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MODIFIED THREE-POLE VMD MODEL WITH TWO-BRANCH-POINT ANALYTIC STRUCTURE AND APPROVED ASYMPTOTIC BEHAVIOUR FOR THE PION ELECTROMAGNETIC FORM FACTOR

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In our previous paper[1] the normalized (at $t = 0$, i.e. $q_N = i$) and analytic pion form factor (ff) model

$$F_\pi(t) = \frac{(q - q_Z)(q_N - q_P)}{(q - q_P)(q_N - q_Z)} \sum_{v=\rho,\rho',\rho''} \frac{(q_N - q_v)(q_N - q_{\bar{v}})}{(q - q_v)(q - q_{\bar{v}})} (f_{v\pi\pi}/f_v) \qquad (1)$$

with an elastic cut from $t = 4m_\pi^2$ to $+\infty$ has been constructed from the VMD parametrization

$$F_\pi^{(VMD)}(t) = \sum_{v=\rho,\rho',\rho''} \frac{m_v^2 (f_{v\pi\pi}/f_v)}{m_v^2 - t}, \qquad (2)$$

where m_v means the mass of vector mesons and $t = -Q^2$ is the photon four- momentum transfer squared. First, the parametrization (2) in the pion c.m. momentum variable

$$q = \left[(t - 4m_\pi^2)/4\right]^{1/2} \qquad (3)$$

has been transformed as follows

$$F_\pi^{(VMD)}(t) = \sum_{v=\rho,\rho',\rho''} \frac{(q_{vo}^2 - q_N^2)}{(q_{vo}^2 - q^2)} (f_{v\pi\pi}/f_v) \qquad (4)$$

then by using the well-known properties of Padè-type approximations[2,3] the contribution of the so- called left-hand cut[4,5] from the second Riemann sheet through the normalized factor in front of the sum in (1) has been included, and finally the nonzero values of vector meson width by means of the change ($m_\pi = 1$)

$$-q_{\bar{v}o} = q_{vo} \longrightarrow q_v = \left\{\left[(m_v - i\Gamma_v/2)^2 - 4\right]/4\right\}^{1/2} \qquad (5)$$

have been incorporated. The latter caused a shift of VMD poles from the real axis into the complex conjugate pairs of poles placed on the second Riemann sheet, which is generated by the square-root two-pion-threshold branch-point. The asymptotic behaviour of (1)

$$F_\pi(t) \sim t^{-1}\Big|_{t\to\pm\infty} \qquad (6)$$

coincides incidentally with the quark counting rules prediction[6,7] for the pion ff and it cannot be changed without any generation of additional singularities. Furthermore, none of the inelastic contributions are included into the pion ff model (1). So, even when there is a quite good description[1] of all existing pion ff data[8-10] by means of the parametrization (1) (χ^2/ndf = 531/282) in comparison with earlier analysed Gounaris-Sakurai type models [11,12], one can expect further improvement of the description of the same 293 experimental data points if inelastic contributions are taken into account in some way and a freedom in the choice of the asymptotic behaviour of the pion ff model is granted.

In this paper we realize the latter by using the inverse Zhukovsky transformation

$$W(t) = i\,\frac{(q_1 + q)^{1/2} - (q_1 - q)^{1/2}}{(q_1 + q)^{1/2} + (q_1 - q)^{1/2}} \qquad (7)$$

for the first time used[13] in the pion ff problems more than ten years ago. Really, the transformation (7) generalizes our approach in ref.[1] by introducing another square-root branch-point

$$t_{inel} = 4\,(q_1^2 + 1) \qquad (8)$$

the position of which will be left as a free parameter. As a consequence it will simulate inelastic contributions effectively and in a fitting procedure the data themselves will take as much inelasticity as they need.

Moreover, the same transformation of (4) in W-variable singles out a common normalized factor for all vector mesons in (4) by means of which a specific freedom in the choice of the pion ff asymptotic behaviour is possible to settle.

Now in detail. First, from (7) the relation

$$q^2 = q_1^2\left\{1 - \left(\frac{1 + W^2}{1 - W^2}\right)^2\right\} \qquad (9)$$

is found, and analogically

$$q_N^2 = q_1^2\left\{1 - \left(\frac{1 + W_N^2}{1 - W_N^2}\right)^2\right\} \qquad (10)$$

and

$$q_{vo}^2 = q_1^2\left\{1 - \left(\frac{1 + W_{vo}^2}{1 - W_{vo}^2}\right)^2\right\}. \qquad (11)$$

Then substituting (9) - (11) into (4) one gets

$$F_\pi^{(VMD)}[W(t)] = \sum_{v=\rho,\rho',\rho''} \frac{\left\{\left(\frac{1 + W_N^2}{1 - W_N^2}\right)^2 - \left(\frac{1 + W_{vo}^2}{1 - W_{vo}^2}\right)^2\right\}}{\left\{\left(\frac{1 + W^2}{1 - W^2}\right)^2 - \left(\frac{1 + W_{vo}^2}{1 - W_{vo}^2}\right)^2\right\}}\,(f_{v\pi\pi}/f_v) \qquad (12)$$

or equivalently

$$F_\pi^{(VMD)}(t) = \left(\frac{1 - W^2}{1 - W_N^2}\right)^2 \sum_{v=\rho,\rho',\rho''} \frac{(W_N - W_{vo})(W_N + W_{vo})(W_N - 1/W_{vo})(W_N + 1/W_{vo})}{(W - W_{vo})(W + W_{vo})(W - 1/W_{vo})(W + 1/W_{vo})}(f_{v\pi\pi}/f_v), \qquad (13)$$

where the pion ff asymptotic behaviour (6) is ensured by the power "2" of the common normalized factor in front of the sum. Really, for $t \to \pm\infty$ on the first Riemann sheet, $W \to -1$ and as a consequence all terms under the sum in (13) are, in the limit $t \to \pm\infty$, constant. On the other hand, the relation (9) provides

$$q = 2i\,q_1 \frac{W}{1 - W^2} \qquad (14)$$

from which it is straightforward to see that $q \sim (1 - W^2)^{-1}\big|_{t\to\pm\infty}$ and $t \sim (1 - W^2)^{-2}\big|_{t\to\pm\infty}$, or vice versa, $(1 - W^2)^2 \sim 1/t\big|_{t\to\pm\infty}$. So, the transformations (3) and (7) with a subsequent change of the power "2" in the term placed in front of the sum in (13) to an arbitrary positive integer M, which makes M-fold zero of the ff parametrization at infinity and does not change the ff analytic structure, lead to a very spontaneous generalization of the asymptotic behaviour (6) of VMD model (2) as follows

$$F_\pi(t) \sim t^{-M/2}\big|_{t\to\pm\infty}. \qquad (15)$$

Further arrangement of (13) depends on the vector meson mass values in comparison with t_{inel}. If we take into account an experience from the previous pion ff analysis[14] one can expect that $m_\rho^2 < t_{inel}$ and $m_{\rho'}^2, m_{\rho''}^2 > t_{inel}$. As a result,

$$W_{\rho o} = -W^*_{\rho o} \quad \text{and} \quad |W_{vo}| = 1 \quad \text{i.e.} \quad W_{vo} = 1/W^*_{vo} \text{ for } v=\rho',\rho'' \qquad (16)$$

and the relation (13) can be rewritten into the following form

$$F^{(VMD)}_\pi(t) = \left(\frac{1-W^2}{1-W_N^2}\right)^M \left\{ \frac{(W_N-W_{\rho o})(W_N-W^*_{\rho o})(W_N-1/W_{\rho o})(W_N-1/W^*_{\rho o})}{(W-W_{\rho o})(W-W^*_{\rho o})(W-1/W_{\rho o})(W-1/W^*_{\rho o})}(f_{\rho\pi\pi}/f_\rho) + \right. \qquad (17)$$

$$\left. + \sum_{v=\rho',\rho''} \frac{(W_N-W_{vo})(W_N-W^*_{vo})(W_N+W_{vo})(W_N+W^*_{vo})}{(W-W_{vo})(W-W^*_{vo})(W+W_{vo})(W+W^*_{vo})}(f_{v\pi\pi}/f_v)\right\}$$

which is very suitable for an incorporation of nonzero values of the corresponding vector meson resonance widths. The latter is carried out by means of the change '5) and $W_{vo} \to W_v$ with a subsequent shift of VMD poles from the real axis always into two complex conjugate pairs of poles for every resonance as follows: the poles corresponding to $\rho(770)$ meson are placed on the second and fourth Riemann sheets and the poles corresponding to $\rho'(1250)$ and $\rho''(1600)$ are placed on the third and fourth sheets of the Riemann surface.

If we take into account the left-hand cut contribution [4,5] by a normalized factor consisting of one pole and one zero on the positive real axis inside the unit circle which corresponds to the negative real axis of the second Riemann sheet in t-variable, we finally obtain the real analytic pion ff model

$$F_\pi[W(t)] = \left(\frac{1-W^2}{1-W_N^2}\right)^M \frac{(W-W_Z)(W_N-W_P)}{(W-W_P)(W_N-W_Z)} \times \qquad (18)$$

$$\times \left\{ \frac{(W_N-W_\rho)(W_N-W^*_\rho)(W_N-1/W_\rho)(W_N-1/W^*_\rho)}{(W-W_\rho)(W-W^*_\rho)(W-1/W_\rho)(W-1/W^*_\rho)}(f_{\rho\pi\pi}/f_\rho) + \right.$$

$$\left. + \sum_{v=\rho',\rho''} \frac{(W_N-W_v)(W_N-W^*_v)(W_N+W_v)(W_N+W^*_v)}{(W-W_v)(W-W^*_v)(W+W_v)(W+W^*_v)}(f_{v\pi\pi}/f_v)\right\}$$

defined on the four-sheeted Riemann surface. It includes the inelastic contributions by one effective inelastic cut in a reasonable approximation, through the power M of the first term in the right-hand side it provides a freedom in the choice of the asymptotic behaviour and depends just on the physical parameters m_v, Γ_v, $f_{v\pi\pi}/f_v$ $(v=\rho,\rho',\rho'')$, t_{inel}, M, W_P, W_Z, for which there is a restriction

$$\sum_{v=\rho,\rho',\rho''} (f_{v\pi\pi}/f_v) = 1 \qquad (19)$$

following from the normalization condition of (18). In order to evaluate these parameters from existing pion ff data [8-10] numerically, the isospin violating omega two-pion decay contribution to $e^+e^- \to \pi^+\pi^-$ is in the corresponding cross section taken into account by means of the Breit-Wigner-form term as follows

$$\sigma(e^+e^- \to \pi^+\pi^-) = \frac{\pi\alpha^2\beta^3}{3t}\left| F_\pi[W(t)] + Re^{i\phi}\frac{m^2_\omega}{m^2_\omega - t - im_\omega\Gamma_\omega}\right|, \qquad (20)$$

where

$$R = \frac{6}{\alpha\, m_\omega}\left(\frac{m^2_\omega}{m^2_\omega - 4m^2_\pi}\right)^{3/4}\left[\Gamma(\omega\to e^+e^-)\,\Gamma(\omega\to\pi^+\pi^-)\right]^{1/2}, \qquad (21)$$

$\alpha = 1/137$ is the fine structure constant, $\beta = \left[1 - \frac{4m^2_\pi}{t}\right]^{1/2}$ is the velociry of the outgoing pion and the phase ϕ is given through the ρ and ω mesons parameters by the expression [15]

$$\phi = \text{arc tg}\,\frac{m_\rho\Gamma_\rho}{m^2_\rho - m^2_\omega}. \qquad (22)$$

We find that with (18) the 293 pion ff data points [8-10] are well fitted with $\chi^2/ndf = 456/281$ giving

$$\begin{array}{lll} m_\rho = 762 \pm 2 \text{ MeV}, & \Gamma_\rho = 143 \pm 3 \text{ MeV}, & f_{\rho\pi\pi}/f_\rho = 0.932 \pm 0.061 \\ m_{\rho'} = 1301 \pm 25 \text{ MeV}, & \Gamma_{\rho'} = 121 \pm 80 \text{ MeV}, & \\ m_{\rho''} = 1749 \pm 122 \text{ MeV}, & \Gamma_{\rho''} = 830 \pm 290 \text{ MeV}, & f_{\rho''\pi\pi}/f_{\rho''} = 0.065 \pm 0.008 \\ t_{inel} = 1.44 \pm 0.02 \text{ GeV}^2, & R = 0.0138 \pm 0.0034 & \\ W_Z = 0.60 \pm 0.03, & W_P = 0.87 \pm 0.06. & \end{array} \qquad (23)$$

By using the relation (19) and (22) one can calculate the values of $f_{\rho'\pi\pi}/f_{\rho'}$ and ϕ as follows

$$f_{\rho'\pi\pi}/f_{\rho'} = 0.003 \qquad (24)$$

$$\phi = 106.4^\circ . \qquad (25)$$

The value (24) indicates a very weak coupling of ρ'(1250) to two pions and there is a question of a presence of ρ'(1250) in $e^+e^- \to \pi^+\pi^-$ at all. In order to solve this problem, we have fitted the same pion ff data by means of (18), however without the term corresponding to ρ'(1250) and leaving there only the contributions of ρ (770) and ρ''(1600). The obtained results are as follows:

$$\chi^2/\mathrm{ndf} = 539/284$$

$$m_\rho = 761 \pm 2\ \mathrm{MeV}, \quad \Gamma_\rho = 152 \pm 3\ \mathrm{MeV}, \quad f_{\rho\pi\pi}/f_\rho = 0.934 \pm 0.053$$
$$m_{\rho''} = 1752 \pm 106\ \mathrm{MeV}, \quad \Gamma_{\rho''} = 891 \pm 265\ \mathrm{MeV}, \qquad (26)$$
$$t_{inel} = 1.48 \pm 0.02\ \mathrm{GeV}^2, \quad R = 0.0143 \pm 0.0026$$
$$W_Z = 0.23 \pm 0.02, \quad W_P = 0.28 \pm 0.03 .$$

The values of $f_{\rho''\pi\pi}/f_{\rho''}$ and ϕ calculated from (19) and (22) respectively are the following:

$$f_{\rho''\pi\pi}/f_{\rho''} = 0.066 \quad \text{and} \quad \phi = 106.2^\circ . \qquad (27)$$

Comparing the values of parameters (23) – (25) with the values given by (26) and (27) one can see their consistency. However missing out the resonance ρ'(1250) in (18) the value of χ^2 is increased by a value of about 80 in describing the same number of pion ff data points. The latter clearly shows that the inclusion of ρ'(1250) essentially improves the description of the data and simultaneously confirms the presence of this resonance in $e^+e^- \to \pi^+\pi^-$.

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Дубничка С., Фурдик И., Мещеряков В.А. E2-87-433
Модифицированная трехполюсная ВМД модель электромагнитного формфактора пиона с двумя точками ветвления

Предложена модификация трехполюсной ВМД модели для электромагнитного формфактора пиона. Сначала ВМД модель преобразуют в переменную импульса пиона в системе центра масс и потом, используя обратное преобразование Жуковского, в другую переменную. В результате получается для всех трех векторных мезонов $\rho(770)$, $\rho'(1250)$ и $\rho''(1600)$ общий нормированный множитель, при помощи которого можно обеспечить произвол в выборе асимптотического поведения пионного формфактора. Явное включение ненулевых ширин векторных мезонов порождает реальную аналитическую модель, определенную на четырехлистной поверхности Римана. Оно эффективным способом учитывает вклады неупругих каналов, зависит только от параметров с явным физическим смыслом, сохраняет нормировку исходной ВМД параметризации и дает превосходный фит всех существующих данных по пионному формфактору, в котором все параметры модели определяются и тем самым подтверждается присутствие $\rho'(1250)$ мезона в $e^+e^- \to \pi^+\pi^-$.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Dubnička S., Furdík I., Meshcheryakov V.A. E2-87-433
Modified Three-Pole VMD Model with Two-Branch-Point Analytic Structure and Approved Asymptotic Behaviour for the Pion Electromagnetic Form Factor

A modification of three-pole VMD model for electromagnetic pion form factor was carried out, first by means of its transformation into the pion c.m. momentum variable and subsequently by using the inverse Zhukovsky transformation into another variable. In such a procedure a common normalized factor for all three vector mesons, $\rho(770)$, $\rho'(1250)$ and $\rho''(1600)$, is singled out, by means of which it is possible to settle a specific freedom in the choice of the pion form factor asymptotic behaviour. An explicit incorporation of nonzero vector meson widths creates a real analytic model, defined on the four-sheeted Riemann surface. It includes inelastic contributions effectively, depends just on the parameters with a clear physical meaning, conserves the normalization of the original VMD parametrization, and provides a perfect fit of all existing pion form factor data, in which all parameters of the model are determined and the presence of $\rho'(1250)$ in $e^+e^- \to \pi^+\pi^-$ is again established.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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