объединенный
институт
ядерных
исследований
дубна

E2-87-420

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OPEN BOSONIC STRING IN BACKGROUND ELECTROMAGNETIC FIELD

Submitted to "Nuclear Physics B"

1987

1. Introduction

The theory of superstrings with the Planck dimension permits probably to unify all the fundamental interactions [1-3]. The characteristic peculiarities of the string dynamics should be displayed on the Panck scales. In the energy region accessible to experiment now and in the nearest future, the string theory is reduced to an infinite set of the local fields, the mass of the corresponding quanta rising from zero to infinity. It is this field-theoretic content of the string theories that is interesting first of all for the contemporary elementary particle physics. For the investigation of the local fields which arise in the low energy limit in the interacting string field theory the background-field method appears to be convenient [4,5]. In particular, it has been shown that the requirement of the conformal invariance in quantum theory of the string propagating in external fields results in proper equations of motion for these fields [5].

It turns out that the investigation of the classical dynamics of the string in background fields enables one to obtain some results on the local fields generated by interacting strings.

The relativistic string in external electromagnetic field as the model for hadrons has been considered in [6-10]. Recently the interest in this problem has arisen again on the basis of the modern string approach the unification of all the fundamental interactions in the elementary particle physics [11, 12].

This paper is devoted to the classical and quantum theory of an open bosonic string propagating in the constant homogeneous electromagnetic field in D-dimensional space-time. Open strings of two types are considered: neutral strings with charges at the ends obeying the relation $q_1 + q_2 = 0$ and charged strings for which $q_1 + q_2 \neq 0$. This paper is organized as follows. In the next section it is shown in a general way that the strength of an external electric field must be less than its critical value in order to present the superlight velocities of the charges at the string ends (see eq.(2.12)). The third section is devoted to the generalization of the light-like gauge in the free string theory to the case of a neutral string $(q_1 + q_2 = 0)$ propagating in an external electromagnetic field. In the fourth section the dynamics of the neutral string in the external electromagnetic field is investigated. Independent physical variables are singled out (the transverse string coordinates) and the squared mass of the string M^2 is expressed in terms of them. If the strength of the external electric field does not exceed its critical value, the string oscillations give a positive contribution to M^2. The distance between equidistant levels of the operator M^2 is smaller by a factor $(1-e^2)$ than that of a free string. Here e is a dimensionless strength of the electric field (see eq.(4.6)). The magnetic field does not affect the distance between mass levels. It is essential that the squared mass of the string has a tachyonic contribution at the classical level also due to the motion of the string as a whole in transverse directions. This contribution depends on the electric and magnetic fields (eq.(4.33)). The tachyonic term disappears if one considers, instead of M^2 the string energy

in a special reference frame where the projection of the total canonical momentum of the string onto the electric field p^1 vanishes. In the fifth section the open string with a net charge $(q_1 + q_2 \neq 0)$ propagating in a background magnetic field is investigated using the light-like gauge conditions from the free string theory. In this case the longitudinal components of the total momentum of the string are only conserved. Therefore, the string energy in the reference frame, where $p^1 = 0$ is considered instead of M^2. At the classical level this energy is stricktly positive. But in quantum theory the zero point fluctuations of the string oscillations give rise to a tachyonic contribution to it. This tachyonic term in the string energy depends on the dimension of the space-time as usual and on the strength of the background magnetic field as well. The frequencies of string oscillations in this case are not integer and depend on the magnetic field (see eq.(5.11)). This results in the corresponding modification of the energy spectrum in comparison with the free string theory (eqs.(5.17) and (5.21)). In the sixth section the solution of the equations of motion for the charged string in external electric and magnetic fields is constructed. In this case there are no conserved quantities analogous to M^2 or the string energy. The Varasoro operators are obtained and the constant $\alpha(0)$ is calculated. In conclusion the obtained results are shortly discussed and the unsolved problems are noted.

2. A critical value of the external electric field

The world sheet swept out by the string in the D-dimensional space-time is described by string coordinates $x^\mu(u^0, u^1)$,

$\mu=0,1,\dots,D-1;\quad u^0=\tau,\quad u^1=\sigma.$ The reparametrization-invariant action for the open bosonic string propagating in a background electromagnetic field $A_\mu(x)$ is

$$S=-\frac{T}{2}\int_{\tau_1}^{\tau_2}d\tau\int_{\sigma_1(\tau)}^{\sigma_2(\tau)}d\sigma\sqrt{|g|}\,g^{ij}\partial_i x^\mu\partial_j x_\mu-\sum_{a=1}^{2}\int_{C_a}dx_\nu A^\nu(x),\qquad(2.1)$$

where T is the string tension, $g^{ij}(u)$ is the auxiliary metric field. The trajectories of the string ends on the (τ,σ) -plane are labelled by $C_a,\ a=1,2$. They are specified by two functions $\sigma_a(\tau),\ a=1,2$. In the embedding space-time the metric with signature $(+,-,-,\dots)$ is used. According to (2.1) with the electromagnetic field there interact only the string ends the electric charges of which are q_1 and q_2, respectively. As a consequence, the external electromagnetic field alters only the boundary conditions in the string dynamics.

As in the free string case one can choose the orthonormal gauge

$$g_{ij}(u)=\eta_{ij}e^{\varphi},\qquad \eta_{ij}=diag(1,-1).\qquad(2.3)$$

In this case the equations of motion for the metric field $g_{ij}(u)$ $\delta S/\delta g_{ij}=0$ give

$$(\dot x\pm x')^2=0,\quad \dot x=\partial_\tau x,\quad x'=\partial_\sigma x.\qquad(2.4)$$

The string coordinates obey the equation

$$\ddot x^\mu-x''^\mu=0,\quad \mu=0,1,\dots,D-1\qquad(2.5)$$

and the boundary conditions

$$x'_\mu+f^1_{\mu\nu}\dot x^\nu+(\dot x_\mu+f^1_{\mu\nu}x'^\nu)\dot\sigma=0,\quad \sigma=\sigma_1(\tau),$$

$$x'_\mu-f^2_{\mu\nu}\dot x^\nu+(\dot x_\mu-f^2_{\mu\nu}x'^\nu)\dot\sigma=0,\quad \sigma=\sigma_2(\tau),\qquad(2.6)$$

$$f^a_{\mu\nu}=(q_a/T)F_{\mu\nu},\quad a=1,2,\quad F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu.$$

The variation of functions $\sigma_a(\tau),\ a=1,2$ in action (2.1) does not give new equations as compared with (2.4)-(2.6). Therefore without loss of generality one can put, as in the free string case, $\sigma_1=0,\ \sigma_2=\pi$. This essentially simplifies the boundary conditions (2.6):

$$x'_\mu+f^1_{\mu\nu}\dot x^\nu=0,\quad \sigma=0,\qquad x'_\mu-f^2_{\mu\nu}\dot x^\nu=0,\quad \sigma=\pi.\qquad(2.7)$$

But as it will be shown further, it is convenient to use the boundary conditions in the form (2.6) when we are looking for the light-like gauge which should be consistent with the boundary conditions.

In addition to eqs. (2.4), (2.5) and (2.7) the following condition should be satisfied at any point of string world surface: the vector $\dot x^\mu$ must be time-like $\dot x^2>0$ and by virtue of (2.4) the vector x'^μ must be space-like $x'^2<0$. It means that in the hyperbolic boundary problem (2.5), (2.4) and (2.7) τ is the evolutionary parameter and σ is the space-like parameter which labeles the points along the string. The fulfilment of these conditions quarantees the absence of superlight velocities in the theory [13].

With the $O(1, D-1)$ -transformation the matrix $F_{\mu\cdot}^{\cdot\nu}$ can be put in the block diagonal form[1)]

$$F_{\mu\cdot}^{\cdot\nu}=\begin{cases} \mathrm{diag}\,(\overset{(1)}{F},\overset{(2)}{F},\dots,\overset{(d)}{F}), & \text{if } D \text{ is even,}\\ \mathrm{diag}\,(\overset{(1)}{F},\overset{(2)}{F},\dots,\overset{(d)}{F},0), & \text{if } D \text{ is odd.}\end{cases} \quad (2.8)$$

Here d is an integer of $(D/2)$ and $\overset{(A)}{F}$, $A=1,2,\dots,d$ are the (2x2)-matrices

$$\overset{(1)}{F}=\begin{pmatrix}0 & E\\ E & 0\end{pmatrix}, \quad \overset{(1+\alpha)}{F}=\begin{pmatrix}0 & H_\alpha\\ -H_\alpha & 0\end{pmatrix}, \quad \alpha=1,2,\dots,d-1. \quad (2.9)$$

Further we put for definitness that D is an even number.

Let us define the matrix F^2 with the elements

$$(F^2)_{\mu\cdot}^{\cdot\nu}=F_{\mu\cdot}^{\cdot\rho}F_{\rho\cdot}^{\cdot\nu}. \quad (2.10)$$

From (2.8) one gets

$$F^2=\mathrm{diag}(E^2, E^2, -H_1^2, -H_1^2, -H_2^2, -H_2^2, \dots, -H_{d-1}^2, -H_{d-1}^2). \quad (2.11)$$

1) We do not consider special configurations of the electromagnetic field which cannot be transformed into the form (2.8). This case $(E^2-H^2=0, \vec{E}\vec{H}=0)$ for $D=4$ is investigated in [6-9].

It is easy to show now that the boundary conditions (2.7) immediately lead to the following restriction on the external electric field

$$\left(\frac{q_a}{T}E\right)^2 < 1, \quad a=1,2. \quad (2.12)$$

Really, from the boundary conditions (2.7) it follows that

$$x'_\mu x'^\mu=-\left(\frac{q_a}{T}\right)^2 F_{\rho\mu}F^{\mu\nu}\dot{x}^\rho\dot{x}_\nu. \quad (2.13)$$

In (2.13) one should put $\sigma=0$ if $a=1$ and $\sigma=\pi$ if $a=2$. In the left-hand side of (2.13) x'^2 can be substituded according (2.4) by $-\dot{x}^2$

$$\left[\delta_\mu^\nu-\left(\frac{q_a}{T}\right)^2 F_{\mu\rho}F^{\rho\nu}\right]\dot{x}^\mu\dot{x}_\nu=0, \quad a=1,2. \quad (2.14)$$

Taking into account (2.11) we obtain

$$\left[1-\left(\frac{q_a}{T}E\right)^2\right]\left[(\dot{x}^0)^2-(\dot{x}^1)^2\right]= \\ =\sum_{\alpha=2}^{d}\left[1+\left(\frac{q_a}{T}H_\alpha\right)^2\right]\left[(\dot{x}^\alpha)^2+(\dot{x}^{\alpha+1})^2\right], \quad a=1,2. \quad (2.15)$$

As it was noted above at any point of the string world sheet including the boundaries the condition

$$\dot{x}^2>0 \quad (2.16)$$

should be satisfied. Therefore $(\dot{x}^0)^2-(\dot{x}^1)^2>0$. As a result, we get from (2.15) the restriction (2.12). Hence the consistent classical theory of the open bosonic string without super light velocities can be formulated only in the case when the external electric field obeys the condition (2.12). It should be noted that this result is obtained without any assumption about the background field $F_{\mu\nu}(x)$.

3. The light-like gauge

It is well known that only using the light-like gauge one can solve the orthonormal gauge conditions (2.4) in such a way that the dependent string coordinates will be expressed as squared functions of the independent (transverse) variables [13]. In particular the positivity of the squared mass in the free string theory can be shown also in the light-like gauge. It turns out that in the presence of an external electric field the light-like gauge cannot be imposed in a way similar to the free string theory

$$n_\mu \dot{x}^\mu = const \neq 0, \qquad n_\mu x'^\mu = 0, \tag{3.1}$$

where n^μ is a constant light-like vector

$$n_\mu n^\mu = 0, \quad n^0 = n^1 = 1, \quad n^2 = n^3 = \ldots = n^{D-1} = 0. \tag{3.2}$$

Indeed, using (2.7) and (3.1) one gets

$$n^\mu \overset{1}{f}_{\mu\nu} \dot{x}^\nu = 0, \; \sigma = 0, \qquad n^\mu \overset{2}{f}_{\mu\nu} \dot{x}^\nu = 0, \; \sigma = \pi. \tag{3.3}$$

Equations (3.3) by virtue of (2.8) and (3.2) give

$$\dot{x}^1 = -\dot{x}^0, \quad \sigma = 0, \pi. \tag{3.4}$$

It means that the vector $\dot{x}^\mu(\tau,\sigma)$ on the boundary is not time-like but is space-like. Hence the gauge conditions (3.1) cannot be used in the presence of an external electric field.

If the charges at the string ends obey the condition $q_1 = -q_2 = q$, then the light-like gauge can be generalised consistently to the case when the external electric field is different from zero. The action of such a string propagating in a background electromagnetic field can be written as

$$S = -\frac{T}{2}\iint_\Sigma d^2u \sqrt{|g|}\, g^{ij} \partial_i x^\mu \partial_j x_\mu + q \iint_\Sigma d^2u\, x'_\mu \dot{x}_\nu F^{\mu\nu}(x). \tag{3.5}$$

An open bosonic string with charges at the ends obeying the condition $q_1 + q_2 = 0$ will be called the neutral string. In the opposite case when $q_1 + q_2 \neq 0$ we shall say that the string is charged.

The boundary conditions (2.6) for the neutral string take the form

$$x'_\nu + f_{\nu\mu}\dot{x}^\mu + (\dot{x}_\nu + f_{\nu\mu} x'^\mu)\dot{\sigma} = 0,$$
$$\sigma = \sigma_a(\tau), \quad a = 1,2; \qquad f_{\mu\nu} = \frac{q}{T} F_{\mu\nu}. \tag{3.6}$$

In order to get consistent light-like gauge conditions for the neutral string in an external electromagnetic field we project the boundary conditions (3.6) on a constant light-like vector n^μ

$$n x' + n f \dot{x} + (n \dot{x} + n f x')\dot{\sigma} = 0, \quad \sigma = \sigma_a(\tau), \quad a = 1,2. \tag{3.7}$$

For simplicity the Lorentz indices are suppressed. Now we choose the gauge imposing the following conditions

$$n x' + n f \dot{x} = 0, \qquad n \dot{x} + n f x' = nP/(T\pi), \tag{3.8}$$

where P^μ is the total canonical momentum of the string

$$P^\mu = \int_0^\pi d\sigma\, p^\mu(\tau,\sigma), \tag{3.9}$$

$$p^\mu(\tau,\sigma) = -\frac{\partial \mathcal{L}}{\partial \dot{x}_\mu} = T(\dot{x}^\mu + f^{\mu\nu} x'_\nu). \tag{3.10}$$

Here $\mathcal{L}$ is the Lagrangian density for the action (3.5). Now we assume that $F_{\mu\nu}(x)$ = const. In this case P^μ in (3.9) is a conserved Noether invariant corresponding to the symmetry of the action (3.5) by the boosts

$$x^\mu \to x^\mu + \delta x^\mu, \quad \delta x^\mu = const. \tag{3.11}$$

It should be noted that such a conserved vector exists only in the theory of the neutral string ($q_1 + q_2 = 0$). If the string has a net charge ($q_1 + q_2 \neq 0$), then the action (2.1) is not invariant under the transformations (3.11).

It is easy to show that the drawback discussed above does not appear in the gauge (3.8). Furthermore, it follows from (3.7) that $\dot{\sigma}_a(\tau) = 0$, $a = 1, 2$, when $nP \neq 0$.

It was shown in [7] that imposing the gauge conditions (3.8) is equivalent to the transition to new parameters by the formula

$$\bar{\tau} \pm \bar{\sigma} = \psi_\pm(\tau \pm \sigma). \tag{3.12}$$

4. The neutral string dynamics in light-like gauge in an external constant homogeneous electromagnetic field

In this case the string dynamics is determined by the equations of motion

$$\ddot{x}^\mu - x''^\mu = 0, \qquad \mu = 0, 1, \dots, D-1, \tag{4.1}$$

the orthonormal gauge conditions

$$(\dot{x} \pm x')^2 = 0, \tag{4.2}$$

and by the light-like gauge conditions

$$n_\mu x'^\mu + n_\nu f^{\nu\rho} \dot{x}_\rho = 0, \quad n_\mu \dot{x}^\mu + n_\nu f^{\nu\rho} x'_\rho = \frac{n_\mu P^\mu}{T\pi},$$
$$n_\mu n^\mu = 0, \quad n^0 = n^1 = 1, \quad n^2 = n^3 = \dots = n^{D-1} = 0. \tag{4.3}$$

In addition, the string coordinates must obey the boundary conditions

$$x'_\mu + f_{\mu\nu} \dot{x}^\nu = 0, \quad \sigma = 0, \pi. \tag{4.4}$$

Let us introduce new variables

$$x^{\pm}=\frac{x^0\pm x^1}{\sqrt{2}},$$
$$\xi^{\alpha}=x^{2\alpha}+ix^{2\alpha+1}, \qquad \alpha=1,2,\ldots,d-1=(D/2)-1. \tag{4.5}$$

The light-like gauge conditions (4.3) now take the form

$$x'^{-}=e\dot{x}^{-}, \quad \dot{x}^{-}=ex'^{-}+\frac{p^{-}}{T\pi}, \qquad e=\frac{q}{T}E. \tag{4.6}$$

The boundary conditions (4.4) can be written as

$$x'^{\pm}\pm e\dot{x}^{\pm}=0, \tag{4.7}$$

$$\xi'^{\alpha}-ih_{\alpha}\dot{\xi}^{\alpha}=0,$$
$$\alpha=1,2,\ldots,(D/2)-1, \qquad h_{\alpha}=\frac{q}{T}H_{\alpha}, \; \sigma=0,\pi. \tag{4.8}$$

In (4.8) there is no summation over α .

The light-like gauge (4.6) enables one to solve eq.(4.2) for $\dot{x}^{\pm}$ and $x'^{\pm}$

$$\dot{x}^{+}=\frac{T\pi}{2p^{-}}\left(\dot{\vec{x}}_{\perp}^{2}+\vec{x}_{\perp}'^{2}-2e\dot{\vec{x}}_{\perp}\vec{x}_{\perp}'\right), \quad \dot{x}^{-}=\frac{p^{-}}{T\pi(1-e^{2})},$$
$$x'^{+}=-\frac{T\pi}{2p^{-}}\left[e(\dot{\vec{x}}_{\perp}^{2}+\vec{x}_{\perp}'^{2})-2\dot{\vec{x}}_{\perp}\vec{x}_{\perp}'\right], \quad x'^{-}=\frac{ep^{-}}{T\pi(1-e^{2})}, \tag{4.9}$$

where $\vec{x}_{\perp}(\tau,\sigma)$ are transverse coordinates of the string

$$\vec{x}_{\perp}=(x_{\perp}^{2},x_{\perp}^{3},\ldots,x^{D-1}). \tag{4.10}$$

Equations (4.9) are in agreement with the equations of motion (4.1) and with the boundary conditions (4.7), (4.8) in the following sense: if the transverse components $\vec{x}_{\perp}(\tau,\sigma)$ obey eq. (4.1) and boundary conditions (4.8), then the longitudinal components $\dot{x}^{\pm}$ and $x'^{\pm}$ expressed in terms of $\dot{\vec{x}}_{\perp}$ and $\vec{x}'_{\perp}$ according (4.9) satisfy eqs.(4.1) and the boundary conditions (4.7). To prove this, it should be taken into account that the product $\dot{\vec{x}}_{\perp}\vec{x}'_{\perp}$ vanishes on the boundaries due to (4.8). This enables us to consider the transverse components $\vec{x}_{\perp}(\tau,\sigma)$ as independent dynamical variables and longitudinal coordinates as dependent ones.

The equations of motion (4.1) and the edge conditions (4.8) for the independent variables $\vec{x}_{\perp}(\tau,\sigma)$ represent $d-1$ independent boundary eigenvalue problems. To obtain the whole solution, it is enough to consider only one of these problems

$$\ddot{\xi}-\xi''=0, \qquad \xi=\xi(\tau,\sigma),$$
$$-\infty<\tau<+\infty, \qquad 0\leq\sigma\leq\pi; \tag{4.11}$$

$$\xi'-ih\dot{\xi}=0, \quad \sigma=0,\pi. \tag{4.12}$$

For simplicity the index α is suppressed. It follows from (4.11) that

$$\xi(\tau,\sigma)=\xi_{+}(\tau+\sigma)+\xi_{-}(\tau-\sigma). \tag{4.13}$$

The boundary conditions (4.12) at $\sigma=0$ give:

$$\xi_{+}'(\tau)(1-ih)=\xi_{-}'(\tau)(1+ih). \tag{4.14}$$

To satisfy (4.14), it is sufficient to put

$$\xi'_{\pm}(\tau)=\omega(\tau)(1\pm ih). \tag{4.15}$$

After substituting (4.15) into the boundary conditions (4.12) at $\sigma=\pi$ we obtain

$$\omega(\tau+\pi)=\omega(\tau-\pi). \tag{4.16}$$

Therefore, the function $\omega(\tau)$ is periodic with period 2π. It allows one to expand $\omega(\tau)$ in the Fourier series and to get for $\xi(\tau,\sigma)$ the following representation

$$\xi(\tau,\sigma)=a\tau+iha\sigma+c+ \frac{i}{\sqrt{\pi T}}\sum_{n=-\infty}^{+\infty}{}' e^{-in\tau}\frac{\xi_n}{n}(\cos n\sigma+h\sin n\sigma), \tag{4.17}$$

where a, c and ξ_n are complex quantities. It should be noted that here there is no usual relation $\bar{\xi}_n=\xi_{-n}$, where the bar denotes the complex conjugation. The prime of the sum symbol in (4.17) that the term with $n=0$ is absent.

Let us introduce the momentum variables in a way analogous to (4.5)

$$\eta^{\alpha}(\tau,\sigma)=p^{2\alpha}+ip^{2\alpha+1}, \quad \alpha=1,2,\dots,d-1. \tag{4.18}$$

Here $(p^2,p^3,\dots,p^{D-1})=\vec{p}_\perp$ are transverse components of the canonical momentum density (3.10). Substituting (3.10) and (4.17) into (4.18) we obtain

$$\eta^{\alpha}(\tau,\sigma)=T(\dot{\xi}^{\alpha}-ih_\alpha\xi'^{\alpha})=Ta^{\alpha}(1+h_\alpha^2)+$$

$$+(1+h_\alpha^2)\sqrt{\frac{T}{\pi}}\sum_{n=-\infty}^{+\infty}{}' e^{-in\tau}\xi_n^{\alpha}\cos n\sigma. \tag{4.19}$$

From (4.19) it follows that

$$a^{\alpha}=\frac{P^{2\alpha}+iP^{2\alpha+1}}{\pi T(1+h_\alpha^2)}, \quad \alpha=1,2,\dots,(D/2)-1, \tag{4.20}$$

where $(P^2,P^3,\dots,P^{D-1})=\vec{P}_\perp$ are transverse components of the total momentum of the string (3.9).

Now we define the squared mass of the string in the usual way

$$M^2=P_\mu P^\mu=2P^+P^- -\vec{P}_\perp^{\,2}, \tag{4.21}$$

where P^μ is the conserved total momentum of the string (3.9). Using (3.9) and (3.10) we obtain the following expression for P^+ in terms of the independent transverse variables

$$P^+=\frac{T^2\pi}{2P^-}(1-e^2)\int_0^{\pi}(\dot{\vec{x}}_\perp^{\,2}+\vec{x}_\perp'^{\,2})d\sigma=$$

$$=\frac{T^2\pi}{2P^-}(1-e^2)\int_0^{\pi}d\sigma\sum_{\alpha=1}^{(D/2)-1}(\dot{\bar{\xi}}^{\alpha}\dot{\xi}^{\alpha}+\bar{\xi}'^{\alpha}\xi'^{\alpha}). \tag{4.22}$$

The substitution of (4.17) and (4.22) into (4.21) gives

$$M^2=-\sum_{\alpha=1}^{(D/2)-1}\frac{e^2+h_\alpha^2}{1+h_\alpha^2}\vec{P}_{\alpha\perp}^{\,2}+T\pi(1-e^2)\sum_{n=-\infty}^{+\infty}{}'\sum_{\alpha=1}^{(D/2)-1}(1+h_\alpha^2)\bar{\xi}_n^{\alpha}\xi_n^{\alpha}, \tag{4.23}$$

where $\vec{P}_{\alpha\perp}^{\,2}=(P^{2\alpha})^2+(P^{2\alpha+1})^2$.

From (4.23) one immediatelly gets two peculiarities of the mass spectrum of the neutral open string in an external electromagnetic field:

i) if the electric field exceeds the critical value $e^2 > 1$, then $M^2 < 0$;

ii) the motion of the string as a whole in the transverse direction gives the tachyonic contribution to M^2 (the first term in the righ-hand side of (4.23)).

In [7,8] an attempt has been made to redefine the mass of the string in the external electromagnetic field in the following way

$$M'^2 = P'^2, \tag{4.24}$$

where $P' = (1-f^2)^{-1} P$. It allowed one to remove the tachyonic contribution from the motion of the string as a whole in the transverse directions. But the physical basis for this definition of the string mass has not been found. Therefore we shall further use eqs. (4.21) and (4.23).

Let us compare (4.23) with the squared mass spectrum of the free open bosonic string [13]

$$M^2_{free} = 2\pi T \sum_{n=1}^{\infty} \sum_{i=1}^{D-2} n \bar{C}^i_n C^i_n, \tag{4.25}$$

$$\{C^i_n, \bar{C}^j_m\} = -i\delta^{ij}\delta_{nm}, \quad i,j = 1,2,\dots,D-2, \tag{4.26}$$

where $\{\dots,\dots\}$ are the Poisson brackets. For this purpose, we introduce, in (4.23) new Fourier amplitudes instead of ξ^α_n. These new amplitudes should obey the same Poisson brackets as in (4.26). The expansions (4.27) and (4.19) can be inverted

$$\xi^\alpha_{\pm n} = e^{\pm in\tau} \int_0^\pi d\sigma \cos n\sigma \Big\{ \Big(\mp i \frac{n}{2}\Big)\Big(\frac{T}{\pi}\Big)^{1/2} [\xi^\alpha(\tau,\sigma) + \xi^\alpha(\tau,-\sigma)] + \frac{1}{\pi(1+h_\alpha^2)}\Big(\frac{\pi}{T}\Big)^{1/2} \eta^\alpha(\tau,\sigma)\Big\}, \quad n > 0. \tag{4.27}$$

The variables $\xi^\alpha(\tau,\sigma)$ and $\eta^\beta(\tau,\sigma)$ obey the usual Poisson brackets

$$\{\xi^\alpha(\sigma), \xi^\beta(\sigma')\} = \{\eta^\alpha(\sigma), \eta^\beta(\sigma')\} = \{\xi^\alpha(\sigma), \eta^\beta(\sigma')\} = \{\bar{\xi}^\alpha(\sigma), \bar{\eta}^\beta(\sigma')\} = 0,$$
$$\{\bar{\xi}^\alpha(\sigma), \eta^\beta(\sigma')\} = \{\xi^\alpha(\sigma), \bar{\eta}^\beta(\sigma')\} = 2\delta_{\alpha\beta}\delta(\sigma-\sigma'). \tag{4.28}$$

The argument τ which is the same in all the functions in (4.28) is dropped for simplicity. From (4.27) and (4.28) it follows that

$$\{\xi^\alpha_{\pm n}, \xi^\beta_{\pm m}\} = \mp i\delta_{\alpha\beta}\delta_{nm}\frac{2n}{1+h_\alpha^2}, \quad n,m > 0. \tag{4.29}$$

Let us introduce new amplitudes

$$a^\alpha_n = \Big(\frac{1+h_\alpha^2}{2n}\Big)^{1/2} \xi^\alpha_n,$$
$$a^{(D/2)-1+\alpha}_n = \Big(\frac{1+h_\alpha^2}{2n}\Big)^{1/2} \bar{\xi}^\alpha_{-n}, \tag{4.30}$$
$$n > 0, \quad \alpha = 1,2,\dots,(D/2)-1$$

with the Poisson brackets

$$\{a^i_n, \bar{a}^j_m\} = -i\delta_{ij}\delta_{nm}, \quad i,j = 1,2,\dots,D-2 \tag{4.31}$$

Substituting (4.30) into (4.23) one obtains

$$M^2 = -M_{tt}^2 + 2\pi T(1-e^2)\sum_{n=1}^{\infty}\sum_{i=1}^{D-2} n\bar{\alpha}_n^i \alpha_n^i , \qquad (4.32)$$

where

$$M_{tt}^2 = \sum_{\alpha=1}^{(D/2)-1} \frac{e^2 + h_\alpha^2}{1 + h_\alpha^2} \vec{P}_{\alpha\perp}^2 . \qquad (4.33)$$

Thus, the distances between mass levels decrease $(1-e^2)$ times as compared with the free string case.

Now we consider the reference frame in which the total canonical momentum of the string has a vanishing projection onto the electric field

$$p^1 = 0. \qquad (4.34)$$

In this reference frame the energy of the string p^0 turns out to be

$$(p^0)^2 = (1-e^2)\left\{\sum_{\alpha=1}^{(D/2)-1} \frac{\vec{P}_{\alpha\perp}^2}{1 + h_\alpha^2} + 2\pi T \sum_{n=1}^{\infty}\sum_{i=1}^{D-2} n\bar{\alpha}_n^i \alpha_n^i\right\} \qquad (4.35)$$

Thus, in the classical theory when $e^2 < 1$ the energy of the neutral string in an external constant homogeneous electromagnetic field is strictly positive.

In quantum theory in eq.(4.35) the tachyonic contribution appears which is caused by zero point fluctuations of the harmonic oscillators

$$E_0^2 = -\pi T(1-e^2)\,\frac{D-2}{12} . \qquad (4.36)$$

5. The energy spectrum of the string with a net charge in an external magnetic field.

In this section we consider the open bosonic string with a net charge $(q_1 + q_2 \neq 0)$ propagating in a background electromagnetic field. In this case the action (2.1) is not invariant with respect to the transformations (3.11). As a consequence, the total canonical momentum of the string p^μ is not conserved. Hence, the squared mass of the string M^2 is not conserved too. In addition, it is not known in which way the light-like gauge conditions in the free string theory should be generalized when the charged string propagates in external electric field.

Therefore let us consider as a background field a constant homogeneous magnetic field. Without loss of generality one can choose a gauge for the electromagnetic potential $A_\mu(x)$ such that $A_0(x) = A_1(x) = 0$. In this case the components p^0 and p^1 of the total canonical momentum of the string are obviously conserved. Instead of the squared mass of the string we shall consider the total energy of the string in the reference frame where $p^1 = 0$.

Thus we go now to the consideration of the basic equations in the problem under consideration. The equations of motion (4.1) and the orthonormal gauge conditions (4.2) remain unchanged.

The boundary conditions (2.7) are written as

$$x'^0 = x'^1 = 0, \quad \sigma = 0, \pi, \tag{5.1}$$

$$\xi'^\alpha - i h_{1\alpha} \dot{\xi}^\alpha = 0, \quad \sigma = 0, \tag{5.2}$$

$$\xi'^\alpha + i h_{1\alpha} \dot{\xi}^\alpha = 0, \quad \sigma = \pi,$$

$$\alpha = 1, 2, \ldots, (D/2)-1, \quad h_{1\alpha} = q_1 H_\alpha / T, \quad h_{2\alpha} = q_2 H_\alpha / T. \tag{5.3}$$

In (5.2) and (5.3) there is no summation over α.

The light-like gauge is defined in the same way as in the free-string case

$$n_\mu x'^\mu = 0, \quad n_\mu \dot{x}^\mu = n_\mu P^\mu / (T\pi), \tag{5.4}$$

where $n_\mu P^\mu$ is projection of the total canonical momentum of the string onto the constant light-like vector n^μ: $n_\mu n^\mu = 0$, $n^0 = n^1 = 1$, $n^2 = n^3 = \ldots = n^{D-1} = 0$. As it was noted above, this projection is conserved in the case under consideration. The gauge conditions (5.4) are written in terms of light-front variables as follows

$$x'^- = 0, \quad \dot{x}^- = \frac{P^-}{T\pi}. \tag{5.5}$$

The solution of the orthonormal gauge conditions (4.2) with the help of (5.5) gives

$$\dot{x}^+ = \frac{T\pi}{2P^-}(\dot{\vec{x}}_\perp^2 + \vec{x}_\perp'^2), \quad x'^+ = \frac{T\pi}{P^-}\dot{\vec{x}}_\perp \vec{x}_\perp'. \tag{5.6}$$

It is easy to show in the same way as in the preceding section that eqs.(5.6) are consistent with the equations of motion (4.1) and boundary conditions (5.1). Therefore the transverse coordinates of the string $\vec{x}_\perp(\tau,\sigma)$ can again be considered as independent dynamical variables.

To find the solution of boundary conditions (5.2), (5.3) we put

$$\xi(\tau,\sigma) = e^{i\lambda\tau}(C_1 \cos\lambda\sigma + C_2 \sin\lambda\sigma), \tag{5.7}$$

where C_1 and C_2 are constants. Substitution of (5.7) into (5.2) and (5.3) results in the relation between the constants C_1, C_2:

$$C_2 = -h_1 C_1 \tag{5.8}$$

and in the equation defining the string frequencies

$$\sin(\lambda + \varepsilon)\pi = 0, \tag{5.9}$$

where

$$\mathrm{tg}\,\pi\varepsilon = \frac{h_1 + h_2}{1 - h_1 h_2}, \quad |\varepsilon| < \frac{1}{2}. \tag{5.10}$$

From (5.9) one gets

$$\lambda_n = n - \varepsilon, \quad n = 0, \pm 1, \pm 2, \ldots \tag{5.11}$$

Thus, the eigenfunctions of the boundary value problem under consideration are proportional to the following expression

$$e^{i(n-\varepsilon)\tau}\cos[(n-\varepsilon)\sigma+\varepsilon_1],\quad n=0,\pm 1,\dots, \tag{5.12}$$

where $tg\,\varepsilon_1 = h_1$.

For $\xi^{\alpha}(\tau,\sigma)$ we have the expansion

$$\xi^{\alpha}(\tau,\sigma)=Q^{\alpha}+\frac{i}{\sqrt{\pi T}}\sum_{n=-\infty}^{+\infty} e^{i(n-\varepsilon^{\alpha})\tau}\frac{\xi_n^{\alpha}}{n-\varepsilon^{\alpha}}\cos[(n-\varepsilon^{\alpha})\sigma+\varepsilon_1^{\alpha}]. \tag{5.13}$$

Using eq.(4.22) with $e=0$ we obtain in the reference frame where $P^1=0$

$$(P^0)^2 = T^2\pi\int_0^{\pi} d\sigma\sum_{\alpha=1}^{(D/2)-1}(\dot{\bar{\xi}}^{\alpha}\dot{\xi}^{\alpha}+\bar{\xi}'^{\alpha}\xi'^{\alpha}). \tag{5.14}$$

The substitution of (5.13) into (5.14) gives

$$(P^0)^2 = T\pi\sum_{n=-\infty}^{+\infty}\sum_{\alpha=1}^{(D/2)-1}\bar{\xi}_n^{\alpha}\xi_n^{\alpha}. \tag{5.15}$$

Thus, at the classical level the squared energy of the string in the chosen reference frame is strictly positive.

Now we introduce instead of ξ_n^{α} in (5.15) new amplitudes [11] with the following nonvanishing Poisson brackets

$$\{a_n^{\alpha},\bar{a}_{n'}^{\beta}\}=-i\delta_{\alpha\beta}\delta_{nn'},$$
$$\{b_m^{\alpha},\bar{b}_{m'}^{\beta}\}=-i\delta_{\alpha\beta}\delta_{mm'}, \tag{5.16}$$

$\alpha,\beta = 1,2,\dots,(D/2)-1;\quad n,n'=1,2,\dots;\quad m,m'=0,1,2,\dots$

Equation (5.15) turns out to be

$$(P^0)^2=T\pi\sum_{n=1}^{\infty}\sum_{\alpha=1}^{(D/2)-1}\{(n-\varepsilon^{\alpha})(\bar{a}_n^{\alpha}a_n^{\alpha}+\bar{b}_n^{\alpha}b_n^{\alpha})+|\varepsilon^{\alpha}|\bar{b}_0^{\alpha}b_0^{\alpha}\} \tag{5.17}$$

In quantum theory zero point fluctuations have to be taken into account in (5.17). Their contribution E_0^2 to (5.17) is

$$E_0^2 = T\pi\sum_{\alpha=1}^{(D/2)-1}\Big[\sum_{n=1}^{\infty}(n-\varepsilon^{\alpha})+\frac{|\varepsilon^{\alpha}|}{2}\Big]. \tag{5.18}$$

The regularised sum in (5.18) can be obtained with the help of the Rieman zeta-function [14]

$$\zeta(z,q)=\sum_{n=0}^{\infty}(q+n)^{-z}. \tag{5.19}$$

Taking into account that

$$\zeta(-1,q)=\frac{1}{2}q(1-q)-\frac{1}{12} \tag{5.20}$$

we get for (5.18)

$$\frac{E_0^2}{T\pi}=-\frac{D-2}{24}+\frac{1}{2}\sum_{\alpha=1}^{(D/2)-1}[\varepsilon^{\alpha}(1-\varepsilon^{\alpha})+|\varepsilon^{\alpha}|]. \tag{5.21}$$

6. The charged string in background electric and magnetic fields

As was noted above, the light-like gauge from the free string theory cannot be generalized to this case*). Therefore, we may here develop a formalism analogous to the covariant approach in the free string theory [13].

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*) It should be noted that in the theory of a free closed bosonic string there is no complete light-like gauge which enables one to eliminate all the unphysical degrees of freedom. The condition of invariance with respect to the global translations of the parameter σ cannot be solved as the algebraic equation.

For this purpose one has to construct the solution of the equations of motion (2.5), which obeys the boundary conditions (2.7) and then to demand the fulfilment of the orthonormal gauge conditions (2.4), i.e. to construct the Virasoro operators. In this section we consider how these operators are modified in the theory of a charged open bosonic string propagating in background electric and magnetic fields.

The solution for the transverse variables $\vec{x}_\perp(\tau,\sigma)$ obtained in the previous section remains obviously without changes. For the longitudinal components $x^{\pm}(\tau,\sigma)$ we have the following boundary conditions

$$x'^{\pm} \pm e_1 \dot{x}^{\pm} = 0, \quad \sigma = 0, \tag{6.1}$$

$$x'^{\pm} \mp e_2 \dot{x}^{\pm} = 0, \quad \sigma = \pi,$$

$$e_a = q_\alpha E/T, \quad \alpha = 1,2. \tag{6.2}$$

Separating the variables we put

$$x^{+}(\tau,\sigma) = e^{i\lambda\tau}(C_1 e^{i\lambda\sigma} + C_2 e^{-i\lambda\sigma}). \tag{6.3}$$

The frequencies λ_n in this case appear to be complex-valued:

$$\lambda_n = n - i\rho, \quad n = 0, \pm 1, \pm 2, \ldots,$$

$$\rho = \frac{\rho_1 + \rho_2}{2\pi}, \quad \rho_\alpha = \ln\frac{1+e_\alpha}{1-e_\alpha}, \quad \alpha = 1,2. \tag{6.4}$$

The constants C_1 and C_2 satisfy the relation

$$C_2 = e^{\rho_1} C_1 .$$

Thus $x^{\pm}(\tau,\sigma)$ can be expanded in the form

$$x^{\pm}(\tau,\sigma) = Q^{\pm} + \sum_{n=-\infty}^{+\infty} e^{i(n\mp i\rho)} \frac{w_n^{\pm}}{n \mp i\rho}\left(e^{i(n\mp i\rho)\sigma} + e^{\rho_1 - i(n\mp i\rho)\sigma}\right). \tag{6.5}$$

Due to the imaginary part of λ_n this solution increases exponentially when $\tau \to \pm\infty$.

The Varasoro operators L_n can be defined in the usual way

$$(\dot{x} \pm x')^2 = \sum_{n=-\infty}^{+\infty} L_n e^{in(\tau\pm\sigma)} . \tag{6.6}$$

For L_n we have the standard bilinear expressions

$$L_n = -\frac{1}{2}\sum_n \left(w^{+}_{n-m} w^{-}_m + \sum_{\alpha=1}^{(D/2)-1} \xi^{\alpha}_{n-m} \xi^{\alpha}_m\right), \quad n = 0, \pm 1, \ldots . \tag{6.7}$$

The difference from the free-string case consists in the unusual Poisson brackets for amplitudes $w_n^{\pm}$

$$\{w^{\pm}_{\pm n}, w^{\mp}_{\pm m}\} = \mp i\, \delta_{nm} \frac{2(n \mp i\rho)}{1 + e^{2}}, \quad n, m > 0. \tag{6.8}$$

In quantum theory in L_0 the normal ordering of the operators should be postulated. This results in the appearance of a constant in L_0 which is analogous to $\alpha(0)$ in the free string theory. This constant turns out to be

$$-\frac{D-2}{24} + \frac{1}{2}\sum_{\alpha=1}^{(D/2)-1} [\varepsilon^{\alpha}(1 - \varepsilon^{\alpha}) + |\varepsilon^{\alpha}|]. \tag{6.9}$$

7. Conclusion

The important physical result obtained here is the constraint on the strength of the background electric field. It should be noted that this conclusion has been drown in the most general way without using any solutions of the equations of motion. It means that the abelian gauge field generated in the low-energy limit in the interacting string theory has also to obey this constraint. This constraint should result in the modifications of the corresponding abelian gauge field theory. For example, the nature of the ultraviolet divergences in such a theory may be changed.

It is interesting to explore further the role of the tachyonic contribution to M^2 due to the transverse motions of the string as a whole. Probably it is impossible to conclude on this basis that the classical dynamics of an open neutral bosonic string is unstable at any values of external electric and magnetic fields. The positivity of the string energy in a special reference frame supports this conclusion.

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Received by Publishing Department
on June 15, 1987.

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Нестеренко В.В. E2-87-420

Открытая бозонная струна в фоновом электромагнитном поле

Исследована классическая и квантовая динамика открытой бозонной струны в постоянном однородном электромагнитном поле в D-мерном пространстве-времени. Рассматриваются открытые струны двух типов. Нейтральные струны, у которых заряды q_1 и q_2 на концах удовлетворяют условию $q_1+q_2=0$, и заряженные струны, у которых $q_1+q_2\neq 0$. Согласованность теории требует, чтобы фоновое электрическое поле не превышало некоторое критическое значение. Расстояния между массовыми уровнями нейтральной открытой струны уменьшаются в $(1-e^2)$ раз, где e-безразмерная напряженность электрического поля; магнитное поле не влияет на это расстояние. Показано, что уже на классическом уровне в массовом спектре нейтральной открытой струны, есть тахионный вклад, обусловленный трансляционным движением струны как целого в поперечных направлениях. Этот вклад отсутствует, если вместо M^2 рассматривать энергию струны в специальной системе отсчета, в которой проекция полного импульса струны на направление электрического поля равна нулю. Найден вклад в энергетический спектр и в операторы Вирасоро нулевых колебаний нейтральной и заряженной струн. Важным моментом исследования является использование обобщенной светоподобной калибровки.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1987

Nesterenko V.V. E2-87-420

Open Bosonic String in Background Electromegnetic Field

The classical and quantum dynamics of an open bosonic string propagating in the D-dimensional space-time in the presence of a background electromagnetic field is investigated. An important point in this consideration is the use of the generalized light-like gauge. There are considered the strings of two types; the neutral strings with charges at their ends obeying the condition $q_1+q_2=0$ and the charged strings having a net charge $q_1+q_2\neq 0$. The consistency of theory demands that the background electric field does not exceed its critical value. The distance between the mass levels of the neutral open string decreases $(1-e^2)$ times in comparison with the free string, where e is the dimensionless strength of the electric field. The magnetic field does not affect this distance. It is shown that at a classical level the squared mass of the neutral open string has a tachyonic contribution due to the motion of the string as a whole in transverse directions. The tachyonic term disappears if one considers, instead of M^2, the string energy in a special reference frame where the projection of the total canonical momentum of the string onto the electric field vanishes. The contributions due to zero point fluctuations to the energy spectrum of the neutral string and to the Virasoro operators in the theory of charged string are found.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1987