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CORRELATIONS AMONG STATIC QUARK MASSES

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1. INTRODUCTION

As is well known, more or less theoretically motivated nonrelativistic quarkonia potentials fit flavour invariantly charmonium and bottonium data with surprising accuracy (see review articles [1,2]). The potential acting between a quark and antiquark is usually assumed to be local, central and velocity independent in the nonrelativistic limit. We consider the class of potentials obeying the QCD motivated limiting constraints

$$r\cdot V(r) \xrightarrow[r\to 0]{} const \qquad V(r)/r \xrightarrow[r\to\infty]{} const. \tag{1}$$

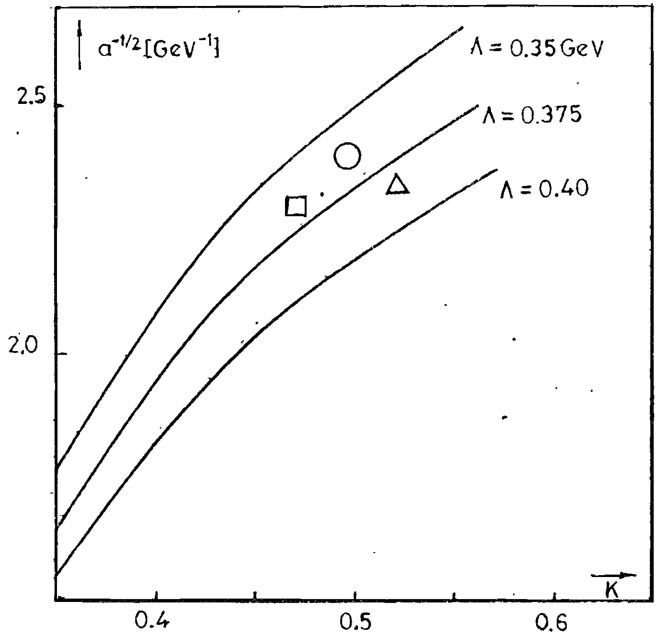
The more exact small distance behaviour due to perturbative loop corrections is beyond our discussion. Between the limiting structure (1) the potentials $V(r)$ are initially arbitrary containing one or more open parameters to be fitted to the $c\bar{c}$ and $b\bar{b}$ data in the intermediate region $0.1 \lesssim r \lesssim 1.0$ fm. The static quark masses m_c and m_b appear as additional fit parameters in the equation

$$E_n(m_q) + 2m_q = M_n(q\bar{q}) \qquad (q=b,c) \tag{2}$$

relating the Schroedinger energy levels E_n to the quarkonia masses M_n. Different potentials reproduce the data optimally if their quark masses deviate from one another by energies up to an order of 0.1 GeV.

We proceed as follows: Fine deviations between data fits of different potentials will be interpreted as "perturbative" effects. Thus choosing two arbitrary potentials $V_A(r)$ and $V_B(r)$, we state the simple relation

$$V_A(r) = V_B(r) + W_{AB}(r), \tag{3}$$

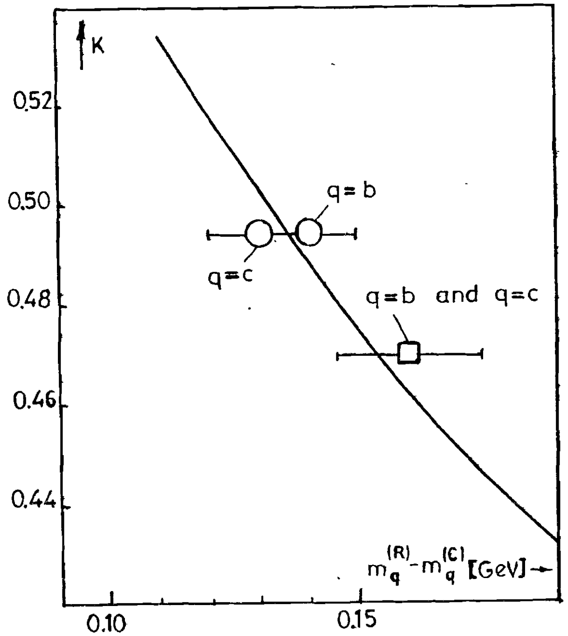


F i g. 1. Correlation between the Cornell fit parameters a and K as prescribed by Richardson's potential for three values of its parameter Λ. Comparison with three Cornell fits of Eichten et al. ([3], △), Hagiwara et al. ([8], □) and Miller and Olsson ([9], ○).

where W_{AB} appears as perturbative potential. At first sight such a simple additive relation between two potentials of rather different structure is not obvious. We arrive at eq.(3) using a method in which the simplest successful potential of the class, the two-parameter Cornell potential [3]

$$V_c(r) = ar - \frac{k}{r} \tag{4}$$

appearing as a superposition of the limiting constraints (1), plays a special role. Considering a potential $V_A(r) \equiv f_A(r)/r$ we extract the structure (4) and a constant term by Taylor expansion of $f_A(r)$ at a nonzero point r_0 which is not fixed initially but should be restricted to the physically relevant region between 0.1 and 1.0 fm. Because of their common dependence on r_0 the extracted terms are correlated with one another and with the rest of the series and can be tuned to such a degree that for a special value of r_0 one gets



F i g. 2. Flavour-invariant correlation between the Cornell fit parameter K and the quark mass difference $m_q^{(R)} - m_q^{(C)}$ from Richardson's potential. Comparison with the Cornell fits of refs. [8,9] (The third fit of Fig.1 is eliminated here because of its extreme mass m_c = 1.84 GeV). Uncertainties of the mass fits are indicated.

$V_A = V_C + W_{AC}$. Subtracting a second arbitrary potential $V_B = V_C + W_{BC}$ we obtain $V_A = V_B + W_{AC} - W_{BC} \equiv V_B + W_{AB}$ and have reproduced eq.(3).

Because of our combined study of two potentials we get correlations between the parameters of different potentials. As a special example, we have calculated how the one-parameter Richardson potential [4] prescribes the correlation between the coulombic term and the string tension of eq.(4) and between K and the quark mass difference $m_q^{(R)} - m_q^{(C)}$ containing the quark masses of the Richardson and the Cornell potential (Figs. 1,2). As a more general application, we study the relation between the constant term extracted by Taylor expansion and mass differences $m_q^{(B)} - m_q^{(A)}$ from arbitrary potentials. Charm-bottom flavour invariance of the potentials entails stability of $m_b - m_c$ against change of the potentials. This confirms indications by earlier work of other authors [5-7].

2. CORRELATIONS

Coming to the details we start with the ansatz

$$V(r) = f(r)/r \,, \tag{5}$$

where $V(r)$ represents some potential obeying eq.(1) and $f(r)$ is assumed to be regular for $r > 0$. Using the decomposition

$$f(r) = f_1(r, r_0) + f_2(r, r_0) \tag{6}$$

$$f_1(r, r_0) = f(r_0) + f^{(1)}(r_0)(r - r_0) + f^{(2)}(r_0)\frac{(r-r_0)^2}{2} \tag{7}$$

we obtain by reordering the terms according to powers of r

$$V(r) = V_1(r, r_0) + W(r, r_0) \tag{8}$$

$$V_1(r, r_0) = u(r_0)\, r + \frac{v(r_0)}{r} + w(r_0) \tag{9}$$

$$u(r_0) = \frac{1}{2} f^{(2)}(r_0) \tag{10}$$

$$v(r_0) = f(r_0) - r_0 f^{(1)}(r_0) + \frac{1}{2} r_0^2 f^{(2)}(r_0) \tag{11}$$

$$w(r_0) = f^{(1)}(r_0) - r_0 f^{(2)}(r_0) \tag{12}$$

and

$$W(r, r_0) = f_2(r - r_0, r_0)/r \,. \tag{13}$$

Only the three-term structure (7) of $f_1(r, r_0)$ guarantees that, on one hand, the Cornell terms can be extracted and, on the other hand,

$$f_2(r, r_0) \equiv f_2(r - r_0, r_0) \tag{14}$$

provides sensitivity of W as a function of r_0 in the region $0.1 \lesssim r, r_0 \lesssim 1.0$ fm which is needed to minimize its effect on data reproduction. Including the mass shift $2m_q$ of eq.(2) in order to get the quarkonia masses M_n as eigenvalues, we obtain

$$V(r) + 2m_q = u(r_0)\,r + \frac{v(r_0)}{r} + \left[w(r_0) + 2m_q\right] + W(r, r_0) \tag{15}$$

having extracted the limiting structure and a constant term from $V(r)$. These three terms are correlated by their common dependence on r_0. We remark that inclusion of a constant term into Cornell fits was discussed already by Hagiwara et al.[8] . It means that a slight variation of one fit parameter requires a corresponding variation of the other ones including the quark mass m_q .

If u , v and w are monotone functions of r_0 , elimination of r_0 yields unique correlations

$$u[r_0(v)] \qquad u[r_0(w)] \qquad v[r_0(w)] \tag{16}$$

which are more suitable for application. Now we compare eq.(15) with the Cornell potential (4) including the corresponding mass shift

$$V_c(r) + 2m_q^{(c)} = ar - \frac{k}{r} + 2m_q^{(c)}. \tag{4a}$$

Writing, according to eq.(3)

$$V(r) = V_c(r) + W(r, r_0^{(c)}) \tag{17}$$

we obtain from eqs. (15), (4a) and (17) the conditions

$$u(r_0^{(c)}) = a \qquad v(r_0^{(c)}) = -k \tag{18}$$

and

$$w(r_0^{(c)}) = 2(m_q^{(c)} - m_q), \tag{19}$$

where r_0 now is fixed at $r_0 = r_0^{(c)}$ minimizing the perturbation W. If one Cornell fit parameter is given, $V(r)$ provides the other ones via eqs. (10)–(12), (18) and (19). We illustrate this in section (4) by a special example.

3. THE STABILITY OF THE DIFFERENCE $m_b - m_c$

Eq.(19) containing the Cornell fit quark mass $m_q^{(c)}$ and the quark mass m_q of some other potential can be generalized to quark

masses $m_q^{(A)}$ and $m_q^{(B)}$ of two arbitrary potentials V_A and V_B . Instead of eq.(19) now we have two equations

$$w_A(r_A^{(c)}) = 2(m_q^{(c)} - m_q^{(A)}) \tag{19a}$$

$$w_B(r_B^{(c)}) = 2(m_q^{(c)} - m_q^{(B)}), \tag{19b}$$

where the points $r_A^{(c)}$ and $r_B^{(c)}$ correspond to $r_0^{(c)}$. Subtraction leads to

$$m_q^{(B)} - m_q^{(A)} = \frac{1}{2}\left[w_A(r_A^{(c)}) - w_B(r_B^{(c)})\right]. \tag{20}$$

This more general equation has a relevant consequence. Provided, the potentials of the class (1) are charm-bottom flavour invariant, the right-hand side of eq.(20) depending on the shape of the potentials alone appears as a flavour-invariant quantity entailing

$$m_b^{(B)} - m_b^{(A)} = m_c^{(B)} - m_c^{(A)} \qquad (q = b, c) \tag{21}$$

or

$$(m_b - m_c)^{(B)} = (m_b - m_c)^{(A)} \tag{22}$$

Eq.(22) exhibits the difference $m_b - m_c$ as a potential-independent quantity. This result appears as a consequence of the charm-bottom flavour invariance of the potentials. It explains the well-known observation that $m_b - m_c$ shows much smaller dependence on the choice of the potential than the fit masses m_b and m_c themselves. Table 1 containing four examples of potentials clearly indicates this effect. The conjecture of relative stability of $m_b - m_c$ is not new. As early as in 1980 Bertlmann and Martin[5] obtained a relatively small interval $3.36 < m_b - m_c < 3.69$ GeV for $1.1 < m_c < 1.7$ GeV by a more direct evaluation of the data. Also the results of Quigg, Rosner and Thacker[6] , $m_b - m_c = 3.455$ GeV, obtained by the inverse scattering method and of the ITEP group[7] , $m_b - m_c = 3.4$ GeV obtained by sum rules from Q^2 duality, fall into these limits. All these investigations a priori more or less avoid potential model assumptions. On the other hand the above approach correlates different potentials which are subjected only to rather general conditions.

Table 1. Examples of quark masses m_b, m_c and their differences $m_b - m_c$ from four potentials indicating the relative stability of $m_b - m_c$ (masses in GeV)

Potential	m_c	m_b	$m_b - m_c$
Cornell [3,8]	1.35	4.77	3.42
Richardson [4,8]	1.50	4.91	3.41
Hagiwara et al. [11]	1.46	4.87	3.41
Miller, Olsson [9]	1.36±0.17	4.77±0.15	3.41±0.02

4. CORRELATIONS BETWEEN THE CORNELL AND THE RICHARDSON POTENTIAL

To illustrate the general mechanism of section 2 by a special example, we study the correlations (16) using as $V(r)$ the Richardson potential [4] which is obtained after Fourier transformation from the momentum space and Wick rotation in the complex p plane in the form

$$V_R(r) \equiv \frac{f_R(r)}{r} \equiv \frac{\Lambda f_R(t)}{t} = \frac{8\pi}{27}\frac{\Lambda}{t}\left\{t^2 - 1 + 4\int_1^\infty \frac{dp}{p}\,\frac{e^{-pt}}{[\ln(p^2-1)]^2+\pi^2}\right\}, \quad (23)$$

where $t = \Lambda r$. Its deviation from the considered class (1) by a logarithmic factor $[\ln \Lambda r]^{-1}$ for $r \to 0$ is not relevant here. Using eqs.(10)-(12) we have calculated the functions u, v and w in a region $0.1 \lesssim r_0 \lesssim 1.0$ fm. The integrals coming from (23) must be evaluated numerically. u, v and w appear as monotone functions of r_0 with $v(r_0) < 0$ as required by the coulombic term. The first of the relations (16) $u(v) = a(k)$ is drawn in Fig.1 for three values of Richardson's fit parameter Λ. The Λ fit of the present data requires a value near 0.375 GeV [8]. Comparison with three Cornell fits [3,8,9] shows that the curve with Λ = = 0.375 GeV goes through the center of the fit region as required by the mechanism of section 2. The third of the correlations (16) relates the Cornell fit parameter K with the mass difference $m_q^{(R)} - m_q^{(C)}$: $v(w) = k(m_q^{(R)} - m_q^{(C)})$. It is drawn in Fig. 2 for Λ = 0.375 GeV and again the curve goes through the

fit region. Its flavour-invariant prediction for a given K clearly remains within the limits of error of the mass fits. Taking, for instance the fit of Miller and Olsson [9] with K = 0.494, the mass fits yield $m_b^{(R)} - m_b^{(C)}$ = 0.14 ± 0.02 and $m_c^{(R)} - m_c^{(C)}$ = 0.13 ± 0.02. Our flavour-invariant prediction is $m_q^{(R)} - m_q^{(C)}$ = 0.137 GeV $(q = b, c)$.

5. SUMMARY

Considering a class of charm-bottom flavour-invariant nonrelativistic potentials which obey conditions (1) we have obtained additive relations of the type (3) between two arbitrary potentials of the class. Thus, small differences between data fits by different potentials in principle can be calculated perturbatively. Such a procedure should be useful to isolate common features of potentials which at first sight are rather different in structure. In addition, it can help to recognize invariance properties against change of the potential, as shown by the example of section 3. We have described analytically correlations between fit parameters of different potentials but also of the same potential, especially between the string tension a and the coefficient of the coulombic term K in the Cornell potential (Fig. 1). Such relations should be of interest also for potentials with more complicated limiting behaviour at small interquark distances.

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Левин К., Моц Г.Б. E2-87-506
Корреляции между статистическими кварковыми массами

Нерелятивистские потенциалы тяжелых кваркониев, ограниченные кулоновским и линейно растущим поведением, аддитивно связываются разложением Тейлора, при этом извлекается постоянный член и предельная структура. Получаются соотношения между параметрами различных потенциалов, в том числе между массами кварков m_b и m_c. Известная стабильность разности m_b-m_c является прямым следствием независимости потенциалов от кварковых ароматов.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Lewin K., Motz G.B. E2-87-506
Correlations Among Static Quark Masses

Nonrelativistic heavy quarkonia potentials with coulombic and linearly rising limiting behaviour are correlated additively by Taylor expansion extracting the limiting structure and a constant term. Relations between fit parameters of different potentials including the quark masses m_b and m_c, are obtained. The known stability of the difference m_b-m_c appears as direct consequence of flavour invariance of the potentials.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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