is constructed directly as $\mathcal{D}\bar\Psi^T\mathcal{D}\Psi^T$, thus simplifying the problem. However, the latter treatment in fact ignores the anomalous nature of the model.

3. The chiral Schwinger model

The chiral version of two-dimensional QED is determined by the following Lagrangian:

$$\mathcal{L} = -\frac14 F_{\mu\nu}F^{\mu\nu} + i\bar\Psi\gamma^\mu\Big[\partial_\mu - ie\frac{1-\gamma_5}{2}A_\mu\Big]\Psi = \frac12 F_{01}^2 + i\Psi_R^+\partial_-\Psi_R + i\Psi_L^+\partial_+\Psi_L + e\Psi_L^+\Psi_L A_+ \, . \tag{18}$$

Though only left current is coupled to the gauge field, the presence of R-fermion term in (18) changes significantly the situation if compared to the one of the previous section.

Let us begin with the case $t = x^0$. The constraint equation is the same as (10) and on its solution (18) becomes

$$\mathcal{L} = \frac{e^2}{2}\mathcal{J}_0^+\frac{1}{\partial_1^2}\mathcal{J}_0^+ + i\bar\Psi\gamma^\mu\partial_\mu\Psi + e\mathcal{J}_0^+\partial_1^{-1}\partial_0A_1 - e\mathcal{J}_1^+A_1 \, , \tag{19}$$

where the notation of section 2 is used. So, for the Noether current in the model we find

$$\partial^\mu j_\mu = -\frac{e}{2\pi}\partial_1A_1 + \frac{e^2}{2\pi}\partial_1^{-1}\mathcal{J}_0^+ \, . \tag{20}$$

For the left and right current we find, respectively,

$$\partial^\mu\mathcal{J}_\mu^+ = \frac{e^2}{2\pi}\partial_1^{-1}\mathcal{J}_0^+$$
$$\partial^\mu\mathcal{J}_\mu^- = -\frac{e}{2\pi}\partial_1A_1 \, . \tag{21}$$

Eq. (21) reflects the presence of the R-fermion term in the free Hamiltonian. Though this current is not coupled to the gauge field, its anomaly can be removed in a suitable gauge. This point is new as compared to the case in the above section as well as to the nonanomalous theories (for example, the situation with the axial current in the ordinary Schwinger model).

The fermion asymmetry suggests two types of "transverse" variables

(A)
$$A_1^T = \vartheta\Big(A_1 + \frac{i}{e}\partial_1\Big)\vartheta^{-1}$$
$$\Psi_L^T = \vartheta\Psi_L$$
$$\Psi_R^T = \vartheta\Psi_R$$

and

(B)
$$A_1^T = \vartheta\Big(A_1 + \frac{i}{e}\partial_1\Big)\vartheta^{-1}$$
$$\Psi_L^T = \vartheta\Psi_L$$
$$\Psi_R^T = \Psi_R \, ,$$

where $\vartheta = \exp\{-ie\,\partial_1^{-1}A_1\}$.

So, we obtain two different Hamiltonians

$$\mathcal{H}^{(T,A)} = i\bar{\Psi}^{(T,A)}\gamma_1\partial_1\Psi^{(T,A)} - \frac{e^2}{2}\mathcal{J}_0^+\frac{1}{\partial_1^2}\mathcal{J}_0^+ - e\mathcal{J}_1^-A_1$$

$$\mathcal{H}^{(T,B)} = i\bar{\Psi}^{(T,B)}\gamma_1\partial_1\Psi^{(T,B)} - \frac{e^2}{2}\mathcal{J}_0^+\frac{1}{\partial_1^2}\mathcal{J}_0^+. \tag{22}$$

Note that in $\mathcal{H}^{(T,A)}$ a new interaction term is present which points the difference between the transverse projection and the gauge fixing in this anomalous model, Of course, the same relations (20), (21) for current divergences take place.

Thus, we are faced with a situation that is analogous to the one in the "left-handed" QED_{1+1}. In fact, Ψ_L's and Ψ_R's propagate separately along X^+ and X^-, respectively. Right fermions do not interact and the interaction between left currents is of a Coulomb type with respect to the frame (x^0, x^1) (case "a"). So, a suitable rotation of the coordinate frame seems natural with X^+ as a time axis (case "b"). The Lagrangian then takes the form

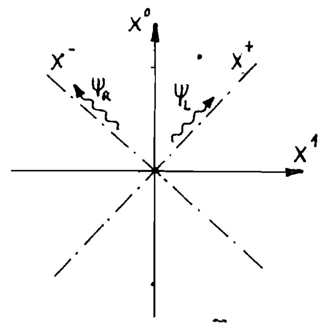

Fig. 2

$$\mathcal{L} = \frac{1}{8}\mathcal{F}_{-+}^2 + i\Psi_L^+\partial_+\Psi_L + i\Psi_R^+\partial_-\Psi_R + e\mathcal{J}_+A_+ \tag{23}$$

and the equations for nondynamical variables A_+ and Ψ_R are

$$\partial_-^2A_+ = \partial_-\partial_+A_- + 4e\mathcal{J}_+$$

$$\partial_-\Psi_R = 0.$$

On their solutions Lagrangian (23) coincides with the corresponding expression (16a) in left-handed QED_{1+1}. So, a massive scalar mode with $m^2 = 4e^2/\pi$ is found.

The R-fermions which propagate freely along the X^- axis give rise to an additional massles mode.

There is one more possible choice: $t = X^-$ (case "c"), that

means A_- and Ψ_L are the nondynamical variables with the following equations of motion :

$$\partial_+A_- = \partial_-A_+ \quad \Rightarrow \mathcal{F}_{-+} \equiv 0$$

$$\partial_+\Psi_L = 0 \quad (\text{or } \partial_+\Psi_L^T = 0, \quad \Psi_L^T = \nu\Psi_L)$$

Lagrangian (23) takes on their solutions the form

$$\mathcal{L} = i\Psi_R^+\partial_-\Psi_R.$$

So, we are dealing with a free fermion theory, which is equivalent, as is known, to a free massless scalar field one.

Conclusions

We have used the ideas of the minimal quantization method in the case of anomalous gauge models in two-dimensional space-time. The fermion asymmetry in the models under consideration puts the problem of a correct choice of the time axis of the system we are going to quantize in. This step is essential since it determines the structure of the constraints in the theory (the equations of motion for its nondynamical variables). Remind that it is just the Gauss law (the constraint that is connected with the nondynamical component of the gauge field) that causes troubles in the anomalous theories.

We have shown that the natural and physically motivated choice is the lightcone frame. In this frame the Coulomb interaction (the only one that is present in these models) is consistent with the fermion propagation direction.

Quantization then leads to the mass spectrum containing

- a massive scalar particle (fermions bound state) with $m^2 = 4e^2/\pi$ in "left-handed" QED_{1+1};
- a massive ($m^2 = 4e^2/\pi$) and a massless scalar modes in the chiral Schwinger model.

We have obtained consistent, unitary theories with uniquely determined mass spectra. The value $a = 2$ of the regularization parameter in the mass formula is singled out (see, also [13]) due to the requirement for self-consistence of the minimal quantization method. So, this requirement plays an analogous role with the gauge-invariance principle which fixes the value of the parameter a ($a = 0$) in the case of anomaly free theory.

I express my gratitude to Dr.V.N.Pervushin for introducing me to this problem and for helpful suggestions. I would also like to thank Profs.B.M.Barbashov and A.V.Efremov for valuable discussions and A.V. Radyushkin and E.A.Ivanov for fruitful conversations.

Appendix

To calculate the anomalous commutators in the models under consideration we use the method of ref. /11/. We consider currents j_μ and the free Hamiltonian $\mathcal{H}_0$ as local limits of the following functionals after their regularization

$$J(F) = \int dx\, dy\, \Psi^+(x) F(x,y) \Psi(y) \Gamma$$

$$H(F_1) = \int dx\, dy\, \Psi^+(x) F_1(x,y) \Psi(y) \Gamma \quad ; \quad F_1(x,y) = \partial_y F(x,y),$$

where

$$F(x_1,y_1) = (2\pi)^{-2} \int dk_1 dq_1\, e^{-ik_1 \frac{x_1+y_1}{2}} e^{-iq_1(x_1-y_1)} \tilde{F}(k_1,q_1)$$

$$\Gamma = \begin{cases} 1 & \text{for} \quad J_0(F) \longrightarrow j_0 \\ \gamma_5 & \text{for} \quad J_1(F) \longrightarrow j_1 \\ \gamma_5 & \text{for} \quad H(F_1) \longrightarrow \mathcal{H}_0 . \end{cases} \tag{A.1}$$

For currents $J_\mu^\pm$ this Γ-matrix acquires additional projection factors $\Gamma_\pm$. The change of the fermion Green function under vector and axial gauge transformations is taken into account

$$\Psi(x) \to e^{i\alpha(x)} \Psi(x)$$

$$S_0(x-y) \to e^{-i\alpha(x)} S_0(x-y) e^{i\alpha(y)}. \tag{A.2}$$

The anomalous term in commutator $[J(F), J(G)]$ is calculated as

$$\mathcal{O}(F,G) = -\,tr[F,G]\,\mathcal{T} \frac{1}{i} S^{inf}(x-y)\Big|_{x^0-y^0=0} \gamma_0 \quad ; \quad \mathcal{T} = \Gamma_F \Gamma_G .$$

So, the following commutation relations are obtained

$$[J_0^+(x), J_0^+(y)] = [J_1^+(x), J_1^+(y)] = \frac{1}{2\pi} \partial_x \delta(x-y)$$

$$[J_0^+(x), J_1^+(y)] = -[J_0^+(x), J_0^+(y)]$$

$$[H_0, J_0^+(x)] = -i\, \partial_1 J_1^+ + \frac{i}{2\pi} \partial_1^2 \alpha(x)$$

$$[H_0, J_1^+(x)] = -i\, \partial_1 J_0^+(x)$$

which have to be used when gauge transformations are performed on the operator level.

Note, that in terms of the transverse variables Ψ^T there are no anomalous terms in commutators with H_0. The reason is in the invariance of Ψ^T under gauge transformations of Ψ's, hence, in the absence of relation (A.2).

Calculations in terms of cone variables with x^+ as a physical time can be performed in a similar way with the following differences taken into account:

i) instead of (A.1) we have

$$F(x^-,y^-) = (2\pi)^{-2} \int dk_-\, dq_-\, e^{ik_- \frac{x^-+y^-}{2}} e^{iq_-(x^- - y^-)} \tilde{F}(k_-,q_-)$$

ii) the equal-time Green function has the form

$$S(x^-)\Big|_{x^+=+0} = -\frac{\gamma_+}{4\pi} \int \theta(p_-)\, e^{\frac{1}{2} i p_- x^-} dp_- .$$

So, we find the only anomalous commutator

$$[J_+(x^-), J_+(y^-)] = -\frac{1}{2\pi i} \partial_- \delta(x^- - y^-) ,$$

which has to be used in solving the corresponding Heisenberg equation

$$\partial_+ \mathcal{O} = \frac{i}{2} [H, \mathcal{O}]$$

that follows from the representation of the wave function as

$$\Psi = \int dp\, F(p_-)\, e^{-\frac{1}{2} i (p_- x^- + p_+ x^+)} .$$

The Klein - Gordon equation in this case reads

$$\left(\partial_+ \partial_- + \frac{1}{4} m^2\right) \Psi = 0 .$$

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Илиева Н. E2-87-588
Минимальное квантование двумерных моделей
с киральными аномалиями

В рамках метода минимального квантования рассмотрены двумерные модели с киральной аномалией - "левая" КЭД и киральная модель Швингера. Обоснован выбор конусного времени как физического времени системы квантования. Получен хорошо известный массовый спектр, но с фиксированным значением регуляризационного параметра а = 2. Такая однозначность является результатом жесткого требования согласованности в методе минимального квантования, отражающегося в физически обоснованном выборе оси времени.

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Ilieva N. E2-87-588
Minimal Quantization of Two-Dimensional
Models with Chiral Anomalies

Two-dimensional gauge models with chiral anomalies - "left-handed" QED and the chiral Schwinger model, are quantized consistently in the frames of the minimal quantization method. The choice of the cone time as a physical time for system of quantization is motivated. The well-known mass spectrum is found but with a fixed value of the regularization parameter a=2. Such a unique solution is obtained due to the strong requirement of consistency of the minimal quantization that reflects in the physically motivated choice of the time axis.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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