объединенный
институт
ядерных
исследований
дубна

E2-87-555

V.K.Mitrjushkin, M.Müller-Preussker*,
A.M.Zadorozhny

MAGNETIC FLUCTUATIONS IN THE QUANTIZED VACUUM OF THE GEORGI-GLASHOW MODEL ON THE LATTICE

Submitted to "Physics Letters B"

* Humboldt-Univesity, Berlin, DDR

1987

1

The quantized vacuum of non-abelian gauge theories is well-known to have a complicated structure, which cannot be recovered by perturbation theory. In spite of the progress in understanding nonperturbative phenomena in QCD, like chiral symmetry breaking, the solution of the $U_A(1)$ -problem or the deconfinement phase transition, we have not yet arrived at a clear pattern of the typical vacuum fluctuations. At present, there are several competing schemes describing the vacuum: instanton gas models[1], the Copenhagen vacuum[2], the chromomagnetic superconductor model[3], and others.

The lattice discretization of gauge theories provides us with a model-independent way of studying the relevant vacuum fields. That instantons as well as monopoles play an important role in the pure Yang-Mills vacuum has been realized very recently by "cooling down" Monte-Carlo generated equilibrium fields[4]. But the immediate investigation of the equilibrium configurations with respect to their (electro) magnetic structure yields valuable information, as well. This has been shown for the Georgi - Glashow model[5] and for pure Yang - Mills theories[6], respectively.

Within this letter we study the influence of electromagnetic fluctuations of the phase structure of the 4D- Georgi - Glashow model on the lattice[5,7,8]. To a certain extent we follow the reasoning given in Ref[5]. In more detail we are going to characterize the fluctuations by investigating correlations between different components of the electromagnetic field tensor. Moreover, we want to see whether monopole-like objects can really be substantiated in different phases.

2

The (Euclidean) action of the Georgi - Glashow model on the lattice can be defined as

$$S = \sum_n \mathcal{S}_n = \beta \sum_{n;\mu>\nu} \left(1 - \tfrac{1}{2} tr U_{n\mu\nu}\right) + \sum tr\left(\Phi_n^+\Phi_n - \Phi_n U_{n\mu} \Phi_{n+\hat\mu} U_{n\mu}^+\right) + \tag{2.1}$$

$$+ \lambda \sum_n' \left(tr(\Phi_n^+\Phi_n) - 2f^2\right)^2,$$

where $\Phi_n = i\vec{\Phi}_n\vec{\sigma}$ denotes the Higgs field (in the adjoint representation) at lattice site n ; $U_{n\mu} \in SU(2)$ and $U_{n\mu\nu} = U_{n\mu} U_{n+\hat\mu,\nu} U^+_{n+\hat\nu,\mu} U^+_{n\nu}$ are the link and plaquette variables, respectively. Additionally to the (bare) couplings $\beta = 4/g^2$, λ and f^2 let us introduce $m^2 = -8\lambda f^2$ taken to be negative. We study a finite 6^4 lattice with periodic boundary conditions. The model is quantized by functional integration, i.e.

$$\langle \Omega \rangle = Z^{-1} \int \prod_n d^3\vec{\Phi}_n \prod_{n,\mu} \{dU_{n\mu}\}\, \Omega \exp\{-S\}\,; \quad \text{with } \langle 1 \rangle = 1 \tag{2.2}$$

$\{dU_{n\mu}\}$ representing the Haar measure of $SU(2)$. The phase structure w.r. to the bare couplings has been studied by measuring the order parameters $\langle tr U_{n\mu\nu}\rangle$, $\langle R^2 \rangle = \frac{1}{2}\langle tr \Phi_n^+\Phi_n\rangle$ and $\langle tr \Phi_n^+ U_{n\mu}\Phi_{n+\hat\mu} U_{n\mu}^+\rangle$ /5,8/. Fig. 1 presents the phase diagram at fixed λ = 0.2 /8/ . The solid line corresponds to a transition of the first order established by a hysteresis behaviour of the order parameters /8/.

At sufficiently large values of β the jump becomes still lower until it is comparable with the statistical errors. In this case we cannot state of which order (second or higher) a transition takes place (if any). The "horizontal" dashed lines in Fig.1 correspond to this case. The order of the other transitions ("vertical" lines) has not been exactly determined, so far. Thus, we have four phases denoted by A,B,C and D . Phases C and D are characterized by fluctuations of the Higgs field modulus $R = (\frac{1}{2} tr \Phi^+\Phi)^{1/2}$ around $\langle R \rangle \sim f$, i.e. near the minimum of the classical Higgs potential. On the contrary, in phases A and B, $\langle R \rangle$ is definitely smaller than f but yet different from zero due to the measure $R^2 dR$ in (2.2). Moreover, phase C shows so-called spontaneous symmetry breaking /9/ . Therefore, we call it Higgs phase. The transition C $\longrightarrow$ D "restores" the symmetry.

In the following we are going to investigate the phase structure with respect to the behaviour of magnetic and electric fluxes related

to that component of the non-Abelian gauge field which is associated with the $U(1)$ group of rotations around $\vec{\Phi}$. We define the electromagnetic field tensor on the lattice by ($\hat\Phi_n^a = \Phi_n^a / |\vec{\Phi}_n|$)

$$F_{\mu\nu}(n) = \frac{1}{a^2 g}\left[tr(\hat\Phi_n U_{n\mu\nu}) - \tfrac{1}{2} tr(\hat\Phi_n U_{n\mu} \hat\Phi_{n+\hat\mu} U_{n\mu}^+ U_{n\nu} \Phi_{n+\hat\nu} U_{n\nu}^+)\right]. \tag{2.3}$$

Within the naive continuum limit (lattice spacing $a \to 0$) this gauge invariant expression corresponds to the definition originally introduced by 't Hooft /10/ (see, e.g. /11/). Thus it should be the mostly suitable definition for detecting monopole-like fluctuations of the 't Hooft - Polyakov type on the lattice. Using expression (2.3) we define the magnetic and the electric fluxes out of an elementary 3D cube at site n , respectively, by

$$M_c(n) = -\frac{g}{2}\int_{\Sigma_c} dS_i\, \varepsilon_{ijk} F_{jk} = g a^2 \left[-F_{23}(n) + F_{23}(n+\hat1) + F_{13}(n) - F_{13}(n+\hat2) - F_{12}(n) + F_{12}(n+\hat3)\right] \tag{2.4}$$

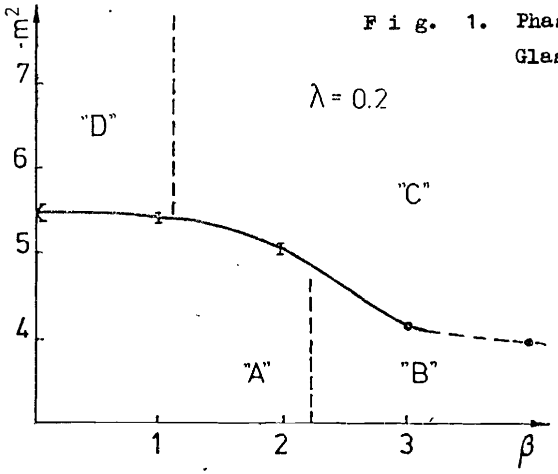
$$E_c(n) = g \int_{\Sigma_c} dS_i F_{i4} = g a^2 \left[F_{14}(n) - F_{14}(n+\hat1) + F_{24}(n) - F_{24}(n+\hat2) + F_{34}(n) - F_{34}(n+\hat3)\right].$$

Within the continuum the magnetic flux M becomes quantized by homotopy arguments $M \to 4\pi N$, $N = 0, \pm1, \pm2, \ldots$ if the closed surface Σ completely lies in regions, where the (energy) density $\mathcal{S}$ (cf. (2.1)) vanishes (i.e. classical Higgs vacuum). On the lattice there is no quantization for the definition M_c given above. In the following we shall measure distributions for detecting M_c- and E_c -values as well as simply for values of the flux $M_\square = g a^2 F_{\mu\nu}$ through elementary plaquettes. The latter, of course, does not depend on the (space-like or time-like) orientation of the plaquettes considered. Let us denote the distributions by $P_c(M_c)$, $P_c(E_c)$ and $P_\square(M_\square)$, respectively. In order to see indications of the existence of monopole-like fluctuations we will measure simultaneously the energy (action) of 3D cubes $\mathcal{S}_n$ and the Higgs field modulus R_n .

The Monte - Carlo simulations with respect to the distribution defined in (2.2) have been carried out by employing the standard Metropolis algorithm.

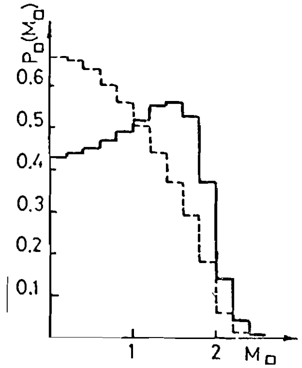
3

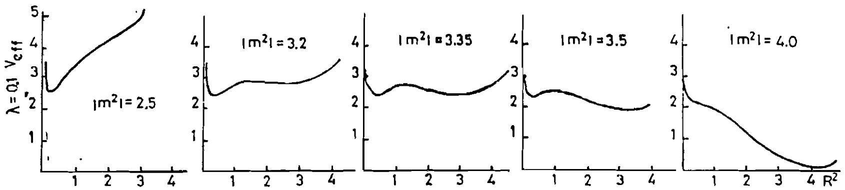
First, let us study the plaquette-flux distribution $P_\square(M_\square)$ for points $(\lambda = 0.2, m^2, \beta)$ (cf. Fig. 2). The numerical results for the



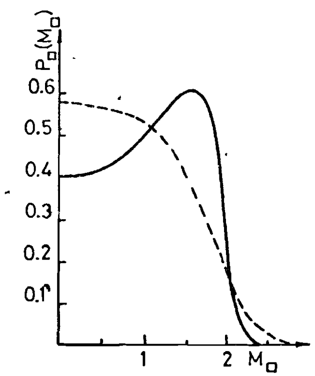
F i g. 1. Phase diagram of the Georgi - Glashow model at λ = 0.2.

Fig. 2. Plaquette-flux distributions $P_\square(M_\square)$ in two phases: solid line for phase D (β = 0.5, m^2= -6.02) and dashed line for phase A(β = 0.5; m^2= -4.8).





F i g. 3. Effective potential at λ = 0.1 and different values of m^2 .



F i g. 4. Plaquette-flux distributions, obtained with the use of (3.2). Dashed line corresponds to $\bar R$ = 1 and solid line corresponds to $\bar R$ = 3.

phases A,B and C look similar (B and C are not included in Fig.2) and produce maxima at zero flux $M_\square$. They can nicely be fitted by simple Gaussian distributions.On the contrary, phase D is singled out by the appearance of a maximum at a nonzero value of $|M_\square|$. This effect can be understood as a strong coupling one. In order to see this, we apply the effective potential method at β = 0 /8,13/ . In this limit the integrations w.r. to the gauge field degrees of freedom as well as for the "angular" variables of the Higgs field can analytically be done. Then, one finds the effective potential of the Coleman-Weinberg type as a function of $\bar R$ /8/.

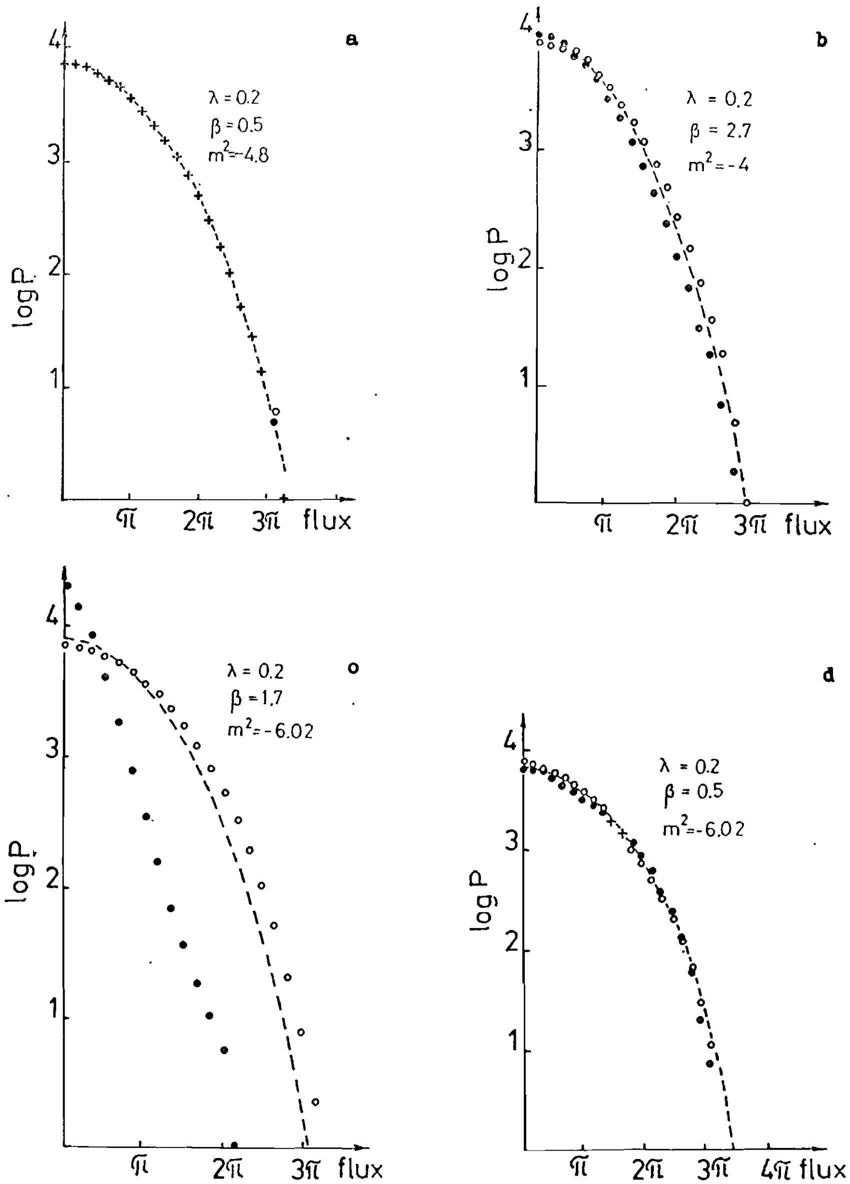
$$V_{eff}(\bar R) = (8+m^2)\bar R^2 + 4\lambda \bar R^4 - 4\ln\frac{sh(2\bar R^2)}{2\bar R^2} - \ln \bar R^2, \qquad (3.1)$$

where the last term is due to the integration measure $d^3\vec\phi$. In Fig.3 the characteristic behaviour of Veff at sufficiently small ($\lambda \lesssim 0.22$) is shown for different values m^2 (for definiteness λ = 0.1 is chosen).The occurrence of a second minimum of Veff with growing $|m^2|$ indicates the first-order phase transition at $m^2 = m_c^2 \simeq$ $\simeq -3.35$ from phase A to D . Let us fix now values m^2 in both the phases, but sufficiently far from m_c^2 , such that the corresponding single minima of Veff determine the average values $\langle R \rangle$. Then, the plaquette-flux distribution can be approximated by

$$P_\square(M_\square) \sim \int \prod_{\ell\in\square_{n\mu\nu}} dU_\ell \, exp\Big(\bar R^2 \sum_{\ell\in\square_{n\mu\nu}} tr(\sigma_3 U_\ell \sigma_3 U_\ell^\dagger)\Big) \times \qquad (3.2)$$
$$\times\, \delta\Big(M_\square - tr\{ig U_{n\mu\nu} + \tfrac{i}{2}\sigma_3 U_{n\mu}\sigma_3 U_{n\mu}^\dagger U_{n\nu}\sigma_3 U_{n\nu}^\dagger\}\Big),$$

where $\square_{n\mu\nu}$ denotes a definite plaquette at (arbitrarily chosen) n , μ , ν . Numerically we find the behaviour as shown in Fig .4 for $\bar R = 1$ and $\bar R = 3$, respectively, which explains our findings in Fig.2 qualitatively.

It is interesting to discuss now the distributions $P_c(M_c)$ and $P_c(E_c)$. The Monte - Carlo data for them are presented in Figs. 5 $a \div d$ for the phases $A \div D$. For comparison, we calculate the cube-flux distribution by folding the single plaquette-flux distributions under the assumption that the fluxes through neighbour plaquettes are completely uncorrelated,



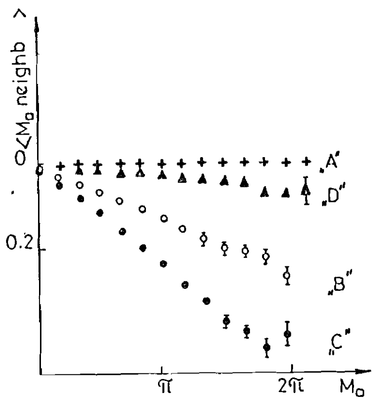
F i g. 5. Magnetic (black circles) and electric (white circles) cube-flux distributions at different phases. Crosses mean coincidence of $P_c(M_c)$ and $P_c(E_c)$ and dashed lines correspond to $P_{uncorr}(M_c)$.

$$P_{uncorr}(M_c) = \int \prod_{\square\in cube}^{6} dM_\square \, \delta\Big(M_c - \sum_{\square\in cube}^{6} M_\square\Big) \prod_{\square\in cube} P_\square(M_\square), \qquad (3.3)$$

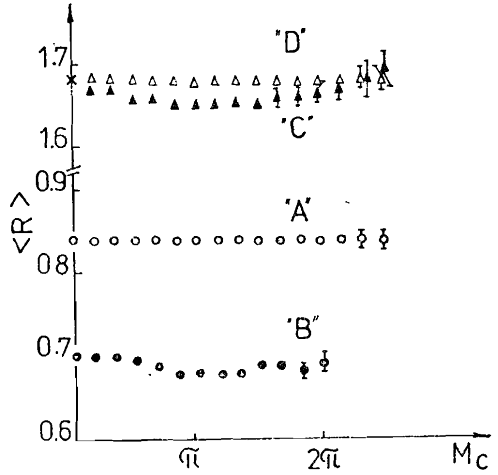
where fluxes in the argument of the δ -function are summed taking into account their orientation. We find $P_c(M_c) = P_c(E_c) = P_{uncorr}(M_c)$ in phase A, (cf. Fig. 5a), which means that this phase is dominated by uncorrelated random noise. On the contrary, the other phases show up correlations which become very strong in the Higgs phase C.

The comparison of Figs. 5a-d makes clear that the magnetic flux out of cubes is well suited to distinguish between the phases in contrast with the electric flux, the distributions for which do not differ very much. Phase C is characterized (in contrast to other three phases) by a very strong suppression of a large magnetic flux (see also[5]) compared with the electric one. It is due to an average cancellation between magnetic fluxes through nearest-neigbour plaquetts at common lattice sites and with common links. In this sence, a strong discrepancy between $P_c(M_c)$ and $P_{uncorr}(M_c)$ is highly significant. This effect is demonstrated by Fig. 6 showing the average magnetic flux through the neighbour plaquettes as a function of the flux through a given plaquette.

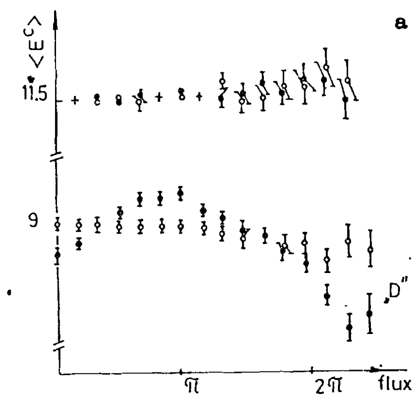
One is now tempted to ask whether large magnetic fluxes are really connected with monopole-like configurations. In order to answer this question we have studied the correlations between magnetic flux M_c and the Higgs field modulus R at given elementary cubes. In Fig.7 the average energy of the Higgs field is plotted as a function of the magnetic flux M_c . With increasing M_c deviations of the average Higgs field towards lower values is seen in phases C and B . Let us remind here that classical monopoles of the 't Hooft and Polyakov type are accompanied by zeros of the Higgs field at their centres. Therefore, it is natural that with increasing flux the value of $\langle R \rangle$ decreases. At the same time, large values of M_c (which are comparable with 4π) may be produced only by point-like monopoles (more exactly, monopoles of very small sizes). Consequently, with further increasing M_c the value of $\langle R \rangle$ should also increase. Just this dependence is observed in phases C and B . In phase C a strong suppression of magnetic fluxes out of the cubes in the presence of large plaquette-flux fluctuations allows one to assume that in this phase a noticeable role is played by monopole-antimonopole pairs (dipoles) of small sizes. In phases A and D (at least for

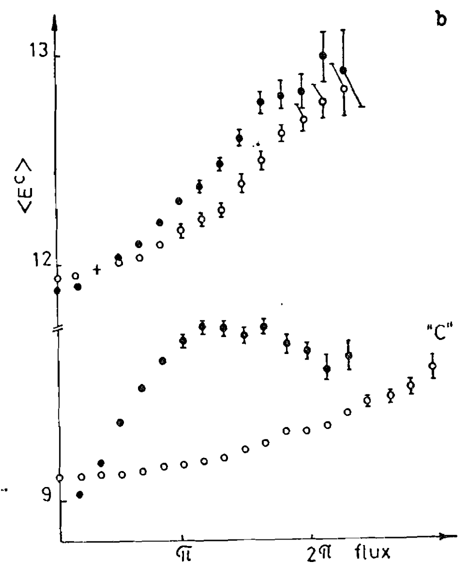


F i g. 6. Plaquette-flux correlations at different phases.



F i g. 7. Dependence of the average Higgs field on the magnetic cube-flux.





F i g. 8. Dependence of the average energy per cube on magnetic (black circles) and electric (white circles) fluxes.

the values of parameters we have chosen) the Higgs field is practically independent of the magnetic flux.

The dependence of average energy in an elementary cube on the magnetic flux in four different phases is represented in Fig. 8.

4

We draw the following conclusions. The four phases of the Georgi-Glashow model observed in the given range of couplings[8] can be well characterized by the behavior of magnetic flux out of 3D cube as well as by the behavior of magnetic flux through plaquette. The vacuum in the Higgs phase C may be assumed to be a "medium" of magnetic dipoles of small sizes. At the transition to phase D this dipole structure is destroyed and the symmetry is "restored". It may be assumed that in phase D a vacuum state analogous to that discussed in ref. 2 is realised.

In phase A vacuum configurations have the nature of a random Gaussian noise. In the transition from A to B vacuum configurations bear out some monopole-antimonopole structure.

Further thorough study is required in order to understand more clearly vacuum in different phases of this model and the types of phase transitions between different phases. At present, these problems are under investigation.

In conclusion, we would like to express our gratitude to Meshcheryakov V.A. and Sissakian A.N. for useful discussions and interest in the work. One of us (M.M.-P) would like to express his gratitude to the Directorates and colleagues of the Laboratory of Computing Technique and Automation and of the Laboratory of Theoretical Physics for kind hospitality extended to him. He warmly acknowledges very useful discussions with G.Schierholz and M.Lüscher at DESY.

Note added in proof.

After having finished this work we heard about similar investigations by M.Laursen, G.Schierholz and U.Wiese. We are indebted to them for informing us prior to publication.

REFERENCES

1. Callan C., Dashen R., Gross D.Phys.Rev.,1978,17D,p.2717;Phys.Rev., 1979, 19D, p. 1826;
   Ilgenfritz E.-M., Muller-Preussker M. Nucl.Phys., 1981, 184B,p443;
   Shuryak E.V. Nucl.Phys., 1982, 203B, p. 93, 116, 140; Nucl.Phys., 1983, 214B, p. 237;
   Dyakonov D.I., Petrov V.Yu. Nucl.Phys., 1984, 245B, p. 259; Nucl. Phys., 1986, 272B, p. 457.

2. Nielsen H.B.Olesen P.Nucl.Phys., 1979, 160B, p. 57;
   Ambjorn J., Olesen P. Nucl.Phys., 1980, 170B, p. 60;
   Olesen P. Nucl.Phys., 1982, 200B, p. 381.

3. 't Hooft CERN-preprint TH-1902 (1974); Phys.Scr., 1982, 25, p.133;
   Mandelstam S. Phys.Rev., 1979, 19D, p. 2391;
   Nair V.P., Rosenzweig C. Phys.Rev., 1985, 31D, p. 401.

4. Teper M. Phys.Lett., 1985, 162B, p. 357;
   Ilgenfritz E.-M., Laursen M.L., Muller-Preussker M., Schierholz G., Schiller H. Nucl.Phys., 1986, 268B, p. 693;
   Teper M. Phys.Lett., 1986, 171B, p. 81,86;
   Barbour I., Teper M. Preprint SHEP 85/86;
   Schierholz G. DESY-preprint 86-09;
   Polikarpov M.I., Veselov A.I. ITEP-preprint 87-41; Pisma v JhETF, 1987, 45, p. 113.

5. Schierholz G., Seixas J., Teper M. Phys.Lett., 1985, 151B, p. 69; 1985, 157B, p. 209.

6. Polonyi J. Nucl.Phys., 1986,461A, p. 279.

7. Olynyk K., Shigemitsu J. Nucl.Phys., 1985, 251B, p. 472;
   Lee L.-H., Shigemitsu J. Nucl.Phys., 1986, 263B, p. 280;
   Baier R., Reusch H.-J. Bielefeld University preprint BI-TP 86/25.

8. Mitrjushkin V.K., Zadorozhny A.M. Phys.Lett., 1986, 181B, p. 111.

9. Lang C.B., Rebbi C., Virasoro M. Phys.Lett., 1981, 104B, p. 294;
   Brower R.C., Kessler D.A., Schalk T., Levine H., Nauenberg M. Phys. Rev., 1982, 25D, p. 3319;
   Karsh F., Seiler E., Stamatescu I. CERN-preprint TH. 3612.

10. 't Hooft G. Nucl.Phys., 1974, 79B, p. 176.

11. See e.g. Goddard P., Olive D.I. Rep.Progr.Phys., 1978, 41, p.1357.

12. Polyakov A.M. JETP Lett., 1974, 20, p. 194.

13. Gerdt V.P., Ilchev A.S., Mitrjushkin V.K., Sobolev I.K., Zadorozhny A.M., JINR-preprint E2-84-313; Nucl.Phys., 1986, 265B, p. 145.

Received by Publishing Department on July 16, 1987.

SUBJECT CATEGORIES OF THE JINR PUBLICATIONS

Index	Subject
1.	High energy experimental physics
2.	High energy theoretical physics
3.	Low energy experimental physics
4.	Low energy theoretical physics
5.	Mathematics
6.	Nuclear spectroscopy and radiochemistry
7.	Heavy ion physics
8.	Cryogenics
9.	Accelerators
10.	Automatization of data processing
11.	Computing mathematics and technique
12.	Chemistry
13.	Experimental techniques and methods
14.	Solid state physics. Liquids
15.	Experimental physics of nuclear reactions at low energies
16.	Health physics. Shieldings
17.	Theory of condenced matter
18.	Applied researches
19.	Biophysics

Митрюшкин В.К., Мюллер-Пройскер М., Задорожный А.М. E2-87-555

Магнитные флуктуации и структура вакуума в модели Джорджи-Глэшоу на решетке

В работе исследуется связь между электромагнитными флуктуациями и фазовой структурой SO(3)-симметричной модели Джорджи-Глэшоу на решетке, имеющей (по крайней мере при достаточно малых значениях константы скалярного самодействия) четыре фазы. Методом Монте-Карло вычислялись распределения электрических и магнитных потоков, а также различные корреляторы.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1987

Mitrjushkin V.K., Müller-Preussker M., Zadorozhny A.M. E2-87-555

Magnetic Fluctuations in the Quantized Vacuum of the Georgi-Glashow Model on the Lattice

We study the influence of (electro)magnetic fluctuations on the phase structure of the 4D-Georgi-Glashow model on the lattice. The distributions of (electro)magnetic fluxes and different correlations were measured using the Monte-Carlo method.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1987