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исследований
дубна

E2-87-484

A.I.Golokhvastov

ACCURATE MULTIPLICITY SCALING

Submitted to "Physics Letters B"

1987

INTRODUCTION. Koba, Nielsen and Olesen have formulated the statement of independence of the multiplicity distribution shape of the energy of primary particles [1]. This statement was formulated for very high energies, i.e. very large multiplicities, when one can operate with multiplicity distribution as with continuous function. Figure 1a depicts a possible picture of these functions for various primary energies. The area under each curve is equal to unity since it is the sum of all the probabilities. The average multiplicity increases with energy. Each curve can be compressed along the horizontal axis proportionally to any of its horizontal dimensions, e.g. $\langle n \rangle$ as in fig.1b, and stretched along the vertical axis by the same factor in order to make the areas equal again (fig.1c). The statement of KNO scaling consists in that the curves coincide at each point [2]. Figure 1c can be written in the form

$$P_n=1/\langle n\rangle\, \psi(n/\langle n\rangle), \tag{1}$$

where $\psi(z)$ is an energy-independent function normalized by the conditions

$$\int \psi(z)dz=1, \tag{2}$$

which follows from the equality of the sum of all probabilities to unity and

$$\int z\psi(z)dz=1, \tag{3}$$

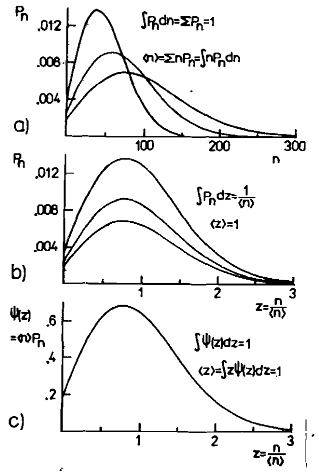
because we compressed the functions P_n until the average value of each function reached unity. Formula (1) puts no restraints, except (2) and (3), in the shape of the function $\psi(z)$. It is merely a definition of the concept of similarity for continuous functions.

The multiplicity distributions of all charged particles are commonly studied. However, some problems arise in this case - the consideration of protons and π-mesons together seems to be incorrect; it is unclear whether leading particles should be included in the distributions; there appears a trivial nonuniformity in the distribution due to the charge conservation: all odd probabilities are equal to zero. In order not to solve these problems, let us consider the multiplicity distributions of negative hadrons (in fact, π^--mesons) for PP and e^+e^- interactions. They are one-to-one related to the distributions of charged particles

$$n_{ch}=2\,n_{neg}+2 \qquad (4)$$

for PP interactions and $n_{ch}=2\,n_{neg}$ for e^+e^- interactions. Further the multiplicity of negative particles is designated as n.

Fig.1. Definition of the concept of similarity for continuous functions (KNO scaling). The normalized functions (a) are similar if after linear compression of each function along the horizontal axis in proportion to any of its horizontal dimensions, e.g. $\langle n\rangle$ (b), and linear stretching along the vertical axis by the same factor (c), they coincide at each point.



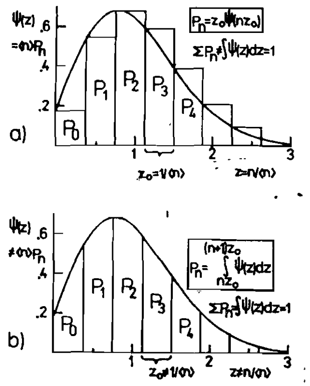


Fig.2. Obtaining of the discrete multiplicity distribution from the continuous normalized universal function $\Psi(z)$. (a) - according to the commonly used recipe $P_n=1/\langle n\rangle\,\Psi(n/\langle n\rangle)$, then $\Sigma P_n \neq 1$. (b) - according to the correct recipe.

CONTRADICTION. For present-day accelerator energies the function P_n is essentially discrete: the condition $\langle n\rangle \gg 1$ ($\langle n_{ch}\rangle \gg 2$) is not fulfilled. For example, $\langle n\rangle\approx 2$ at $P_{lab.}$=100 GeV/c and $\langle n\rangle\approx 5$ at 2000 GeV/c. In this case, irrespective of any physical considerations, formula (1) becomes mathematically incorrect because it contradicts the condition $\Sigma P_n=1$ as shown in fig.2a. To obtain some multiplicity distribution having a given value of $\langle n\rangle$ from the continuous universal function $\Psi(z)$ in fig.2a, the inverse operation to that in fig.1 should be done, i.e. the scale $z_o=1/\langle n\rangle$ should be chosen on the z axis. Then the probability P_n is equal to the area of the rectangle which touches the curve $\Psi(z)$ by its left vertex at the point $z=nz_o=n/\langle n\rangle$. The height of the rectangle is $\Psi(n/\langle n\rangle)=\langle n\rangle P_n$ and its base $1/\langle n\rangle$. For very small values of z_o the sum of the areas of the rectangles (total probability) equals the area under the curve, i.e. it is equal to unity. However, with increasing z_o these areas cannot remain equal at each value of z_o. Our "numerical integration" becomes too rough. Figure 2a approximately corresponds to the multiplicity distribution in PP interactions at $P_{lab.}$=100 GeV/c.

Thus, in order to test the hypothesis of similarity of multiplicity distributions, the concept of similarity for discrete functions is first to be defined.

GENERALIZATION. An obvious generalization of the recipe of obtaining all multiplicity distributions from one universal function $\Psi(z)$ is shown in fig.2b. It is seen that the sum of probabilities is always equal to unity, and for $z\to 0$ the figures a and b coincide. This can be expressed as [3]

$$P=\int_{nz_o}^{(n+1)z_o}\Psi(z)dz. \qquad (5)$$

If one introduces a continuous parameter $m\equiv z/z_o$ which fills up gaps on the discrete axis n in fig.1a, formula (5) can be rewritten in the form

$$P_n=\int_{n}^{n+1}P(m)dm, \qquad (6)$$

where

$$P(m)=1/\langle m\rangle\,\psi(m/\langle m\rangle), \qquad (7)$$

with

$$\langle m\rangle=\int mP(m)dm=1/z_o\ . \qquad (8)$$

Thus, the discrete multiplicity distribution is presented as a histogram from the continuous function having KNO-invariant properties. One can say that the definition of the concept of similarity remained the definition for continuous functions. Only the recipe of obtaining the discrete distribution from the continuous function was changed. Instead of the inconsistent recipe actually used in (1): $P_n=P(m)\big|_{m=n}$, we deal now with the correct recipe (6). Almost the same method of obtaining multiplicity distributions from the continuous normalized functions, which was not yet KNO-invariant, was used in papers [4].

COMPARISON WITH EXPERIMENT. Multiplicity distributions in inelastic PP interactions at $P_{Lab.}=1.5\div2000$ GeV/c ([5,6] and references in [6,7]) and in e^+e^- annihilation at $\sqrt{s}=3\div35$ GeV ([8] and references there) are used for comparison in this paper. A detailed comparison is also made in [7].

As seen from fig.2b, if we have an experimental multiplicity distribution at some energy, we can obtain a distribution for a lower energy corresponding to z_o which is twice as much. In this case $P_0'=P_0+P_1$; $P_1'=P_2+P_3$; $P_2'=P_4+P_5$ and so on. The same can be repeated for $z_o''=3z_o$: $P_n''=P_{3n}+P_{3n+1}+P_{3n+2}$ and so on. A comparison of the points obtained by such a method from ISR data with those at lower energies is made in figs.3 and 4. One can see that they coincide down to the lowest energies. Figure 4 also shows the lower limits of values of D_q: D_q is minimum for a given value of $\langle n\rangle$ when only two neighbouring probabilities P_n are not equal to zero [9].

The data of fig.4 from $\langle n\rangle\geq1$ are well described by a linear dependence $D_q\propto(\langle n\rangle+0.5)$. This dependence for all charged particles looks like [10] $D_q^{ch}\propto(\langle n_{ch}\rangle-1)$ taking (4) into account. It is easy

to show that the central moments of the continuous KNO invariant function P(m) obey similar relations $(\mu_q)^{1/q}\equiv(\int(m-\langle m\rangle)^q P(m)dm)^{1/q}\propto\langle m\rangle$. And from formula (6), for not too small values of $\langle n\rangle$, one can get the following approximate equalities [7]

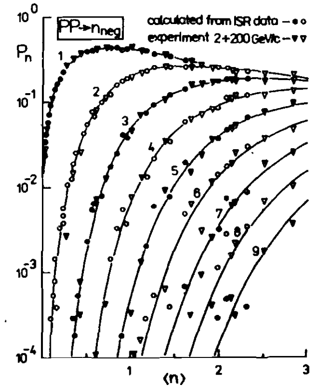
$$\langle m\rangle=\int mP(m)dm=\sum_n\int_n^{n+1}mP(m)dm\approx\sum_n(n+0.5)\int_n^{n+1}P(m)dm=\sum_n(n+0.5)P_n=$$
$$=\langle n\rangle+0.5; \qquad (9)$$

and also

$$\mu_q=\int(m-\langle m\rangle)^q P(m)dm=\sum_n\int_n^{n+1}(m-\langle m\rangle)^q P(m)dm\approx$$
$$\approx\sum_n((n+0.5)-(\langle n\rangle+0.5))^q\int_n^{n+1}P(m)dm=\sum_n(n-\langle n\rangle)^q P_n=D_q^q. \qquad (10)$$

Therefore, contrary to the commonly used quantities $C_q=\langle n^q\rangle/\langle n\rangle^q$, the ratios $(\langle n\rangle+0.5)/D_2$ and D_q/D_2 go fastly to the plateau with increasing the collision energy if the multiplicity scaling (5) is valid as seen in figs.5 and 6. The presented errors are calculated

Fig.3. Comparison of the experimental multiplicity distributions with the multiplicity distributions calculated from those obtained for higher energies according to the recipe $P_n'=P_{2n}+P_{2n+1}$; $P_n''=P_{3n}+P_{3n+1}+P_{3n+2}$ and so on (see fig.2b).



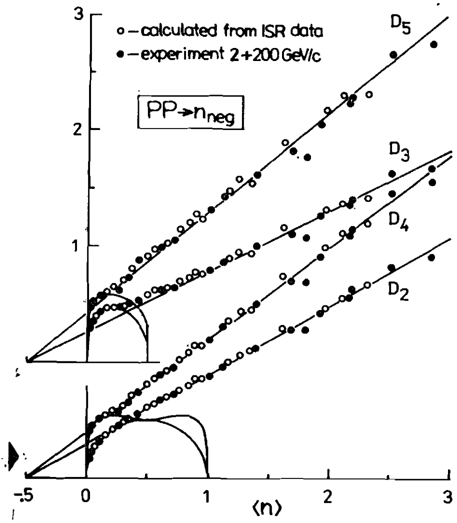
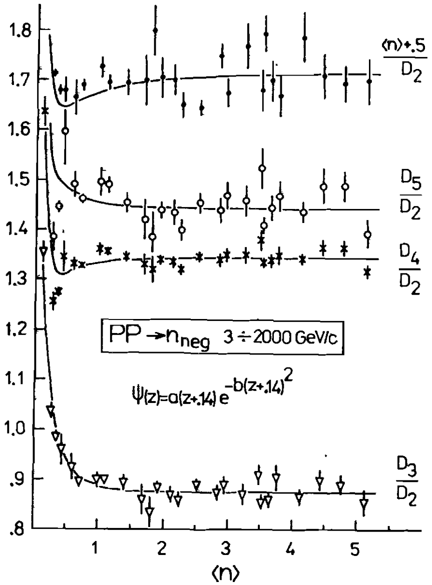


Fig.4. $D_q=(\sum(n-\langle n\rangle)^q P_n)^{1/q}$ versus $\langle n\rangle$. The calculated points are obtained as in fig.3.

Fig.5. The quantities which should go fastly to the plateau with increasing energy if the accurate multiplicity scaling is valid. The curves are obtained by formula (5) with $\psi(z)$ presented in the figure. The coefficients a and b calculated from the conditions (2) and (3) are equal to 1.251 and 0.618, respectively.



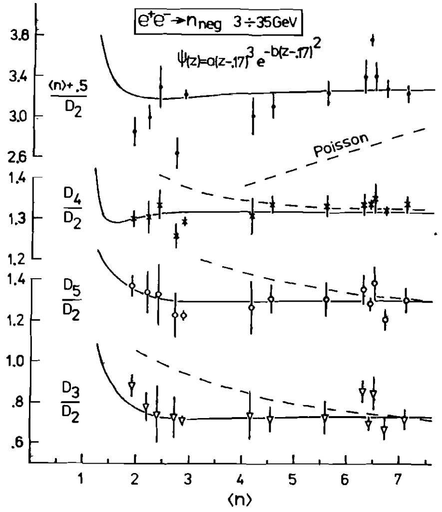
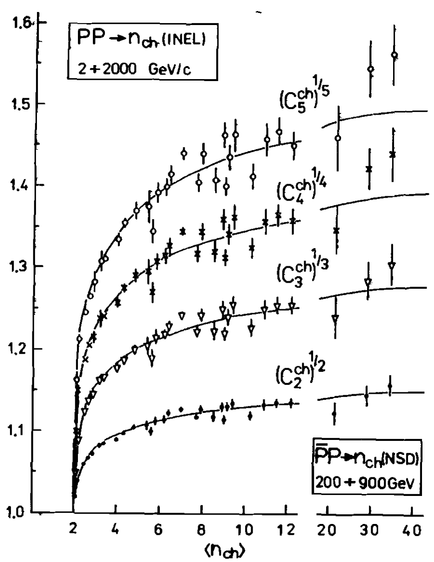


Fig.6. The same for e^+e^- interactions. For $z < 0.17$ $\psi(z)=0$. The coefficients a and b are equal to 13.16 and 2.565. Calculating the curves, P_0 was assumed to be equal to zero since it was not measured experimentally. The Poisson distribution is denoted by the dashed lines.

under the assumption of normality and independence of the published errors of the cross sections σ_n. The curves are obtained according to formula (5) by using the functions $\psi(z)$ presented in the figures. The curves of fig.3 are obtained in the same manner. A few different functions $\psi(z)$, which describe the data well too, have been found. However, all of them contain z² in the exponent [11]. The presented functions differ from those [12] only in a shift along the z axis.

Fig.7. $(C_q^{ch})^{1/q} = \langle n_{ch}^q\rangle^{1/q}/\langle n_{ch}\rangle$ versus $\langle n_{ch}\rangle$. The quantities C_q are raised to the 1/q power for stretching the scale at small q. The curves are obtained from the scaling for negative particles as in fig.5 when passing to all charged particles according to (4). Using the UA5 data on the ratio between inelastic and nondiffractive interactions [14], we obtain that the Collider points rise approximately by 1.5 errors when passing to inelastic interactions.



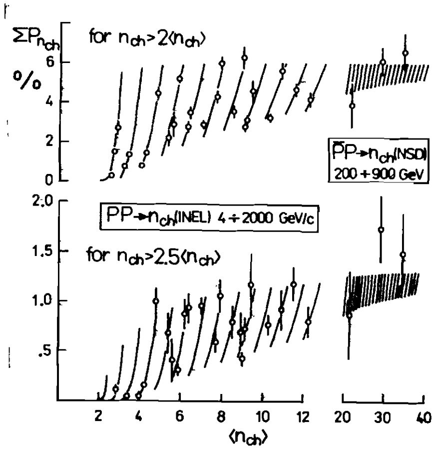


Fig.8. Percentage of events having $n_{ch} > 2\langle n_{ch}\rangle$ (or $2.5\langle n_{ch}\rangle$) versus $\langle n_{ch}\rangle$. The curves are obtained as in fig.7. Jumps of the function occurs when $2\langle n_{ch}\rangle$ ($2.5\langle n_{ch}\rangle$) becomes equal to an even integer. In this case the next probability $P_{n_{ch}}$ does not already enter into the sum.

Unfortunately, for the present there are no Collider data on all inelastic interactions. Therefore, nondiffractive Collider data [13] are presented in figs.7 and 8 for comparison. These points are likely to rise to some extent as one passes to inelastic interactions. The curves are obtained using the scaling for negative particles and formula (4).

The author is grateful to S.A.Khorozov and R.Szwed for many fruitful discussions.

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Received by Publishing Department
on June 29, 1987.

Голохвастов А.И. E2-87-484
Точный скейлинг по множественности

Показано, что используемая обычно формула KNO скейлинга $\langle n\rangle P_n = \Psi(n/\langle n\rangle)$ при конечных $\langle n\rangle$ противоречит условию нормировки $\Sigma P_n = 1$. Приведено непротиворечивое обобщение понятия подобия распределений по множественности. Анализ экспериментальных данных по PP- и e^+e^--взаимодействиям показывает, что распределения по множественности отрицательных частиц подобны во всем экспериментально исследованном интервале энергий.

Работа выполнена в Лаборатории высоких энергий ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1987

Golokhvastov A.I. E2-87-484
Accurate Multiplicity Scaling

The commonly used formula of KNO scaling $\langle n\rangle P_n = \Psi(n/\langle n\rangle)$ at finite $\langle n\rangle$ is shown to contradict the normalization condition $\Sigma P_n = 1$. A consistent generalization of the concept of similarity for multiplicity distribution is presented. Analysis of the experimental data on PP and e^+e^- interactions shows that the multiplicity distributions of negative particles are similar over the whole experimentally studied energy range.

The investigation has been performed at the Laboratory of High Energies, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1987