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PHENOMENOLOGY AND THEORY OF CONFINEMENT

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Introduction

At present a lot of diverse opinions exist about confinement. In recent years the idea of separation of effects of hadronization and confinement has got popular. By this idea, the confinement is nonessential for the hadronization dynamics. However, from a phenomenological point of view, the confinement problem has to explain why we cannot observe quarks as leptons, and to substantiate the experimental method of measurement of quark quantum numbers.

In this paper, we discuss phenomenological and theoretical arguments of the separation of the hadronization dynamics from confinement and the idea of the "kinematic" confinement.

1. Phenomenology

Quarks were proposed as elements of the unitary classification of hadrons/1/ and in the first static models it was not clear whether the quark is a physical reality or a mathematical tool/2/.

The first results confirming the reality of quarks as dynamical consistents of hadrons have been obtained in deep-inelastic scattering experiments/3/.

The observation of quarks and measurement of their quantum number in these experiments are essentially based on the unitary relation for the S-matrix.

$$S S'^{+} = I \qquad ; \qquad S = I + iT$$

$$\sum_h {}_{ph}\langle f | T | h \rangle_{ph} \; {}_{ph}\langle h | T | i \rangle_{ph} = 2\,\mathrm{Im}\; {}_{ph}\langle f | T | i \rangle_{ph} \tag{1}$$

(where $|h\rangle_{ph}$ are all physical hadron states; $f, i \in h$).

All cross-sections of measurable processes (like $e^+e^- \to$ hadrons, $e^+e^-p \to$ hadrons, etc.) behave as if they are imaginary parts of the quark-gluon diagrams. The experimental momentum distributions of hadrons in the left-hand side of eq.(1) well reproduce the distributions of quarks, antiquarks, and gluons whose dynamics is completely controlled by the right-hand side of eq.(1) (i.e., by QCD-perturbation theory). In this way the quark and gluon spins, their charges and number of colors have been measured[4,5].

However, unlike leptons quantum numbers of which are measured in elastic processes, the "observation" of quarks with the help of the inelastic hadron reactions (1) has somewhat a paradoxal character.

On the one hand, the complete set of physical states $|h\rangle_{ph}\,{}_{ph}\langle h|$ in eq.(1) does not contain quarks and gluons as if their production amplitudes are equal to zero(or the residues of the color Green functions are equal to zero)

$$ {}_{ph}\langle f\,|\,T\,|\,c\rangle_{ph} = 0 \qquad \left(\mathrm{Res}\; G^{(c)} = 0 \right), \tag{2}$$

where $|c\rangle_{ph}$ is a quark or a gluon physical state

Really, nobody has observed the quark production and eq.(2) is a "phenomenological criterion" of confinement.

On the other hand, for the measurement of quark quantum number in the phenomenology one essentially uses the fact that the imaginary part of hadron amplitudes in the energy region far from resonances is factorized: it is expressed by a product of the quark-parton creation amplitudes

$$ \mathrm{Im}\; {}_{ph}\langle f\,|\,T\,|\,i\rangle_{ph} \simeq {}_{ph}\langle f\,|\,T_{①}\,|\,c\rangle_{part}\; {}_{part}\langle c\,|\,T_{②}\,|\,i\rangle_{ph}. $$

That is the "observation" of quarks is based on the analytical properties of the "elastic" hadron amplitude, which in the usual quantum field theory would be interpreted as a nonzero probability of creation of quarks as partons

$$ {}_{ph}\langle f\,|\,T\,|\,c\rangle_{part} \neq 0\,. \tag{3}$$

It should be noted that for leptons in perturbative QED the physical and "parton" states coincide $|\ell\rangle_{ph} \simeq |\ell\rangle_{part}$, which signifies that the "nonobservability" (2) and "observability" (3) takes place in the same energy region of the Minkowski space.

Thus, the high-energies phenomenology widely uses imaginary parts of quark diagrams and the restriction of all physical states only to hadron states.

An analogous situation can be observed now in the low-energies quark phenomenology, where for the construction of the chiral effective meson Lagrangian from QCD one uses the real part of one-loop quark diagrams, and at the same time, the quarks are removed from the unitarity relation (see refs.[6] and references therein). The hypothesis of hadron physical states (2) is not accompanied by the consideration of dynamics that underlines that hypothesis. Thus the confinement process is separated from the quark hadronization process. Analogous conclusion is made in the phenomenology of the sum-rule method[7] where the confinement effects do not influence the description of experimental data.

Thus, in the recent quark phenomenology these is a tendency to separate the hadronization process from the confinement one, and to consider the latter as a purely "kinematic" effect.

2. Theory

In QCD the equality (1) is called the principle of quark-hadron duality (QHD): the global one (if eq.(1) is used in the sense of averaging over energy) and the local one (if eq.(1) is used without averaging).

The "observation" of quarks in QCD is explained by the asymptotic freedom phenomenon [8], i.e. by an effective decrease of the coupling constant in the deep-Euclidean region. The QCD-perturbation theory is valid only in the Euclidean space, where all diagrams are calculated. Then, dispersion relations (i.e. the integration over energy) are used to establish the relation of theoretical values with the realistic experimentally measurable cross-sections in the Minkowski space [7]. However, in this way, one can explain only the global QHD (where the averaging over energy represents a dispersion integral), but cannot explain the fact that the cross-section of the process $e^+e^- \to$ into hadrons in the energy region far from resonances pointwise coincides with the imaginary part of a quark loop (i.e., cannot explain the principle of the local QHD based on the QCD-perturbation theory in the Minkowski space.) In the case of local QHD we are forced to admit that the imaginary parts of quark loops are not equal to zero, and the quarks manifested as analytical properties of the "elastic" hadron amplitudes that are calculated by the QCD-perturbation theory in the Minkowski space. But the question arises why we do not observe quarks as physical states in the same energy region.

We see that the "asymptotical freedom" is not sufficient for explaining all the phenomenology of the observation of quarks and gluons and for a complete understanding of this phenomenology it is desirable to explain confinement.

The explanation of confinement is usually connected in QCD with the proof of the Wilson criterion [9] or with a linear-rising quark-quark potential that is found by a computer calculation [10].

Historically, the Wilson criterion was inspired by the Schwinger model [11] (QED_{1+1}) with a linear-rising Coulomb potential. Recently it has been established [12] that the version of the Schwinger model quantization does not lead to confinement in the sense of disappearance of the quark Green function residue (2).

It is well known [13] that the linear-rising vector potential also does not lead to confinement (the Klein paradox).

The relativistic version of the linear-rising potential is well studied in the $1/N$ -approximation in refs. [14] where it is shown that this version explains the hadronization process rather than confinement. In the last years the confinement potentials have been successfully used as the potentials of hadronization and of spontaneous breaking of chiral symmetry [14,15]. We see that in QCD-theory these is a tendency to consider the dynamics, which is traditionally connected with confinement, as only the dynamics of hadronization. (From this point of view, it is useful to consider the Wilson criterion and the confinement potentials as a criterion of hadronization and as potentials of hadronization). In the theory, like in the phenomenology, the dynamics hadronization is separated from confinement. The latter means only the restriction of the physical states (or of the Green functions) only to colorless (colorscalar) sector (2). Here two questions arise:

1) How to "restrict" the physical sector, so that this restriction (2) does not influence the hadronization dynamics?
2) Why analytical properties of "elastic" colorless amplitudes (for example, the color scalar current correlators) are reproduced by the bare quark diagrams, or why the sum over hadrons states forgets about hadronization potential in the energy far from resonances, or in the sense of the energy averaging?

There are a lot of theoretical [16-19] and phenomenological [6,7] arguments pointing out that if we answer the first question, the unitary relation (1) will answer the second question automatically. In the presence of bound states (hadrons) the unitary relation should be understood as one of the self-consistency conditions of the theory used for normalizing the bound-state wave functions and their interaction constants [20]. If for some reasons the probability of the color channels disappears, the probability of other hadron channels increases so that the total probability is equal to unity. In this context the "parton" states (3) are manifestation of the analytical properties of hadron amplitudes, which follows from the unitary and do not contradict the confinement (2). Thus, for proving the QHD formulae (1) which contains only hadron amplitudes and which is used for the measurement of quark quantum number it is sufficient to prove only the "kinematic" confinement forbidding the color particle production.

3."Kinematic" confinement

The confinement mechanism which does influence the hadronization dynamics has been proposed in ref.[21,22]. The main idea consists in the explicit solution of constraints (i.e., the Gauss equation and the gauge condition) and in the explicit construction of physical variables as unlocal gauge invariant functionals of the initial fields. For QED such variables are the nonlocal transverse ones

$$\left.\begin{aligned} ieA_j^T &= \mathcal{U}(A)\,(ieA_j+\partial_j)\,\mathcal{U}(A)^{-1} \\ \psi^T &= \mathcal{U}(A)\,\psi \end{aligned}\right\} ;\ \mathcal{U}(A)=\exp\left\{ie\frac{1}{\partial^2}\partial_i A_i\right\}. \tag{4}$$

The gauge of these variables is not fixed under the relativistic transformation. Due to the nonlocality in the new Lorentz reference frame the fields (4) become transverse with respect to the new time axis. In terms of the variables (4) one can solve the old problem of the completely relativistic covariant construction of the path integral that does not depend on a gauge choice even for a one-particle fermion Green function[21].

The quantization with the explicit constraint solution contains also the new physical information that is usually omitted in the ordinary methods. Nonlocal variables of the type of (4) are defined up to stationary gauge factors $v(\vec{x})$ with the phase being solu-

tions of the Gauss equation in the "empty space" ($A_i=0;\psi=0$)

$$v(\vec{x}) = \exp\{i\hat{\lambda}(\vec{x})\} \quad ; \quad \left(\frac{\partial}{\partial x_i}\right)^2 \hat{\lambda}(\vec{x})=0 \tag{5}$$

which are zeroes of the operator ∂^2 in eq.(4).

In the general case the gauge factors $v(\vec{x})$ should describe smooth nonsingular maps of the (D-1)-dimensional space onto the gauge group G. If these maps exist, the physical fields (4) are degenerated: the same physical state corresponds to the fields ψ and $v(\vec{x})\psi$.

This degeneration arises in the gauge theories in a finite space $|\vec{x}|\leqslant R$ with the topologically nontrivial homotopy group of the stationary gauge factors.

$$\pi_{D-1}(G)=\mathbb{Z} \quad (\ \mathbb{Z} \text{ is the group of integers under addition}) \tag{6}$$

The condition (6) is satisfied for the Schwinger model $(\pi_1(U(1))=\mathbb{Z})$ and for QCD $(\pi_3(SU(3))=\mathbb{Z})$, but not for $QED_{3+1}\ (\pi_3(U(1))=0)$.

A nontrivial solution of eq.(5) for QCD is the matrix

$$v^{(n)}(\vec{x}) = \exp\left\{i\frac{x^a\tau^a}{R}\pi n\right\} \quad ; \quad \tau^1=\lambda^2;\ \tau^2=\lambda^5;\ \tau^3=\lambda^7 \qquad n=0,\pm1,\pm2,\dots \tag{7}$$

(where λ^a are the Gell-Mann matrices, τ^a are the matrices of the minimal $SU(2)$-subgroup, where the $SU(3)$-fundamental representation is irreducible under $SU(2)$-transformations). This matrix describes a nonsigular map with the index n calculated by the formula

$$n=\frac{1}{24\pi^2}\int d^3_x\, \varepsilon_{ijk}\, tr(V_i V_j V_k)\ ; \quad V_i = v^{(n)}\partial_i v^{(n)^{-1}} \tag{8}$$

The index n does not disappear even in the infinite-volume limit and represents an example of topological quantum anomalies of the type of the axial current divergence: both these quantities (the index and divergence) after removing the regularization are not equal to zero despite the disappearance of the initial elements of their construction (the field V_i , or the Pauli--Willars propagators).

Generally speaking, here we have also the degeneration of the matrix $\mathcal{V}(\vec{x})$ with respect to angles describing the color co-ordinate - orientation with respect to space co-ordinates. Due to the degeneration the generating function of the Green functions should be averaged over all degeneration parameters, for example (n).

$$Z_{Conf}(\eta,\bar{\eta},\mathcal{J}) = \lim_{R,T\to\infty} \lim_{N\to\infty} \frac{1}{2N} \sum_{n=-N}^{N} Z_{RT}\left(\mathcal{V}^{(n)}\eta;\ \bar{\eta}\mathcal{V}^{(n)^{-1}};\ \mathcal{V}^{(n)}\mathcal{J}\mathcal{V}^{(n)^{-1}}\right), \tag{9}$$

where $Z_{RT}(\eta,\bar{\eta},\mathcal{J})$ is the usual Faddeev-Popov path integral, and $\eta, \bar{\eta}, \mathcal{J}$ are the color field sources that have in eq.(9) the degeneration gauge factor. As it has been shown in detail in ref./22/ (p.p. 43-46), after averaging over the infrared degeneration parameters all the Green functions which are not scalar under color gauge transformations disappear. But the colorless Green functions of the type of correlators between electromagnetic and weak currents coincide with the usual QCD perturbative Green functions.

We would like to emphasize the noncommutativity of the limit procedures in (9) determined as in quantum statistics/23/.

Thus, all the color Green functions disappear by virtue of the quantum interference of an infinite number of the gauge factors of the topological degeneration, i.e. the confinement criterion is fulfilled "kinematically". This is just the manifestation quantum

anomalies (8). So the following physical picture arises: In a hadron lepton collision all particles are created (hadrons, quark, gluons, etc.) however, due to the degeneration of color physical states in experiment we can observe a superposition of the amplitudes with different topological numbers (8), therefore the total probability of color particle productions happens to be equal to zero. In accordance with the probability conservation law the probabilities of the hadron channels increases so that the unitarity relation is fulfilled and just this relation allows us to observe quarks and gluons by means of the imaginary parts of "elastic" hadron amplitudes.

The quark hadronization goes independently of the confinement process. The description of the hadronization dynamics as a rule accomplished dividing the gluon propagator into two parts: a perturbative and a nonperturbative part given in different function classes and dominating in different regions of interaction. In ref./24/ it is shown that the appearance of the nonperturbative hadronization propagator can be connected with the zero modes of gluon field of a finite energy density/25/.

Conclusion

History of the development of the quark theory of hadrons can be expressed in three words: in the sixties dominated was the problem of quark "existence"; in the seventies, the "confinement" problem; in the eighties, the "hadronization" problem.

The statement of the problem of "confinement" of quarks is dictated by experiments proving their "existence". The theoretical proof of confinement in QCD means to give answers to two questions:

1) Why are physical states of the theory limited by a hadron (coloreless) sector?

2) Why does the sum over finite hadron states reproduce the analytical properties of quark-gluon diagrams?

In the last years the phenomenological and theoretical facts appear pointing out that the answer to the first question is also the answer to the second question (through the unitary relation), and that the very "confinement" does not influence the hadronization dynamics. From a phenomenological and theoretical point of view, it is useful to consider the "confinement" as a purely "kinematic" effect. Just, this confinement is explained by the destructuve interference of the gauge (phase) factor of the topological degeneration of the physical variables which takes place at the explicit solution of the gauge theory constraint.

The main problem of the last years is to find a theoretical foundation of the "hadronization" dynamics and to explain the appearance of dimensional QCθ-parameters.

Up to now it has been assumed that all dimensional parameters are connected with the only Λ -parameter appearing in the theory as the infrared boundary condition for the renormalization group equations (dimensional transmutation phenomenon).

The very fact of existence of a finite theory without ultraviolet divergences where the renormalization group equations become simple identities permits another interpretation of the Λ -parameter.

If we shall consider QCD as a part of a unification theory without ultraviolet divergences (i.e. with the physical Pauli-Villars regularization with the mass of an order of the asymptotical desert) then the Λ -parameter can be calculated and expressed in terms of the asymptotical desert parameters, i.e. the Λ -parameter is the ultraviolet (but not infrared) one /21/.

From this point of view the question about the real infrared dimensional transmutation still remains open.

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Феноменология и теория конфайнмента

В последнее время стали убеждаться, что описание экспериментальных данных не зависит от механизма конфайнмента. С точки зрения современной феноменологии динамика адронизации отделена от конфайнмента, который полезно рассматривать как чисто "кинематический" эффект. С другой стороны в современной теории появились результаты, которые указывают, что критерий Вильсона и потенциалы конфайнмента не достаточны для объяснения феноменологического конфайнмента в смысле нулевых цветных амплитуд и функций Грина. Однако, эти потенциалы хорошо объясняют спектр адронов и спонтанное нарушение киральной инвариантности, т.е. динамику адронизации. Кинематический конфайнмент может быть объяснен топологическим вырождением всех цветных состояний. Это вырождение возникает в КХД если мы будем квантовать теорию путем явного решения уравнения связи; тогда все цветные состояния определены с точностью до калибровочных факторов, описывающих отображение $\pi_3(SU(3)) = Z$. Полная вероятность рождения цветных амплитуд равна нулю из-за деструктивной интерференции этих факторов. В результате в КХД остается только адронный сектор, используемый в феноменологии.

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Phenomenology and Theory of Confinement

In the recent phenomenology, the confinement effects are not essential for the description of experimental data. From a point of view of the phenomenology the hadronization dynamics is separated from the confinement that is useful to consider as a purely "kinematic" effect. On the other hand, the recent theory contains results which point out that the Wilson criterion and the confinement potentials are not sufficient for explaining the phenomenological confinement in the sense of zero color amplitudes or Green functions. However, these potentials well explain the hadron spectrum and spontaneous breaking of chiral symmetry, i.e., the hadronization dynamics. The "kinematic" confinement can be explained by the topological degeneration of all color-particle physical states in QCD. This degeneration arises if we quantize the theory by explicitly solving the gauge and dynamic constraints: all color states are defined up to the gauge (phase) factors describing the map of the three-dimensional space onto $SU(3)_c$-group ($\pi_3(SU(3)_c = Z$). The total probability of the color particle generation is equal to zero due to the destructive interference of these phase factors. As a result, in QCD there remains only a hadron sector used in the phenomenology.

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