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# PION AS GOLDSTONE PARTICLE IN $QCD_m$

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## Introduction

One of the actual problems of QCD is the foundation of spontaneous chiral symmetry breaking and the description of the pion as the Goldstone particle. There are two different approaches to this problem.

The first of them is to construct chiral phenomenological Lagrangians directly from QCD[1] or from the intermediate low-energy quark models of the Nambu-Jona-Lazinio (NJL)[2] type. A major achievement here is, in comparison with the initial chiral Lagrangian, the reduction of the number of phenomenological parameters[3]. However, these works corroborate the spontaneous symmetry breaking by their numerous results rather than substantiate this phenomenon. Besides, this approach does not describe the $J/\psi$ - particle - spectroscopy. The object of critics of the NJL model is also the ultraviolet divergences on which the physical parameters depend.

The second approach[4-7] deals with proving spontaneous symmetry breaking by means of the "confinement" potentials, i.e. by the potentials to which lattice calculations[8], heavy quarkonium spectrum[9], ect., testify. The potentials

$$K(x) = \gamma_0 V(\vec{x}) \gamma_0 \delta(x_0),$$

$$V(\vec{x}) = -\frac{\alpha}{|\vec{x}|} + \alpha' |\vec{x}| + V_0 |\vec{x}|^2$$

are given in the class of functions opposite to that for potentials corresponding to the NJL model, where $K(x) \sim \delta^4(x)$ .

Unlike the first approach, these works have proved that the "increasing" potentials lead to the spontaneous chiral symmetry breaking with a dynamical quark mass (depending on the momentum), and explained the light and heavy quarkonium spectrum having a rich nonlocal structure /6/. However, the defect of such a potential approach is a manifestly nonrelativistic formulation that makes difficult the description of the low-energy dynamics.

The central question, we would like here to discuss, consists in the following: is it possible to formulate the model which would consistently unify the nonlocal spectroscopy and local chiral Lagrangians into the "bilocal chiral Lagrangians"?

## 1. Bilocal Lagrangians

The Lagrangian describing the strong interactions of quark (q) and gluon (A) has the form

$$L = -\frac{1}{4}\left(F^{i}_{\mu\nu}\right)^2 + \bar{q}^{A}_{a\alpha}\left[\delta_{ab}\,\delta^{AB}\,\gamma_{\alpha\beta} - \delta_{ab}\,\hat{A}^{AB}_{\alpha\beta} - (\hat{m}_0)_{ab}\,\delta^{AB}\,\delta_{\alpha\beta}\right] q^{B}_{b\beta} \quad (1)$$

where $(A, B)$, $(a, b)$ and $(\alpha, \beta, \mu, \nu)$ are the color, flavour and space-time indices, respectively, $\hat{m}_0 = \mathrm{diag}(m_0^1, \dots, m_0^n)$ is the bare quark mass matrix.

We use the effective action where gluon-quark interactions are taken into account by the quark-quark "potential"

$$S_{eff} = -(\bar{q}, G_0^{-1} q) + \frac{1}{2}(q\bar{q}, K q\bar{q}) \,. \quad (2)$$



Here the notation of review /10/ is used,

$$G_0^{-1} \equiv (\hat{m}_0 - i\not{\partial})\,\delta^4(x-y),$$

$$(\bar{q}, G_0^{-1} q) \equiv \int d^4x\, d^4y\; \bar{q}(x)\, G_0^{-1}(x,y)\, q(y), \quad (3)$$

$$(q\bar{q}, K q\bar{q}) \equiv \int d^4x_1 d^4y_1 d^4x_2 d^4y_2\; q_{N_1}(y_1)\,\bar{q}_{L_1}(x_1)\cdot K_{L_1N_1|L_2N_2}(x_1 y_1|x_2 y_2)\, q_{N_2}(y_2)\,\bar{q}_{L_2}(x_2) \doteq \int d^4x\, d^4y\;\; j^i_\mu(x)\, D_{\mu\nu}(x-y)\, j^i_\nu(y) \quad (4)$$

$$\left(L_i \equiv (A_i, a_i, \alpha_i),\; N_i \equiv (B_i, b_i, \beta_i),\; i=1,2\right)$$

$$K_{L_1N_1|L_2N_2}(x_1 y_1|x_2 y_2) = \delta_{a_1b_2}\,\delta_{a_2b_1}\,(\gamma_\mu)_{\alpha_1\beta_2}(\gamma_\nu)_{\alpha_2\beta_1} \cdot \left(\frac{\lambda_i}{2}\right)^{A_1B_2}\left(\frac{\lambda_i}{2}\right)^{A_2B_1}\delta^4(x_1-y_2)\,\delta^4(x_2-y_1)\,D_{\mu\nu}(x_1-x_2)\,. \quad (5)$$

Since $\{N_c\}\otimes\{N_c^*\} = \{1\}\oplus\{N_c^2-1\}$, one can decompose the potential $K$ in the color singlet and the $(N_c^2-1)$- plet components through the projective operators $P_1$ and $P_{N_c^2-1}$

$$\sum_{i=1}^{N_c^2-1}\left(\frac{\lambda_i}{2}\right)^{A_1B_2}\left(\frac{\lambda_i}{2}\right)^{A_2B_1} = \frac{1}{2}\left(\frac{N_c^2-1}{N_c}P_1 - \frac{1}{N_c}P_{N_c^2-1}\right). \quad (6)$$

In the following, we shall consider only the singlet channel, i.e. $K = K_1$, where

$$(P_1)^{A_1B_2|A_2B_1} = \delta^{A_1B_1}\,\delta^{A_2B_2}. \quad (7)$$

For our aim of unification of the spectroscopy and chiral hadron dynamics in the framework of the NJL model it is useful to introduce the colorless bilocal variables $\mathcal{M}(x,y)$ as suggested in refs.[11,12] (see also review[10]). In these variables the four-quark part of the action (2) is written as

$$\exp\{\tfrac{i}{2}(q\bar q, Kq\bar q)\} = \int d\mathcal{M}\exp\{-\tfrac{i}{2}(\mathcal{M}, K^{-1}\mathcal{M}) + i(q\bar q\,\mathcal{M})\} \tag{8}$$

where

$$[\mathcal{M}(x,y)]_{ab;\alpha\beta} = \sum_{i=0}^{n^2-1}\Big[S^i(x,y) + \gamma_5 P^i(x,y) + \gamma_\mu V^i_\mu(x,y) + \gamma_\mu\gamma_5 A^i_\mu(x,y) + \tfrac{i}{2}[\gamma_\mu,\gamma_\nu]\,T^i_{\mu\nu}(x,y)\Big]_{\alpha\beta}\Big(\tfrac{\lambda_i}{2}\Big)_{ab}, \tag{9}$$

$d\mathcal{M} \equiv dS\,dP\,dV\,dA\,dT$ , $S$ , $P$ , $V$ , $A$ , and $T$ are, respectively, scalar, pseudoscalar vector, axial and tensor parts of $\mathcal{M}$. Substituting (8) into the generating functional for the Green functions with the action $S_{eff}$ at $K = K_1$,

$$Z[\eta,\bar\eta,J] = \int dq\,d\bar q\,\exp(iS_{eff} + iS[\eta,\bar\eta,J]) \tag{10}$$

( $\eta$, $\bar\eta$, $J$ are the sources)
and performing integration over the quark fields one can express $S_{eff}$ only in terms of the bilocal fields $\mathcal{M}$,

$$Z[\eta,\bar\eta,J] = \int d\mathcal{M}\,\exp(iS_{eff}(\mathcal{M}) + iS[\eta,\bar\eta,J|\mathcal{M}]), \tag{11}$$

where

$$S_{eff}(\mathcal{M}) = -\tfrac{1}{2}(\mathcal{M}, K^{-1}\mathcal{M}) - i\,\mathrm{tr}\ln(G_0^{-1} + \mathcal{M}). \tag{12}$$

Remark, the effective action can be written in the variables $M \equiv \mathcal{M} + \hat m_0$ too,

$$S_{eff}(M) = -\tfrac{1}{2}\big((M - \hat m_0), K^{-1}(M - \hat m_0)\big) - i\,\mathrm{tr}\ln(i\not\partial + M). \tag{13}$$



The classical equation for the fields $M$, due to

$$\frac{\delta S_{eff}(M)}{\delta M} = 0$$

is the well-known Schwinger-Dyson equation,

$$\Sigma = \hat m_0 - i K_1 G_\Sigma, \tag{14}$$

where $\Sigma$ is the solution of (14) and

$$G_\Sigma(x,y) = \frac{1}{\Sigma - i\not\partial}\,\delta^4(x-y).$$

Equation (14) defines the spectrum of the fermions and is the main tool of the investigation of the spontaneous chiral symmetry breaking phenomenon.

The second variation of $S_{eff}$ over $M$ at the minimum point, $M = \Sigma$ , defines the spectrum of mesons, i.e. the second variation condition gives the Bethe-Salpeter equation for the vertex function of the quark-antiquark bound state

$$\mathcal{M}' = i K_1 G_\Sigma\, \mathcal{M}'\, G_\Sigma \tag{15}$$

with $\mathcal{M}' = M - \Sigma$.

The effective action in terms of the bilocal meson fields variables is given by

$$S_{eff}(M) \equiv S_{eff}(\Sigma + \mathcal{M}') = S(\Sigma) + S_{free}(\mathcal{M}') + S_{int}(\mathcal{M}'), \tag{16}$$

where $\mathcal{M}'$ is a solution of (15) and

$$S_{free} \equiv -\tfrac{1}{2}(\mathcal{M}', K^{-1}\mathcal{M}') + \tfrac{i}{2}G_\Sigma\mathcal{M}'G_\Sigma\mathcal{M}', \tag{17}$$

$$S_{int} \equiv i\sum_{k=3}^{\infty}\frac{(-1)^k}{k}\,[G_\Sigma\,\mathcal{M}']^k. \tag{18}$$

The actions (17) and (18) on the solutions of (13) and (15) contain the whole dynamics of the mesons defined directly from QCD.

The next step in constructing our model is to fix the form of the quark-quark interaction potential. We shall here concentrate on the potential

$$K_1(x) = \frac{4}{3}\gamma_0^{(1)}\gamma_0^{(2)}\delta(x_0)V(\vec{x}),$$
$$V(\vec{x}) = -\frac{\alpha_s}{|\vec{x}|} + V_0|\vec{x}|^2, \tag{19}$$

$$\alpha_s = \frac{e_s}{4\pi} \simeq 0.3,$$
$$\left(\frac{4}{3}V_0\right)^{1/3} \simeq (250-300)\ \mathrm{MeV}, \tag{20}$$

which qualitatively reflects the spectroscopy light and heavy quarkonia[6,9]. For light quarks $\left(m_0 \ll \left(\frac{4}{3}V_0\right)^{1/3}\right)$, with the compton length much larger than the "sizes" of the potential, the main role plays the oscillator potential, $V_0|\vec{x}|^2$; while for heavy quarks $\left(m_0 \gtrsim \left(\frac{4}{3}V_0\right)^{1/3}\right)$, the Coulomb potential, $\frac{\alpha_s}{|\vec{x}|}$.

## 2. The oscillator potential

Let us consider an oscillator potential. In ref.[6] for this potential solutions of the Schwinger-Dyson (14) and Bethe-Salpeter (15) equations were found in the rest frame of the mesons, for the massless quarks $\hat{m}_0 = 0$.

The Schwinger-Dyson equation (14) in the momentum representation can be written as

$$\Sigma_{\alpha\beta}(p_\mu) = i\int\frac{d^4q}{(2\pi)^4}(K_1)_{\alpha\alpha_1|\beta\beta_1}(q)\left[\frac{1}{\not p+\not q-\Sigma(p+q)}\right]_{\alpha_1\beta_1} \tag{21}$$



and substitution

$$\Sigma(p_0,\vec{p}) = E(p)\sin\vartheta(p) + \frac{\vec{p}\vec{\gamma}}{p}\left[E(p)\cos\vartheta(p) - p\right] \tag{22}$$

(with $p=|\vec{p}|$)

leads to an equation of the sine-Gordon-type for the function $\vartheta(p)$,

$$\frac{4}{3}V_0(p^2\vartheta')' = 2p^3\sin\vartheta - \frac{4}{3}V_0\sin 2\vartheta \tag{23}$$

(the prime corresponds to the $p$-derivative),
where $E$ can be expressed via the solution of (23)

$$E(p) = p\cos\vartheta - \frac{4}{3}V_0\frac{1}{p^2}\cos^2\vartheta - \frac{2}{3}V_0(\vartheta')^2. \tag{24}$$

The solution of equation (23) has the following asymptotic behaviour[6]

$$\vartheta(p\to\infty) \sim \exp\left(-\frac{2\sqrt{2}}{3}\underline{p}^{3/2}\right),$$
$$E(p\to\infty) \sim p,$$
$$\vartheta(p\to 0) \sim \frac{\pi}{2} + c\,\underline{p}, \quad (c\approx -2) \tag{25}$$
$$E(p\to 0) \sim -\frac{2}{3}c^2 V_0^{-1/3};$$

where $\underline{p} \equiv \left(\frac{4}{3}V_0\right)^{-1/3}p$ is the dimensionless variable,

$$|\Sigma(p\to 0)| \sim \frac{2}{3}c^2 V_0^{1/3},$$
$$\Sigma(p\to\infty) \sim p\exp\left(-\frac{2\sqrt{2}}{3}\underline{p}^{3/2}\right). \tag{26}$$

In ref.[6] it has been shown that this solution is more energetic-

-favourable than the trivial solution $\Sigma = 0$ ; it leads to the spontaneous chiral symmetry breaking and to the quark mass vanishing at large momenta.

In ref.[6] the Bethe-Salpeter equation,

$$G_{\Sigma}^{-1} \chi G_{\Sigma}^{-1} = i K_1 \chi \qquad (27)$$

has been solved for the wave function $\chi$ connected with the bilocal meson field, $\mathcal{M}'$, (see eqs. (15), (27)), by the relation

$$\mathcal{M}' = G_{\Sigma}^{-1} \chi G_{\Sigma}^{-1} \equiv i K_1 \chi . \qquad (28)$$

In particular, the pion wave function coincides with the dynamical quark mass function (see ref.[6]),

$$\tilde{\chi}_{\pi}(\vec{p}) \sim \gamma_5 \sin \vartheta(\vec{p}). \qquad (29)$$

The spontaneous chiral symmetry breaking induces a large mass splitting of $\pi$ - and $\rho$ - mesons without any spin-spin interaction. In ref.[6] by neglecting the Coulomb interaction results have been obtained, consistent qulitatively with the experimental mass spectrum of mesons. As we have noted, taking account of the Coulomb interaction (20) is dictated by the spectroscopy of heavy quarkoniua[9]

Usually, the calculation of the meson mass is made in the rest frame, where

$$\langle 0|\mathcal{M}'(z|X)|h\rangle \sim \exp\left(i P_0^{(h)} X_0\right) K_1(\vec{z}, z_0) \chi_h(\vec{z}|P_0^{(h)}) \delta^3(\vec{X}). \qquad (30)$$

The defect of ref.[6] is the incorrect description of a moving quarkonium which leads to a wrong dispersion law ( $P_0(\vec{P}\to 0) \sim -2|\vec{P}|$).



Naturally, such a solution one cannot use for describing the meson relativistic interaction by means of the effective bilocal Lagrangian.

Let us consider QED as an example in order to give a correct description of the quarkonium, in analogy with the positronium.

## 3. Relativistic description of quarkonium and chiral bilocal Lagrangian

There is an opinion that the relativistic description of positronium in QED could be achieved by passing to the manifest-relativistic-covariant gauge or by taking into account the higher orders of perturbation theory. But in the relativistic gauge there exists an unsolved problem of equal-time bound-states of two particles.

In recent years, the description of bound-states in QED has been realized only in the Coulomb gauge[13,14]. Significant results have been got in the calculation of corrections up to the order $O(\alpha^6)$. However, by improving the accuracy of calculations in the Coulomb gauge one cannot solve the problem of the relativistic description. The correct statement of the problem consists in the way of restoring, within a given accuracy, the relativistic covariance of the positronium wave function.

It turns out, that this can be made in the scheme of quantization of gauge fields in which the Poincare algebra is fulfilled at the level of operators rather than at the level of matrix elements alone.

Such a quantization scheme is formulated and described in detail in the works[15] by Schwinger. It differs from the conventional one by relativistic transformations of the spinors accompanied by an additional gauge rotation:

$$\delta_L \psi(x) = \delta_L^0 \psi + ie\Lambda_{Schw.}\psi, \tag{31}$$

where $\delta_L^0$ is the usual transformation with the parameter $\varepsilon_k$,

$$\Lambda_{Schw.} = \varepsilon_k \frac{1}{\partial_n^2}\left(\dot{A}_k^T - \partial_k \frac{1}{\partial_n^2} e j_0\right).$$

$\dot{A}^T$ is the time derivative of the transverse part of the electromagnetic field, $j_0$ is the time current component. This transformation realized the motion of the Coulomb field together with the bound state (that is formed by this field). In the other words, in the Schwinger scheme the very field decomposition into the Coulomb and transverse parts has really a covariant structure.

In the given quantization scheme after the Lorentz transformation the wave function can be written in the following covariant form:

$$\langle 0|\mathcal{M}'(z|X)|h\rangle \sim \exp(iP'X)\, K_1(z_\perp^{(P')}|z_\parallel^{(P')})\, \chi_h(z_\perp^{(P')}|P'). \tag{32}$$

where $z$ is the relative coordinate, separated into the longitudinal $(z_\parallel^{(P')})$ and transverse $(z_\perp^{(P')})$ parts relative to the vector $P' = (P_0', \vec{P})$, and $P'^2 = P^2$, $z_{\parallel\mu}^{(P')} = P_\mu'(P'z)/P'^2$, $z_\perp^{(P')} = z - z_\parallel^{(P')}$. The expression (32) is the unknown relativistic wave function of the bound-state (positronium) in QED.

In refs[16,17] it was shown that the Schwinger operator quantization is founded by the classical formulation of electrodynamics with the explicit solution of the equation for the temporal field component (the Gauss equation)

$$\partial_k^2 A_0 = \partial_i \dot{A}_i + e j_0 \quad \left(A_0 = \frac{1}{\partial_k^2}(\partial_i \dot{A}_i + e j_0)\right) \tag{33}$$



The Lagrangian and the energy-momentum tensor at the solution of eq. (33) are expressed only in the terms of the nonlocal variables $A_i^T$, $\psi^T$:

$$\left.\begin{aligned} ieA_i^T &= V(ieA_i + \partial_i)V \\ \psi^T &= V\psi \end{aligned}\right\} V = \exp\left(ie\frac{1}{\partial_k^2}\partial_i A_i\right) \tag{34}$$

the classical relativistic transformations of which coincide with the Schwinger ones

$$\delta_L^0\left(\frac{1}{\partial_k^2}\partial_i A_i\right) = \Lambda_{Schw.}\ .$$

Such dynamical fixation of the gauge by the explicit solution of the Gauss-equation seems to be a necessary step for the consistent relativistic quantization. According to ref.[19], an explicit solution is allowed by an oscillator-like potential (20) in the framework of an infrared degeneration. In fact the equation

$$\partial_i^2 A_0^{T*} = j_0$$

contains the solution

$$A_0^T(x) = \int d^3y \left[-\frac{1}{4\pi|\vec{x}-\vec{y}|} - \frac{V_0}{e^2}\vec{y}(\vec{x}-\vec{y})\right] e j_0(y) \equiv \int d^3y\, V(\vec{x},\vec{y})\, j_0(y)$$

which leads, in the Lagrangian

$$\frac{e}{2}\int d^3x\, A_0^T(x)\, j_0(x) = \int d^3x\, d^3y\, j_0(x)\, V(x,y)\, j_0(y) \equiv$$

$$\int d^3x\, d^3y\, \frac{1}{2}\left[V(x,y) + V(y,x)\right] j_0(x)$$

to the oscillator-like potential (20) used in this paper as a basic interaction for the describing of quarkonium (as a time-equal relativistic system). Thus, the scheme of relativistic operator quantization not only recognized the covariance of the bound-state wave functions, but also justifies the use of the oscillator-like potential (20) for the quarkonium description.

After such relativization for the wave functions of ref. /6/, we can decompose the bilocal meson field; taking into account (27) and (28) we have

$$\mathcal{M}'(x,y) = i K_1(z) \int \frac{d^4P}{(2\pi)^4} \sum_h \delta^+(P^2-\mu_h^2) \cdot$$
$$\left\{ e^{-iPX} \underset{\sim}{\mathcal{X}}_h(z|P) N_{(h)} a_h(P) + e^{iPX} \underset{\sim}{\bar{\mathcal{X}}}_h(z|-P) N_{(h)} a_h^+(P) \right\}. \tag{35}$$

Here $z \equiv x+y$ and $X \equiv \frac{1}{2}(x-y)$ are the relative and absolute coordinates; $N_{(h)}$ is the normalization factor, the creation and annihilation operators are $a_h$ and $a_h^+$ with the hadron quantum numbers $h$. Within such a description the full relativistic bilocal "time-equal" quarkonium function can be expressed in the following factorizable terms

$$\mathcal{M}'(z|X) = \sum_h i K_1(z_\perp^{(D)}|z_\parallel^{(D)}) \left[ \mathcal{X}_h(z|D) \Phi(X) + \bar{\mathcal{X}}_h(z|-D) \Phi^+(X) \right]. \tag{36}$$

The expressions (16)-(18), (32), (35) and (36) with the solutions $\Sigma$ and $\mathcal{X}$ from ref. /6/, describe the relativistic version of the chiral bilocal theory. It is important to note that the model proposed in this paper corresponds to taking into account the planar diagrams ($1/N_c$-decomposition) and can describe, as it is shown in refs. /11,18/ the Veneziano amplitude.



In connection with the above formulated construction scheme for the unique description of the spectroscopy and local chiral theory consider also the local limit. As is known, the choice $K_1 \sim \delta^4(z)$ for the NJL-potential leads to a local chiral Lagrangian instead of the bilocal Lagrangian (16). It is easy to see that not only such a modification of the potential gives local Lagrangians. Really, the bilocal field interesting for us is at the same time proportional to the potential and to the Bethe-Salpeter wave function which degenerates into a $\delta$-function in the limit $V_0 \to 0$ due to the normalization factor $V_0^{-1}$. Physically, it is equivalent to that at low momenta (and large wave lengths) the bilocal hadrons interact as point-like objects without structure. Thus, the constructed model gives a unique description of the nonlocal spectroscopy and local chiral theory.

## 4. The confinement problem

A chiral effective meson Lagrangian is constructed from QCD by using one-loop quark diagrams and at the same time, the quarks as physical states being removed from the unitarity relation. This relation, as is known, is formulated only in terms of hadron amplitudes (i.e. one supposes that all amplitudes of color-particle generation are equal to zero, and the probability of the total hadronization is equal to unity).

The chiral spectroscopy /4-6/ historically has come from the problem of explanation of the quark unobservability by solving the Schwinger-Dyson equation for the "confinement" potential. However, instead of the confinement one meets the phenomenon of spontaneous chiral symmetry breaking. Really, the propagator $G_\Sigma$ in (14), (21) and (28) cannot explain the difference between the physical (unobservable) quarks and the perturbative (bare) ones, used for constructing the loops. The original explanation that difference between the bare and physical (dressed) quarks has been proposed by t'Hooft[18] in the framework of two-dimensional chromodynamics, where due to the infrared divergences the physical quarks requier an infinite mass, whereas in the loop diagrams the infrared divergences cancel out. As it may be concluded from the final result, an analogous process of "dressing" of the physical quarks is allowed by the scheme used here for the relativistic operator quantization based on the explicit solution of the Gauss equation. Really, the variables (33) are given up to nonsingular phase factors defined by the infrared solutions of the Laplace equation

$$ie A_i^{ph} = u(x) \left( ie A_i^T + \partial_i \right) u^{-1}(x)$$

$$\psi^{ph} = u(x) \psi^T,$$

where $u(x) \equiv \exp(i \lambda(x)), \ \partial_i^2 \lambda(x) = 0.$

For the gauge theories SU(2), SU(3), $SU(N \geq 2)$ (in perturbation theory) in a finite space $|x| \leq R$ such nontrivial nonsingular factors exist. For example, the factor

$$u_{(n)}(x) = \exp(i x_a \tau_a \pi n / R)$$

(where $|n| = 0, 1, 2, \ldots$)

gives a smooth map of the space $R(3)$ on to SU(2), while in QED such smooth (nonsingular) factors do not exist. In fact, taking account of these factors means that all physical color fields as if are in an external purely gauge fields $U_i^{(n)} = u_{(n)} \partial_i u_{(n)}^{-1}$ which suppress all color-particle generation amplitudes, after averaging procedure over the infrared degeneration parameters



(due to the destructive interference of the indicated phase factors). In the same time in quark loops all phase factors cancel out, and the loop diagrams are effectively constructed from the bare quark propagators. The process of such a dressing is described in detail in refs[16,17,19].

## Conclusion

In this paper, we discussed the possibility of constructing, directly from QCD, the bilocal hadron model which includes as limit cases both the local chiral phenomenological Lagrangians and the quarkonium spectroscopy with the spontaneous chiral symmetry breaking.

That construction has required the relativistic description of quarkonium. As the example of QED indicates, such a description is possible only in the quantization scheme where noncovariant decomposition of the gauge field into the Coulomb and transverse parts has the covariant character, i.e. Lorentz transformations simultaneously change the gauge. Such a covariant scheme is the Schwinger relativistic operator quantization which can be proved by explicit solution of the Gauss equation. Just this solution contains, due to infrared ambiguities, an additional information such as the modified Coulomb law and phase factors of topological degenerations the interference of which suppresses the color amplitudes and gives foundation of the unit hadronization probability [16,19].

This "minimal" modified QCD (QCDm) realised on explicit solution of the Gauss equation gives the above relativistic bilocal hadron model. The formulation of the model is based on the assumption of a small coupling constant in the low-energy region and ignores the formula of asymptotical freedom $\alpha_s(q^2) \sim [\beta \ln(q^2/\Lambda^2)]^{-1}$ which is correct only in the region of a small $\alpha_s$.

In QCDm, like in QED, nonperturbative phenomena can be interpreted by the static potential; whereas perturbative ones, by the relativistic quark-gluon interaction.

To clear up the status of QCDm, one needs a calculation of these relativistic corrections and a detailed study of the chiral symmetry spontaneous breaking with taking into account the $1/N_c$ - expansion, the Coulomb interaction, and the quark masses. In addition, such a description is interesting for pure gluodynamics.

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Пион как голдстоуновская частица в КХД$_М$

Обсуждается возможность формулировки КХД, позволяющей дать единое описание физики адронов во всей энергетической шкале. КХД рассматривается в полной аналогии с КЭД. В КЭД непертурбативные явления объясняются исключительно статическим потенциалом, а для корректного релятивистского описания связанных состояний необходимо релятивистское операторное квантование. Показано, что такое квантование хромодинамики допускает инфракрасные доопределения статического потенциала, которые могут описать спектроскопию легких и тяжелых кварkониев, киральные лагранжианы, конфайнмент и амплитуду Венециано.

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Pion as Goldstone Particle in QCD$_m$

The possibility of constructing the QCD-model (QCD$_m$) that allows us to give a unique description of the hadron physics at all energies is discussed. QCD$_m$ is constructed fully in analogy with QED. In QED, nonperturbative phenomena are explained solely by the static potential, and the relativistic operator quantization is necessary for correct relativistic description of the bound-states. It is shown that such quantization of the chromodynamics allows the infrared redefinitions of the static potential that would explain the spectroscopy of light and heavy quarkonia, the chiral phenomenological Lagrangians, the confinement and the Veneziano amplitude.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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