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DEEP INELASTIC SCATTERING IN THE FORMALISM WITH THE WAVE FUNCTIONS OF COMPOSITE SYSTEMS AT REST

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I. Introduction

The interaction processes at high energies and large transfer momenta are very important in studying the strong interaction dynamics and elementary particle structure. At present, a regular method for describing these processes is perturbation theory that is applicable owing to the property of asymptotic freedom of quantum chromodynamics /1,2/ . However, the inclusion of a composite structure of hadrons results in a representation in which only a part corresponding to the scattering of escaped constituents from a bound state is calculated by perturbation theory /3/ . In the total expression for the cross section, this part is integrated in the product with the wave functions of a bound state, the determination of which is beyond the scope of perturbation theory. In quantum field theory, those functions describing transformation of a physical particle into constituents imply the dependence on the total momentum variable defined by the interaction dynamics. In general, this dependence can be taken into account by the perturbation theory method in the coupling constant proposed in ref. /4/ . However, in the case of deep inelastic processes the problem is solved by choosing a reference frame. For this purpose, the system of "infinite momentum" $P_z \to \infty$ is usually used /5/ . In such an approach all physical quantities are expressed through the wave functions of a composite particle moving with infinite momentum.

In the present paper, a deep inelastic process is studied when a composite particle is at rest; as a result, the corresponding cross section is expressed through usual, from the point of view of nonrelativistic quantum mechanics, wave functions. A new version of expanding structure functions over a series in the coupling constant, each term in it possessing a spectral property due to a correct inclusion of a conservation law of energy in any order of perturbation theory, is suggested. The performed analysis shows that in the rest frame of a bound state ($\vec{P} = 0$) an impulse approximation is insufficient for a correct description of the elastic limit $x_{Bj} \to 1$ in contrast with the system $P_z \to \infty$. To obtain leading terms in the asymptotic region $x_{Bj} \to 1$, one should take into account the interaction of components in the final state. The relevant diagrams are pointed out, whose calculations in the QCD model are in agreement with the earlier obtained results, e.g. [6] .

2. Perturbation Theory

Let us consider the deep inelastic scattering of an electron on a hadron. The cross section of such a process is defined by the tensor

$$W_{\mu\nu} = \frac{1}{2\pi}\int d^4x\, \exp(iq\cdot x)\langle P|\mathbb{J}_\mu(x)\,\mathbb{J}_\nu(0)|P\rangle\ , \qquad (1)$$

where $|P\rangle$ is the eigenstate of the total Hamiltonian H, corresponding to a hadron with four-momentum, normalized by the condition $\langle P|Q\rangle = (2\pi)^3 2P^0\delta^{(3)}(\vec{P}-\vec{Q})$ x); $\mathbb{J}_\mu(x)$ is the electromagnetic current in the Heisenberg representation. For definiteness we assume that at the zero moment of time the Heisenberg pictures and interactions coincide

$$\mathbb{J}_\mu(x) = J_\mu(x) \qquad \text{if} \qquad x^0 = 0\ .$$

If the dressed current $\mathbb{J}_\mu(x)$ is expanded over the interaction constant

$$\mathbb{J}_\mu(\vec{x},t) = \left\{T\exp i\int_0^t dt' H_I(t')\right\}^+ J_\mu(\vec{x},t)\left\{T\exp i\int_0^t dt' H_I(t')\right\}\ , \qquad (2)$$

we obtain one of the possible versions of perturbation theory for structure functions of deep inelastic scattering. In the zero order we have the known expression with free currents

$$W_{\mu\nu} = \frac{1}{2\pi}\int d^4x\, \exp(iq\cdot x)\langle P|J_\mu(x)J_\nu(0)|P\rangle\ , \qquad (3)$$

which expounds the main drawback of perturbation theory (2) – it lost such an important property of structure functions as spectrality connected with a correct inclusion of the energy conservation law in constructing the deep inelastic scattering cross section. Indeed, one may verify that in the zero approximation (3) $W_{\mu\nu} \neq 0$ below the reaction threshold $(P+q)^2 = M^2$, $x_{Bj} = -q^2/2P\cdot q > 1$. It is clear that no total set of states $|N\rangle$ between currents in (3) will lead to the δ-function with respect to energy in the expression

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x) In what follows, to denote momenta of eigenvectors of the total and free Hamiltonian, we shall use capital and small letters, respectively.

$$T_\mu = \int d^4x\, \exp(iq\cdot x)\langle P|J_\mu(x)|N\rangle\ , \qquad (4)$$

since $|P\rangle$ is the eigenstate of the total Hamiltonian H and the time translations of current $J_\mu(x)$ are defined by the free Hamiltonian. Since the violation of the spectral property is known to imply distortion of the behaviour of structure functions in the vicinity of $x_{Bj} \to 1$, representation (3) becomes useless for studying this region. To restore the spectral property, in (3) one uses the parton picture in which the following two moments are important: transition to the system $P_z \to \infty$ and assumption of a limited transverse motion of quarks in a hadron. In this case, of importance are the projection properties of the wave function of a bound state with respect to a longitudinal fraction of the momentum of composites which occur only in the system $P_z \to \infty$. Since our consideration proceeds in the rest frame of a composite, one should have a perturbation theory in which the spectral property is conserved in each expansion term.

We rewrite the tensor $W_{\mu\nu}$ in the form

$$W_{\mu\nu} = \int d^3\vec{x}\, \exp(-i\vec{q}\cdot\vec{x})\langle P|J_\mu(\vec{x},0)\,\delta(P^0+q^0-\hat{H})\,J_\nu(0)|P\rangle\ . \qquad (5)$$

A symbolic notation of the δ– function with the operator argument is interpreted by the change in (5)

$$\delta(z-\hat{H}) \Rightarrow \sum \delta(z-E_N)|N\rangle\langle N|\ , \quad z = P^0+q^0,$$

where $|N\rangle$ is the total set of eigenstates of the Hamiltonian $\hat{H}$. Since both the currents in (5) are free, the construction of perturbation theory is reduced to the expansion of the δ-function in the coupling constant. For this purpose we use the representation

$$2\pi i\,\delta(z-\hat{H}) = [z-\hat{H}-i\epsilon]^{-1} - [z-\hat{H}+i\epsilon]^{-1}$$

the definition of the T matrix operator

$$[z-\hat{H}+i\epsilon]^{-1} = [z-\hat{H}_0+i\epsilon]^{-1} + [z-\hat{H}_0+i\epsilon]^{-1}\hat{T}(z)[z-\hat{H}_0+i\epsilon]^{-1}$$

$$[z-\hat{H}-i\epsilon]^{-1} = [z-\hat{H}_0-i\epsilon]^{-1} + [z-\hat{H}_0-i\epsilon]^{-1}\hat{T}^+(z)[z-\hat{H}_0-i\epsilon]^{-1}$$

and the unitarity condition x)

$$\hat{T}^{+}(z) - \hat{T}(z) = 2\pi i\, \hat{T}(z)\delta(z-\hat{H}_0)\hat{T}^{+}(z). \qquad (6)$$

Then, after simple calculations we get

$$W_{\mu\nu} = \int d^3\vec{x}\, \exp(-i\vec{q}\cdot\vec{x}) \langle P | J_\mu(\vec{x},0) \{ I + [z-H_0+i\epsilon]^{-1} \hat{T}(z) \} \cdot$$
$$\cdot\, \delta(z-\hat{H}_0) \{ I + [z-H_0+i\epsilon]^{-1} T(z) \}^{+} J_\nu(0) | P \rangle . \qquad (7)$$

Using the total set of bare states $|n\rangle$ (i.e. eigenstates of the Hamiltonian H_0) and integrating over $d\vec{x}$, we have

$$W_{\mu\nu} = (2\pi)^3 \sum_n \delta^{(4)}(P+q-p_{(n)})\, T_\mu T_\nu^{+} , \qquad (8)$$

where

$$T_\mu = \langle P | J_\mu(0) \{ I + [z-\hat{H}_0+i\epsilon]^{-1} \hat{T}(z) \} | n \rangle$$

and $P_{(n)}$ is the total 4-momentum of states $|n\rangle$.

We propose a perturbation theory for the functions $W_{\mu\nu}$ that is based on the expansion in the coupling constant of the operator $\hat{T}(z)$ in relations (7) and (8). Then, the presence in (8) of the four-dimensional δ -function in any order of the suggested version of perturbation theory provides the above mentioned spectral property.

3. Impulse approximation and threshold behaviour

Represent a bound state $|P\rangle$ as the Fock column with the components $\Phi_P^{(n)}(p_1,\dots p_n)$ to be called below the n-particle wave functions

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x) Provided that bound states exist in the theory, the right-hand side of the unitarity condition (6) has an additional term $2\pi i \sum_\beta (z-H_0) \times |\beta\rangle \delta(z-E_\beta) \langle \beta | (z-H_0)$. Moreover, in one of the schemes of quark confinement just this becomes essential. These problems will be discussed elsewhere.

$$\langle p_1,\dots p_n | P \rangle = \delta^{(3)}(\vec{P} - \sum_{l=1}^{n} \vec{p}_l)\, \Phi^{(n)}_{\vec{P}}(p_1 \dots p_n).$$

Let us write down (8) in the impulse approximation corresponding to $\hat{T}(z)=0$

$$W_{\mu\nu} = \sum_i \int \frac{d\vec{p}_i}{2p_i^0} \rho_P^i(\vec{p}_i,\alpha) \langle p_i | J_\mu(0) | p_i' \rangle \langle p_i' | J_\nu(0) | p_i \rangle (2p_i^{0\prime})^{-1}, \qquad (9)$$

where

$$\vec{p}_i' = \vec{p}_i + \vec{q}\,, \quad \alpha = P^2 + P^{+}(q^{-} + p_i^{-} - p_i^{-\prime})\,, \qquad p_i^2 = m_i^2 \qquad p^{\pm} = p^0 \pm p^3 .$$

The quantity $\rho_P^i(\vec{p}_i,\alpha)$ defined by the formula

$$\rho_P^i(\vec{p}_i,\alpha) = \sum_n \frac{(2\pi)^{-3(n+1)}}{2p_i^0 \cdot P^{+}} \int \prod_{l\neq i}^{n} \frac{d\vec{p}_l}{2p_l^0}\, \delta\Big(\alpha - (\sum_1^n p_l)^2\Big)\, \delta^{(3)}\Big(\vec{P} - \sum_1^n \vec{p}_l\Big) \Big| \Phi_P^{(n)}(p_1,\dots,p_n) \Big|^2 , \qquad (10)$$

makes it probable that the i-th component of hadron P has momentum in the interval $\vec{p}_i\,,\ \vec{p}_i + d\vec{p}_i$ whereas the square of the effective mass of all the constituents is in the interval $\alpha\,,\ \alpha + d\alpha$ the following normalization condition being fulfilled

$$\int d\alpha \int d\vec{p}_i\, \rho_P^i(\vec{p}_i,\alpha) = 1 .$$

We rewrite formula (9) in a more compact form

$$W_{\mu\nu} = \sum_i \int \rho_P^i(p)\, \delta^{(+)}\big((P+q-p)^2 - m_i^2\big)\, \omega_{\mu\nu}\, d^4 p\ , \qquad (11)$$

where the $\rho_P^i(p)$ function of the four-dimensional argument $p = (p_0,\vec{p})$ is related with the distribution (10) as follows.

$$\rho_P^i(p) = \rho_P^i\Big(\vec{P}-\vec{p}\,,\ (p_0 + \sqrt{(\vec{P}-\vec{p})^2 + m_i^2}\,)\Big).$$

The quantity $\rho_P^i(p)$ is the probability for the total four-momentum of all the constituents but the i-th one to be in the interval $p\,,\ p+dp$ in a hadron with momentum P

$$\rho^i_{\underline{P}}(p) = \sum_n (2\pi)^{-3(n+1)} \int \prod_{\ell=1}^{n} \frac{d\vec{p}_\ell}{2p^0_\ell}\, \delta^{(4)}\Big(p - \sum_{\ell\neq i}^{n} p_\ell\Big)\, \delta^{(3)}\Big(\vec{P} - \sum_{l=1}^{n} \vec{p}_l\Big) \left| \Phi^{(n)}_{\underline{P}}(p_1, \dots, p_{i-1}, p_{i+1}, \dots p_n) \right|^2 . \qquad (12)$$

To avoid a complex dependence of the wave functions on the hadron momentum P, arising due to the interaction dynamics, we shall consider only the special reference frames $\vec{P}=0$ and $P_z \to \infty$. We make rough estimates of expression (14) that lead to a standard parton picture of interaction. In the system $\vec{P}=0$, without loss of generality one can direct the z axis along the vector $\vec{q}$ ($q_z = |\vec{q}|$), then the δ-function in (11) becomes

$$\delta((P+q-p)^2 - m_i^2) = \delta\Big(W^2 - \frac{W^2}{M\xi}p^- - M\xi p^+ + p^2 - m_i^2\Big) =$$

$$= \frac{M\xi}{W^2}\,\delta\Big(p^- - M\xi + \frac{M\xi}{W^2}\big(M\xi p^+ + m_i^2 - p^2\big)\Big),$$

where $W^2 = (P+q)^2$, $M\xi = M + \nu - |\vec{q}| = M + \nu - \sqrt{\nu^2 - q^2}$.

The neglect of the third term in the δ-function argument corresponds to the parton model. Since the rejected term $M\xi(M\xi p^+ + m_i^2 - p^2)/W^2$ has the order of smallness $\langle M^2\rangle/\nu$, the parton model means taking the limit $\nu \to \infty$ and the structure tensor acquires the form

$$W_{\mu\nu} = \frac{M\xi}{W^2} \sum_i \int \rho^i_{\vec{0}}(p)\, \delta(p^- - M\xi)\, \omega_{\mu\nu}\, d^4p . \qquad (13)$$

Integration in (13) over the angular variables of vector $\vec{p}$, on which $\rho^i_{\vec{0}}(p)$ does not depend, leads to the following expression

$$W_{\mu\nu} = \pi \frac{M\xi}{W^2} \sum_i \int_0^\infty dp^2 \int_{p^2/2M\xi + M\xi/2}^{\infty} dp_0\, \rho^i(p_0, p^2)\, \omega_{\mu\nu} . \qquad (14)$$

Analogous calculations for the system $P_z \to \infty$ ($q^+ < 0$, $\vec{q}_\perp = 0$) lead to the known parton distribution over the longitudinal fractions of momenta

$$W_{\mu\nu} = \pi \frac{M\xi}{W^2} \sum_i \rho^i_\infty(1-\xi)\, \omega_{\mu\nu} , \qquad (15)$$

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x) Note that $1-\xi = \dfrac{2x_{Bj}}{1+\sqrt{1+\dfrac{4M^2 x_{Bj}}{-q^2}}}$ is the Nachtman variable /7/.

where

$$\rho^i_\infty(x) = \int_0^\infty dp^2 \int_0^\infty d\vec{p}_\perp^{\,2}\, \rho^i_\infty(p^+, \vec{p}_\perp^{\,2}, p^2). \qquad (16)$$

It is to be noted that according to the representation (12) $\rho_{\vec{0}}(p)$ is the function of two arguments $p^\circ, |\vec{p}|$ and in the system $P_z \to \infty$ depends on three arguments $p^+, \vec{p}_\perp^{\,2}$ and p^2 invariant with respect to rotations in the plane perpendicular to the z axis. Approximate representations (13) and (15) corresponding to the well-known picture, have been obtained by neglecting small components (of an order of M^2/ν) in the δ-function argument (11). To elucidate how good is the approximation, a more accurate analysis is necessary with the conservation of the integration limits over the variables $\vec{p}_\perp^{\,2}$ and p^2 following from the exact δ-function in (11) /7,8/.

Using the representation (13) one can investigate the behaviour of the structure functions near the exclusive threshold $\xi \to 0$. It is easily seen that it is defined by the asymptotic behaviour of the wave functions of a hadron at rest in the region of large momenta of all constituents ($p_i^- < M\xi$). Analogously, in the system $P_z \to \infty$ the elastic limit of the structure functions is defined, according to (16), by the behaviour of the wave functions of the light front at $x \to 1$.

The asymptotic analysis of equations for n-particle wave functions of bound states in the QCD /4,6/

$$\Phi^{(n)}_0(p_1, \dots p_n) \to |\vec{p}|^{-3(n-1)+\frac{n}{2}}, \quad |\vec{p}_1| \sim |\vec{p}_2| \sim \dots \sim |\vec{p}_n| \sim |\vec{p}| \to \infty ,$$

$$\Phi^{(n)}_\infty(\vec{p}_{1\perp}, x_1, \dots \vec{p}_{n\perp}, x_n) \to (1-x_i)^{\frac{1}{2}(2n-3)+|\Delta\lambda|}, \quad x_i \to 1 .$$

($\Delta\lambda$ is the difference between helicity of a bound state and that of an active quark) shows that the impulse approximation provides different results depending on a reference frame. Indeed, the formalism with hadrons at rest gives a fall at $\xi \to 0$ $\nu W_2 \simeq \xi^{5n-6}$ sharper than the generally accepted one $\nu W_2 \simeq \xi^{2n-3+2|\Delta\lambda|}$ that is in agreement with the consequence of the impulse approximation in the reference frame $P_z \to \infty$. It is the case when the zero order in perturbation theory does not provide a correct description of the regularity under consideration; therefore, one should take into account the subsequent terms of expansion $\hat{T}(z)$ in (8). Note that a similar situation takes place while analysing the asymptotic behaviour of the elastic form factor of the composite system

$$F_\mu = \langle Q | J_\mu(0) | P \rangle .$$

The amplitude of the deep inelastic process (8) has the same field-theoretical axpression as the form factor F_μ , thus being its inelastic analog. The variable $W^2 = (P+q)^2 = Q^2$ plays the role of a final state mass. An essential difference from the elastic form factor is in that the final state expands in a standard way in powers of the coupling constant

$$|Q\rangle = \{ I + [Q^0 - M_0 + i\epsilon]^{-1} \hat{T}(Q^0) \} |n\rangle$$

and has the zero free limit as the Fock state. In the formalism the leading asymptotics of the structure functions with respect to $1 - x_{Bj}$ and of the form factor with respect to transfer momentum is defined in the impulse approximation, i.e. in the zero order in perturbation theory. However, in our approach $\vec{P} = 0$, as it will be seen from a further consideration, the leading asymptotics with respect to $1 - x_{Bj}$ manifests itself in the next to zero order of perturbation theory. An analogous situation is observed in analysing the representation for the elastic form factor written with the help of the wave functions of hadrons at rest. In this case, one should also take account of the subsequent orders of perturbation theory /4/ in the coupling constant for the boost operator defining the final state $|Q\rangle$ with arbitrary momentum Q through the state of the composite system $|\vec{0}\rangle$ at rest.

We shall show that the leading asymptotics at $\xi \to 0$ in the system $\vec{P} = 0$ corresponds to diagrams of the type of Fig.1 and the results of calculations are in agreement with the impulse approximation in the system $P_z \to \infty$. For a qualitative explanation

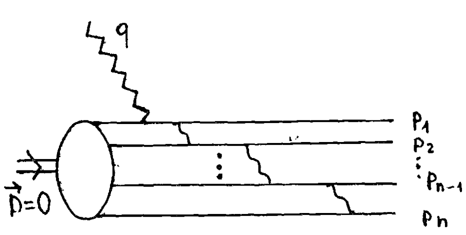

Fig.1

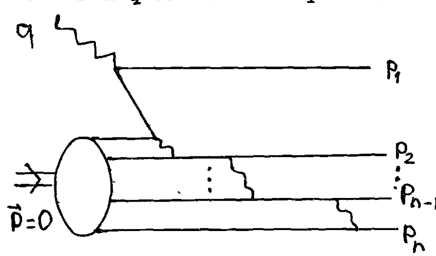

Fig.2

of the afore-said we should like to note that part of any diagram (of the type of Fig.2) to the left from the photon vertex corresponds to the scattering of constituents with total momentum $\vec{q}$. In the limit $|\vec{q}| \to \infty$ (this is the region under investigation) the nuclei corresponding to these diagrams are defined by perturbation theory in the system of "infinite momentum" with the Z axis directed along the vector $\vec{q}$. This is the reason for cross sections at $\xi \to 0$ to decrease more slowly due to a specific in $P_z \to \infty$ cancellation of large terms in the energy denominators.

In the reference frame $\vec{P} = 0$ one should also allow for the diagrams describing the photon production of pairs from vacuum (Fig.2). The calculations show that they give the same contribution as the diagrams of Fig.I. Since the calculation of diagrams in Fig.2 encounters no additional difficulties, we shall investigate only the diagrams of Fig.1.

Let us consider the case of a meson with two valent constituents. Using the second order expansion in coupling constant of $\hat{T}(Z)$ from representation (8) we have

$$T_\mu = \int \Phi^*_{\vec{0}}(\vec{\ell}_1 \vec{\ell}_2)(d\vec{\ell})_2 \langle \ell_1, \ell_2 | J_\mu(0) \{ [Z - M_0 + i\epsilon]^{-1} H_I \}^2 | P_1 P_2 \rangle , \qquad (17)$$

$$(2\pi)^{3K} (d\vec{\ell})_K = \delta^{(3)}\Big(\sum_1^K \vec{\ell}_i\Big) \prod_{i=1}^K d\vec{\ell}_i / 2\ell_i^0 , \quad Z = M + q^0, \quad H_I = g \bar{\Psi} \gamma^\mu \Psi G^a_\mu \hat{T}_a .$$

In the QCD the connected diagrams corresponding to (17) are shown in Fig.3.

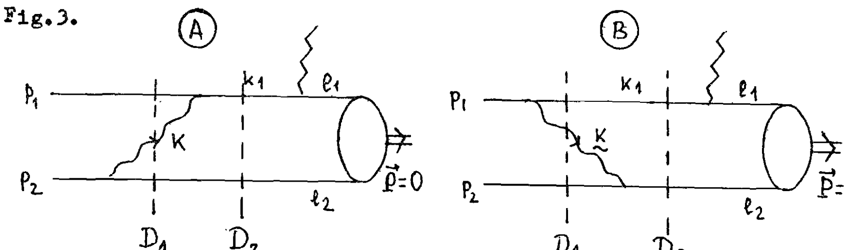

Fig.3

Using the representation (8) one can formulate the diagram technique /9/ , according to which each dashed line between the interaction vertices is associated with the energy denominator

$$D = M + q^0 - \sum_\ell \sqrt{\vec{k}_\ell^{\,2} + m_\ell^2} , \qquad (18)$$

where $\vec{K}_\ell$ is momentum ascribed to the line intersected by the dahsed line. In the region under investigation $|\vec{q}| \to \infty$ due to an explicit form of (18) one becomes aware of the above cancellation of large terms as it occurs in the diagram technique $P_z \to \infty$.

$$M + q^0 \sim |\vec{q}| + W^2/2|\vec{q}| \,,$$

$$\sum \sqrt{\vec{\kappa}_\ell^{\,2} + m_\ell^2} \sim \Big(|\vec{q}| + \sum \frac{\vec{\kappa}_{\ell\perp}^{\,2} + m_\ell^2}{2\kappa_\ell^z} \Big) \prod \theta(\kappa_\ell^z) .$$

Identical change of the energy denominators (18) owing to the condition $\vec{q} = \sum \vec{\kappa}_\ell$

$$D = M + q^- - \sum \kappa_\ell^- \,,$$

where $\kappa_\ell^- = \sqrt{\vec{\kappa}_\ell^{\,2} + m_\ell^2} - \vec{\kappa}_\ell \vec{q}/|\vec{q}|$, takes automatically into account the cancellation of large terms $\sim |\vec{q}|$. In this notation it is easy to compare the contributions to D of different momenta: momenta with large constituents in the direction of $\vec{q}$ $\vec{\kappa}_\ell \cdot \vec{q}/|\vec{q}| \to \infty$ give contribution $\kappa_\ell^- \to 0$, and the small $|\vec{\kappa}_\ell| \sim m$ correspond to $\kappa_\ell^- \sim m$. For large momenta in the opposite to $\vec{q}$ direction $\vec{\kappa}_\ell \cdot \vec{q}/|\vec{q}| \to -\infty$, we have $\kappa_\ell^- \to \infty$.

According to the afore-said, the energy denominators of diagram A have the form

$$D_1 = M\xi - p_1^- - \kappa^- - \ell_2^- \,, \quad D_2 = M\xi - \kappa_1^- - \ell_2^- .$$

The four-dimensional δ-function in (8) at $\vec{P} = 0$ can also be represented in variables p^+, p^- and $\vec{p}_\perp$

$$\delta^{(4)}\Big(P + q - \sum_1^n p_\ell\Big) = \delta\Big(\sum_1^n \vec{p}_{\ell\perp}\Big)\,\delta\Big(\frac{W^2}{M\xi} - \sum_1^n p_\ell^+\Big)\,\delta\Big(M\xi - \sum_1^n p_\ell^-\Big) . \quad (19)$$

Hence, it is seen that at $|\vec{q}| \to \infty$, $\xi \to 0$, $p_1^-, \kappa^- \sim o(\xi)$ and $\kappa_1^- \sim o(m/|\vec{q}|)$, i.e.

$$D_1 \simeq -\ell_2^- \,, \quad D_2 \approx -\ell_2^- . \quad (20)$$

Analogously, for diagram B we have

$$D_1 = M\xi - \kappa_1^- - \underset{\sim}{\kappa}^- - p_2^- \,, \quad D_2 = M\xi - \kappa_1^- - \ell_2^- .$$

However, in contrast with diagram A the energy denominator $D_1 \simeq -\underset{\sim}{\kappa}^- \sim O(1/\xi)$, since $\underset{\sim}{\vec{\kappa}} = -\vec{\kappa}$. Therefore, the contribution of diagram B to the asymptotics of structure functions as $\xi \to 0$ is suppressed. Each inner line denoting the propagation of a particle with mass m and three-dimensional momentum $\vec{\kappa}$ is associated with the factor $\big(2\sqrt{\vec{\kappa}^2 + m^2}\big)^{-1}$. Then, for the wavy line corresponding to a massless gluon with momentum κ this factor in the asymptotic region $\xi \to 0$ can be changed by the following expression:

$$2\sqrt{\vec{\kappa}^2} = 2|\vec{p}_2 - \vec{\ell}_2| \Rightarrow p_2^+ = \frac{\vec{p}_{2\perp}^{\,2} + m_2^2}{p_2^-} \,, \quad (21)$$

where we have taken into account the limitedness of momentum $\vec{\ell}_2$ (which is connected with a rapid decrease of the wave function $\Phi_0(\vec{\ell}_1 \vec{\ell}_2)$ at $|\vec{\ell}_2| \to \infty$) and the condition

$$p_1^- + p_2^- = M\xi .$$

The inclusion of a spin results in the appearance in each inner line of additional factors $\hat{\kappa} + m$ and $d_{s\rho}(\kappa)$ for particles with spin 1/2 and 1, respectively. The gluon projection operator $d_{s\rho}(\kappa)$ depends on the gauge condition fixed in a covariant way

$$d_{s\rho}(\kappa) = -g_{s\rho} .$$

The external legs and vertices are taken into account in a standard way as, for instance, in the Feynman diagram technique.

Thus, with (20) and (21) included, T_μ can be written as follows:

$$T_\mu = e\, g^2 C_F (p_2^+)^{-1} \bar{v}(p_2) \Big\{ \int (d\vec{\ell})_2\, \Phi_0^*(\vec{\ell}_1, \vec{\ell}_2) (\ell_2^-)^{-2} \hat{M}_\mu \Big\} u(p_1) \,, \quad (22)$$

where

$$\hat{M}_\mu = \gamma^\rho v(\ell_2) \otimes \bar{U}(\ell_1) \gamma_\mu \frac{\hat{\kappa}_1 + m_1}{2\kappa_1^0} \gamma_\rho \,, \quad \vec{\kappa}_1 = \vec{\ell}_1 + \vec{q} \,, \quad C_F = \sum \hat{t}_a \hat{t}_a$$

According to the representation (8) and (19), for the tensor $W_{\mu\nu}$ we have

$$W_{\mu\nu} = \frac{1}{(2\pi)^3} \int \frac{d\vec{p}_1 d\vec{p}_2}{2p_1^0\, 2p_2^0}\, T_\mu T_\nu^+\, \delta(\vec{p}_{1\perp} + \vec{p}_{2\perp})\, \delta\Big(p_1^+ + p_2^+ - \frac{W^2}{M\xi}\Big)\, \delta(p_1^- + p_2^- - M\xi) \,, \quad (23)$$

with $p_i^2 = m_i^2$. Due to the presence of the suppression factor $(p_2^+)^{-1}$ in (22), in the scale invariant limit (leading asymptotics in ν^{-1}) the δ-functions in (23) are changed by x)

x) Retaining the exact δ-functions we could obtain an expression in which the dependence on W^2 violating scaling would be taken into account.

$$\delta\left(p_1^+ - \frac{W^2}{M\xi}\right)\delta\left(p_2^- - M\xi\right)\delta\left(\vec{p}_{1\perp} + \vec{p}_{2\perp}\right) \equiv \Delta(p_1, p_2). \tag{24}$$

The corresponding approximation for $\hat{M}_\mu$ gives

$$\hat{M}_\mu \to \frac{1}{2}\gamma^\rho v(\ell_2) \otimes \bar{U}(\ell_1)\gamma_\mu \gamma^- \gamma_\rho$$

Then, (23) becomes

$$W_{\mu\nu} = \frac{(e\cdot C_F)^2}{2\pi}\int (d\vec{\ell})_2 \frac{\overset{*}{\Phi}_{\vec{0}}(\vec{\ell}_1 \vec{\ell}_2)}{(\ell_2^-)^2}\hat{V}_{\mu\nu}\frac{\Phi_{\vec{0}}(\vec{r}_1 \vec{r}_2)}{(r_2^-)^2}(d\vec{r})_2 \frac{\Delta(p_1,p_2)\,\alpha_S(-(p_2-\ell_2)^2)\,\alpha_S(-(p_2-r_2)^2)}{(p_2^+)^2}\frac{d\vec{p}_1}{2p_1^0}\frac{d\vec{p}_2}{2p_2^0}, \tag{25}$$

where α_S is moving coupling constant and

$$\hat{V}_{\mu\nu} = \bar{U}(\ell_1)\gamma_\mu\gamma^-\gamma_\rho(\hat{p}_1+m_1)\gamma^\delta\gamma^-\gamma_\nu U(r_1) \otimes \bar{v}(r_2)\gamma_\delta(\hat{p}_2 - m_2)\gamma^\rho v(\ell_2).$$

After integrating over $d\vec{p}_1 dp_2^-$ in (25) in the leading in ν order, we get

$$W_{\mu\nu} = \xi^2\frac{(e\cdot C_F)^2}{2\pi}\int\frac{d\vec{p}_{2\perp}}{(\vec{p}_{2\perp}^{\,2} + m_2^2)^2}\int(d\vec{\ell})_2\frac{\overset{*}{\Phi}_{\vec{0}}(\vec{\ell}_1\vec{\ell}_2)}{(\ell_2^-)^2}\bar{U}(\ell_1)\gamma_\mu\gamma^-\gamma_\nu U(r_1)\otimes\bar{v}(r_2)\gamma^- v(\ell_2)\frac{\Phi_{\vec{0}}(r_1,r_2)}{(r_2^-)^2}(d\vec{r})_2. \tag{26}$$

Thus, from (26) using the standard definition of the structure functions, we have

$$\frac{\nu}{M}W_2 = \xi^2 e^2 C_F^2\int\frac{d\vec{p}_{2\perp}^{\,2}}{(\vec{p}_{2\perp}^{\,2}+m_2^2)^2}Sp\left(\Gamma^+\frac{\gamma^+\gamma^-}{2}\Gamma\frac{\gamma^-\gamma^+}{2}\right), \tag{27}$$

where

$$\Gamma = \int(d\vec{r})_2\frac{U(r_1)\,\Phi_{\vec{0}}(r_1,r_2)\,v(r_2)}{(r_2^-)^2}\alpha_S\left(r_2^- - \frac{\vec{p}_{2\perp}^{\,2}+m_2^2}{M\xi}\right).$$

The asymptotic expression (27) obtained is in agreement with the calculation of impulse approximation in the field theory on the null plane with an accuracy up to the constant coefficient /6/. Now we shall consider the case of a baryon with three valence constituents. Though the calculation is performed by analogy with the two-particle case, this example exposes some specific features typical of only many-particle composite systems. Consider the diagrams of Fig.4 corresponding to the fourth order terms of expansion $\hat{T}(z)$. For $H_I = g\bar{\Psi}\gamma^\mu\Psi G_\mu^a \hat{T}_a$ this is a minimal order in which connected diagrams arise that describe the three-quark interaction in the final state. (Three- and four-gluon vertices are not considered here).

Analogously to (24) the four-dimensional δ-function in (8) in the scale invariant limit $\nu\to\infty$ is substituted by

$$\delta\left(p_1^+ - \frac{W^2}{M\xi}\right)\delta\left(p_2^- + p_3^- - M\xi\right)\delta\left(\vec{p}_{2\perp} + \vec{p}_{1\perp} + \vec{p}_{3\perp}\right) \equiv \Delta(p_1,p_2,p_3). \tag{28}$$

Among the energy denominators D_ℓ of diagrams A and B in Fig.4, only D_3^A and D_3^B differ from each other

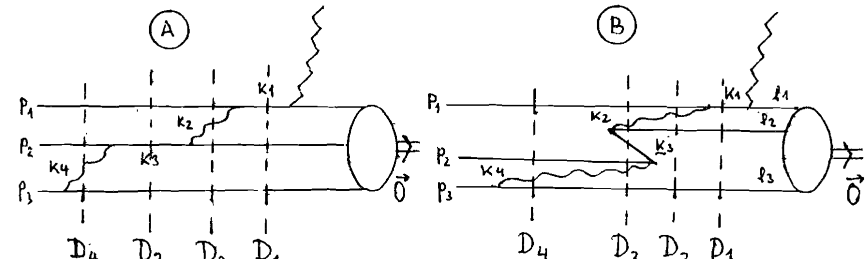

Fig.4

$$D_3^A = M + q^0 - p_1^0 - k_3^0 - \ell_3^0 = M + q^- - p_1^- - k_3^- - \ell_3^-,$$

$$D_3^B = M + q^0 - p_1^0 - k_2^0 - \underset{\sim}{k}_3^0 - p_2^0 - k_4^0 - \ell_3^0 = M + q^- - p_1^- - k_2^- - \underset{\sim}{k}_3^- - p_2^- - k_4^- - \ell_3^-,$$

which, by virtue of (28), in the limit $\nu\to\infty$ and $\xi\to 0$ transform into

$$D_3^A \simeq -\ell_3^-, \qquad D_3^B \simeq -(p_2^+ + p_3^+) \simeq O(m^2/\xi M).$$

Note that Z - diagram (Fig.4) is suppressed by the energy denominator only as $\xi\to 0$ since in the regime $P_z\to\infty$ apart from $|\vec{q}|\to\infty$ an additional condition $(p_1+p_2+p_3)^2/|\vec{q}|\to\infty$ should be satisfied. All the rest of D_ℓ in the limit $\xi\to 0$ like D_3^A turn out to be finite quantities dependent only on momenta of the wave function of a hadron at rest. Therefore, one may think that the contribution from diagram of Fig. 4B is suppressed in comparison with 4A; however, in the case of spinor quarks the decreasing contribution of Z -diagram owing to the energy denominator D_3^B is compensated by the increasing as $\xi\to 0$ projection operator $\hat{\underset{\sim}{k}}_3 + m$. Moreover, in our case just this diagram corresponds to the leading asymptotics of structure functions. Such a cancellation takes place in the field theory of null plane and the contribution from a fermion moving backward is taken into account by an instanteneous propagator part $\gamma^+/\underset{\sim}{k}_3^+$ usually denoted by the vertical spinor line [2,10]. In the leading in order for the tensor $W_{\mu\nu}$ the contribution from diagrams of Fig.4A and B has the following form:

$$W^{A}_{\mu\nu}=\frac{(\frac{1}{3}C_F)^2}{2\pi^2}\int(d\vec{\ell})_3\frac{\overset{*}{\Phi}_{\vec{0}}(\vec{\ell}_1,\vec{\ell}_2\vec{\ell}_3)}{[\ell_3^-(\ell_2^-+\ell_3^-)]^2}\hat{V}^{A}_{\mu\nu}\frac{\Phi_{\vec{0}}(\vec{\tau}_1,\vec{\tau}_2\vec{\tau}_3)}{[\tau_3^-(\tau_2^-+\tau_3^-)]^2}(d\vec{\tau})_3\prod_{i=1}^{3}\frac{d\vec{p}_i}{2p_i^0}\prod_{i=1}^{4}\frac{\alpha_s(c_i/M\varrho)\,\Delta(p_1,p_2,p_3)}{[(p_2^++p_3^+)\,p_3^+]^2},$$

$$W^{B}_{\mu\nu}=\frac{(\frac{1}{3}C_F)^2}{2\pi^2}\int(d\vec{\ell})_3\frac{\overset{*}{\Phi}_{\vec{0}}(\vec{\ell}_1\vec{\ell}_2\vec{\ell}_3)}{\ell_3^-(\ell_2^-+\ell_3^-)^2}\hat{V}^{B}_{\mu\nu}\frac{\Phi_{\vec{0}}(\tau_1\tau_2\tau_3)}{\tau_3^-(\tau_2^-+\tau_3^-)^2}(d\vec{\tau})_3\prod_{i=1}^{3}\frac{d\vec{p}_i}{2p_i^0}\prod_{i=1}^{4}\frac{\alpha_s(c_i/M\varrho)\,\Delta(p_1p_2p_3)}{(p_2^++p_3^+)^4(p_3^+)^2},$$

where

$$\hat{V}^{A}_{\mu\nu}=p_1^+\bar{U}(\ell_1)\gamma_\mu\gamma^-\gamma^s\gamma^-\gamma^\rho\gamma^-\gamma_\nu U(\tau_1)\otimes\bar{U}(\ell_2)\gamma^{s'}\frac{(\hat{K}_3+m_3)}{2K_3^0}\gamma^\delta(\hat{p}_2+m_2)\gamma^\sigma\frac{(\hat{K}_3+m_3)}{2K_3^0}\gamma^{\rho'}U(\tau_2)\otimes$$

$$\otimes\bar{U}(\ell_3)\gamma^{\delta'}(\hat{p}_3+m_3)\gamma^{\sigma'}U(\tau_3)d_{ss'}d_{\rho\rho'}d_{\delta\delta'}d_{\sigma\sigma'},$$

$$\hat{V}^{B}_{\mu\nu}=p_1^+\bar{U}(\ell_1)\gamma_\mu\gamma^-\gamma^s\gamma^-\gamma^\rho\gamma^-\gamma_\nu U(\tau_1)\otimes\bar{U}(\ell_2)\gamma^{s'}\frac{(\hat{\underset{\sim}{K}}_3-m_3)}{2\underset{\sim}{K}_3^0}\gamma^\delta(\hat{p}_2+m_2)\gamma^\sigma\frac{(\underset{\sim}{K}_3-m_3)}{2\underset{\sim}{K}_3^0}\gamma^{\rho'}U(\tau_2)\otimes$$

$$\otimes\bar{U}(\ell_3)\gamma^{\delta'}(\hat{p}_3+m_3)\gamma^{\sigma'}U(\tau_3)d_{ss'}d_{\rho\rho'}d_{\delta\delta'}d_{\sigma\sigma'}.$$

$d_{\rho s}$ is the gluon projection operator.

Note that the first denominator in $W^{A}_{\mu\nu}$ corresponds to the product $D_1D_2D_3D_4$ and the last denominator originates from the multipliers $(K_2^0\cdot K_4^0)^{-1}$ corresponding to the gluon lines. The last multiplier $W^{B}_{\mu\nu}$ has the square energy denominator D_3^B apart from $(K_2^0\cdot K_4^0)^{-1}$. In $\hat{V}_{\mu\nu}$ $\hat{p}_1+m_1\sim p_1^+\gamma^-$ and $\hat{k}_1+m_1\sim k_1^+\gamma^-$ are taken into account. It follows from (28) that $p_2^-,\,p_3^-<M\varrho$ therefore, in the leading in ϱ order we have

$$\hat{V}^{A}_{\mu\nu}=\left(\frac{1}{2}\right)^4(p_1^+p_2^+p_3^+)\,\bar{U}(\ell_1)\gamma_\mu\gamma^-\gamma^s\gamma^-\gamma^\rho\gamma^-\gamma_\nu U(\tau_1)\otimes\bar{U}(\ell_2)\gamma^{s'}\gamma^-\gamma^\delta\gamma^-\gamma^\sigma\gamma^-\gamma^{\rho'}U(\tau_2)\otimes$$

$$\otimes\bar{U}(\ell_3)\gamma^{\delta'}\gamma^-\gamma^{\sigma'}U(\tau_3)d_{ss'}d_{\rho\rho'}d_{\delta\delta'}d_{\sigma\sigma'},$$

$$\hat{V}^{B}_{\mu\nu}=\left(\frac{1}{2}\right)^4(p_1^+p_2^+p_3^+)\,\bar{U}(\ell_1)\gamma_\mu\gamma^-\gamma^s\gamma^-\gamma^\rho\gamma^-\gamma_\nu U(\tau_1)\otimes\bar{U}(\ell_2)\gamma^{s'}\gamma^+\gamma^\delta\gamma^-\gamma^\sigma\gamma^+\gamma^{\rho'}U(\tau_2)\otimes$$

$$\otimes\bar{U}(\ell_3)\gamma^{\delta'}\gamma^-\gamma^{\sigma'}U(\tau_3)d_{ss'}d_{\rho\rho'}d_{\delta\delta'}d_{\sigma\sigma'}.$$

In $\hat{V}^{B}_{\mu\nu}$ we have taken into account that $\vec{\underset{\sim}{K}}_3=-\vec{K}_3$ and therefore $\hat{\underset{\sim}{K}}_3-m\sim\underset{\sim}{K}_3^-\gamma^+=K_3^+\gamma^+$. In the diagonal gauge $d_{\mu\nu}=-g_{\mu\nu}$ by virtue of $(\gamma^-)^2=(\gamma^+)^2=0$ we have $V^{A}_{\mu\nu}=0$, i.e. as $\varrho\to0$ the leading contribution is from diagram of Fig.4B.

$$\hat{V}^{B}_{\mu\nu}=(p_1^+p_2^+p_3^+)\bar{U}(\ell_1)\gamma_\mu\gamma^-\gamma_\nu U(\tau_1)\otimes\bar{U}(\ell_2)\gamma^s\gamma^-\gamma^\rho U(\tau_2)\otimes\bar{U}(\ell_3)\gamma_s\gamma^-\gamma_\rho U(\tau_3).$$

Thus, for the tensor $W_{\mu\nu}$ we get

$$W_{\mu\nu}=\varrho^3\mathcal{M}(\tfrac{1}{3}C_F)^2\int(d\vec{\ell})_3\frac{\overset{*}{\Phi}_{\vec{0}}(\vec{\ell}_1\vec{\ell}_2\vec{\ell}_3)}{\ell_3^-(\ell_2^-+\ell_3^-)^2}\hat{\omega}_{\mu\nu}\frac{\Phi_{\vec{0}}(\tau_1\tau_2\tau_3)}{\tau_3^-(\tau_2^-+\tau_3^-)^2}(d\vec{\tau})_3,$$

where

$$\mathcal{M}=\int\frac{dp_2^-}{2p_2^-}\frac{dp_3^-}{2p_3^-}d\vec{p}_{2\perp}^{\,2}d\vec{p}_{3\perp}^{\,2}\frac{\delta(p_2^-+p_3^--M)(p_3^-)^2\prod^{4}\alpha_s(c_i/M\varrho)}{(\vec{p}_{\perp2}^{\,2}+m_2^2)^2\left[\frac{\vec{p}_{\perp2}^{\,2}+m_2^2}{p_2^-}+\frac{\vec{p}_{3\perp}^{\,2}+m_3^2}{p_3^-}\right]^4}$$

and

$$\omega_{\mu\nu}=\bar{U}(\ell_1)\gamma_\mu\gamma^-\gamma_\nu U(\tau_1)\otimes\bar{U}(\ell_2)\gamma^s\gamma^-\gamma^\rho U(\tau_2)\otimes\bar{U}(\ell_3)\gamma_s\gamma^-\gamma_\rho U(\tau_3).$$

The version of perturbation theory for the structure functions of deep inelastic process (1) suggested in this paper can successfully be applied in the case of an arbitrary number of constituents. As a result, the investigated physical characteristics are written in terms of the wave functions of a bound state at rest that have a clearer physical meaning than in the system $P_z\to\infty$. Of special interest such a formalism should be for studying composite systems (for instance, of nuclei) whose wave functions in the rest system have already been studied in other processes.

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Хведелидзе А.М., Квинихидзе А.Н., Сисакян А.Н. E2-87-543
Глубоконеупругое рассеяние в формализме с волновыми функциями покоящихся составных систем

Глубоконеупругий процесс лептон-адронного рассеяния изучается в системе покоя связанного состояния. Предлагается новый вариант разложения структурных функций по константе взаимодействия, каждый член которого обладает свойством спектральности. Показано, что в системе покоя составной частицы импульсное приближение недостаточно для корректного описания упругого предела $x_{Bj} \to 1$, в отличие от системы $P_z \to \infty$. Для получения ведущей асимптотики структурных функций при $x_{Bj} \to 1$ необходим учет взаимодействия составляющих в конечном состоянии. На примере связанного состояния двух и трех частиц указаны соответствующие диаграммы, расчет которых в модели КХД находится в согласии с результатами, полученными в формализме $P_z \to \infty$.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1987

Khvedelidze A.M., Kvinikhidze A.N., Sissakian A.N. E2-87-543
Deep Inelastic Scattering in the Formalism with the Wave Functions of Composite Systems at Rest

A deep inelastic process of lepton-hadron scattering is studied in the bound-state rest frame. A new version of expansion of structure functions over an interaction constant is proposed, each term in it having spectral properties. It is shown that the impulse approximation is insufficient for a correct description of the elastic limit in the composite particle rest frame in contrast with the system $P_z \to \infty$. The leading asymptotics of the structure functions as $x_{Bj} \to 1$ can be obtained by allowing for the interaction of constituents in a final state. Using as an example a bound state of two and three particles it is shown that the results of calculations of the relevant diagrams in the QCD model are in agreement with those obtained in the formalism $P_z \to \infty$.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1987