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SUMMARY

The spontaneous breakdown of symmetry of the σ-model in the $[1,1]$ representation of $SU(2)\times SU(2)$ group is investigated. It is shown that the spontaneous breakdown is realized in all cases of squared mass μ^2 in mass term in the Lagrangian ($\mu^2>0$, $\mu^2=0$, $\mu^2<0$), unlike the σ-model in $[1/2,1/2]$ representation, in which the spontaneous breakdown only for the case $\mu^2<0$ manifests itself. Further, different but equivalent methods of obtaining of the nonlinear realization for pions in the frame of an extended in such a way σ - model are demonstrated. Finally, it is sketched, that the obtained results can be generalized to all $[N/2,N/2]$ representations of $SU(2)\times SU(2)$ chiral group.

Introduction

In order to construct a linear realization of the chiral symmetry for pions, one has to introduce other fields. The simplest possibility appears via introducing a scalar σ -field with isospin $T=0$, which is needed to complete the four dimensional linear representation $[1/2,1/2]$ of chiral $SU(2)\times SU(2)$ group. The model obtained by such a procedure is known as σ-model /1/.

If one wants to extend this model and to have the scheme with automatically conserved isospin, then the representation must be considered of the type $[N/2, N/2]$ only /2/. Namely these representations contain $T=0$ representation of isospin group.

The natural extension of the aforementioned σ -model (and still sufficiently simple for investigation) seems to be the consideration of $[1,1]$ representation which contains besides the scalar $T=0$ and pseudoscalar $T=1$ particles, also the scalar particles with $T=2$.

The purpose of this paper is to investigate the structure of an extended in such a way version of σ -model, denoted further by Σ and to generalize the results for all $[N/2,N/2]$ representations.

The plan is as follows. After a brief survey of the symmetry of the Σ -model in sect. 1, we are interested (sect. 2) in the spontaneous breakdown /3/ (further we shall prefer to call it the Goldstone mode /4/) of the Σ -model, the nature of which reveals the form invariance of a Hamiltonian. In such aspects of symmetry, physical states form a representation basis of a (classical) Lagrangian. In this case there are Goldstone bosons and the vacuum states are invariant under the subgroup but not under the fullgroup.

On the basis of the obtained results, one can see immediately that Goldstone mode manifests without special assumption unlike the case of the σ -model [3] where the squared mass μ^2 in the mass term in the Lagrangian is negative only. In our case the Goldstone mode is realized for all three possibilities ($\mu^2>0$, $\mu^2=0$, $\mu^2<0$).

It can be generalized that in the framework of renormalizable Lagrangians the Goldstone mode for odd values of N in [N/2,N/2] representations is realized in the same way as in the σ -model and for N -even will manifest like in the Σ -model.

In sect. 3 a few possibilities to carry out the nonlinear realization [2,6] of $SU(2)\times SU(2)$ group are presented in which we do not admit the existence of fields other than that of the pion. This is possible to achieve generally, when partners of Goldstone bosons acquire infinite masses [5].

1. *Symmetry of the Σ -model*

It is known that the Lie algebra of $SU(2)\times SU(2)$ is isomorphic to the algebra of the 4-dimensional rotation group $R(4)$. Let us denote the generators of $R(4)$ by $L_{\mu\nu}$ ($\mu,\nu = 1,...4$; $L_{\mu\nu}=-L_{\nu\mu}$). The correspondence with the generators Q_k , Q_k^5 of Lie algebra of $SU(2)\times SU(2)$ is as follows:

$$L_{ij} \sim \epsilon_{ijk}\, Q_k$$

and

$$L_{4k} \sim Q_k^5 \quad i,j,k=1,2,3\ .$$

Then the basis of the [N/2,N/2] representation of $SU(2)\times SU(2)$ group can be written as an irreducible tensor of $R(4)$ of the rank N, $\phi^{(N)}_{\alpha\beta\sigma}$ $(\alpha,\beta,\gamma,..=1,..4)$ which is traceless and symmetric,

In our applications of particular interest is the [1,1] representation, denoted by $\phi_{\alpha\beta}$,the infinitesimal transformation of which is:

$$\delta\phi_{\alpha\beta}=\frac{i}{2}\,\omega_{\mu\nu}\,(L_{\mu\nu})_{\alpha\bar\alpha,\beta\bar\beta}\,\phi_{\bar\alpha\bar\beta} \tag{1}$$

with parameter $\omega_{\mu\nu}$ and

$$[L_{\mu\nu}]_{\alpha\bar\alpha,\beta\bar\beta}=-i\{\delta_{\beta\bar\beta}[\delta_{\mu\alpha}\delta_{\nu\bar\alpha}-\delta_{\mu\bar\alpha}\delta_{\nu\alpha}]+ \\ +\delta_{\alpha\bar\alpha}[\delta_{\mu\beta}\delta_{\nu\bar\beta}-\delta_{\mu\bar\beta}\delta_{\nu\beta}]\}. \tag{2}$$

From (1) we get

$$\delta\phi_{\alpha\beta}=\omega_{\alpha\sigma}\phi_{\sigma\beta}+\phi_{\alpha\sigma}\,\omega^T_{\sigma\beta} \tag{3}$$

which implies the axial vector and the vector infinitesimal transformation laws respectively

$$\begin{aligned}\delta\phi_{ik}&=a_i\phi_{k4}+a_k\phi_{i4}\\ \delta\phi_{i4}&=a_i\phi_{44}-a_k\phi_{ik}\\ \delta\phi_{44}&=-2(a_i\phi_{i4})\end{aligned} \tag{3.a}$$

where $a_i=\omega_{i4}$ $\phi_{ii}=-\phi_{44}$

$$\begin{aligned}\delta\phi_{ik}&=\omega_{i\ell}\phi_{\ell k}+\omega_{k\ell}\phi_{i\ell}\\ \delta\phi_{i4}&=\omega_{i\ell}\phi_{\ell 4}\\ \delta\phi_{44}&=0\end{aligned} \tag{3.b}$$

with $\omega_{i\ell}=\epsilon_{i\ell k}\,\beta_k$.

Then $Tr(\phi)^2$, $Tr(\phi)^3$, $Tr(\phi)^4$, $(Tr(\phi)^2)^2$... are invariants under (3) and a chiral invariant Lagrangian that we want to consider is

$$\mathfrak{L} = \frac{1}{2}\,\partial_\mu \phi_{\alpha\beta}\,\partial_\mu \phi_{\beta\alpha} - W(\Phi_n)_{n\geq 2} \tag{4}$$

with potential energy

$$W(\Phi_n) = \mu^2 \Phi_2 + \frac{1}{2}\lambda \Phi_2^2 + f_1 \Phi_3 + f_2 \Phi_4 \tag{4.a}$$

where we have used the notation

$$\Phi_n = \mathrm{Tr}\,\underbrace{\phi.\phi.\phi \ldots \phi}_{n\geq 2}$$

$\phi_{\alpha\beta}$ can be expressed through physical fields $\Sigma(x)$, $\pi_i(x)$ and $T_{ik}(x)$ in the following way

$$\phi_{ik} = \frac{1}{2}\{T_{ik} + \delta_{ik}\frac{1}{\sqrt{6}}\Sigma\}$$

$$\phi_{i4} = \frac{1}{2}\pi_i \qquad i,k = 1,2,3 \tag{4.b}$$

$$\phi_{44} = -\frac{1}{2}\sqrt{\frac{3}{2}}\,\Sigma \quad \text{and} \quad T_{ii} = 0$$

Further the vector currents are given by

$$J^{\ell}_\mu = \epsilon_{ik\ell}\{\phi_{k\alpha}\partial_\mu \phi_{\alpha i} - \phi_{k\alpha}\partial_\mu \phi_{\alpha k} + \partial_\mu \phi_{i\alpha}\phi_{\alpha k} - \partial_\mu \phi_{k\alpha}\phi_{\alpha i}\} \tag{5.a}$$

and the axial vector currents by

$$J^{5\ell}_\mu = \phi_{4\alpha}\partial_\mu \phi_{\alpha\ell} - \phi_{\ell\alpha}\partial_\mu \phi_{\alpha 4} + \partial_\mu \phi_{\ell\alpha}\phi_{\alpha 4} - \partial_\mu \phi_{4\alpha}\phi_{\alpha\ell} \tag{5,b}$$

Since Lagrangian (4) is exactly invariant under the group, all currents become divergenceless, i.e.,

$$\partial_\mu J^{\alpha\beta}_\mu = \frac{\delta}{\delta\omega_{\alpha\beta}}\mathfrak{L} = 0. \tag{5.c}$$

Finally, if the symmetry is broken by a term of the form $\mathfrak{L}_{S.B.} = c\,.\,\Phi_2$ then PCAC is conserved.

2. Goldstone Mode of the Σ-Model

A subject of this section is to discuss the Goldstone mode of the Σ-model. Generally the Goldstone [3,8] mode manifests by an exactly symmetric Lagrangian, provided that the physical vacuum is not invariant under the symmetry group.

In the case of noninvariance [8] of the vacuum state under the symmetry group, there would be such relations sufficient that not all fields have zero vacuum expectation value.

The vacuum expectation value of Σ is not forced by any symmetry principle to vanish and thus one can choose Σ to develop vacuum expectation value, i.e., we may write

$$\langle\Sigma\rangle_0 = v \neq 0. \tag{6}$$

This, however,, makes a particle interpretation of the Σ-field impossible. Let us introduce field $\tilde{\Sigma}$ such that $\langle\tilde{\Sigma}\rangle_0 = 0$. Then

$$\phi_{\alpha\beta} = \tilde{\phi}_{\alpha\beta} + (V)_{\alpha\beta} \tag{7}$$

where $(V)_{\alpha\beta} = v\,J_{\alpha\beta}$ and $J_{\alpha\beta} = \frac{1}{2\sqrt{6}}c_{(\alpha)}\,\delta_{\alpha\beta}$ with

$$c_{(\alpha)} = \begin{cases} 1 & \text{if} \quad \alpha = i \\ -3 & \text{if} \quad \alpha = 4. \end{cases}$$

The constant v (one can see immediately from PCAC condition that $v = f_\pi$) is determined by minimizing [3,5] the potential energy (considered as a function of the classical fields Σ, π_i and T_{ik}) $W(\Phi_n) = W(\Sigma, \pi_i, T_{ik})$ introduced in eq. (4.a).

The general conditions for minima look as follows

$$\sum_{n\geq 2}\frac{\partial W(\Phi_n)}{\partial\phi_{\alpha\beta}}\Big|_{\tilde{\phi}=0} = 0 \tag{8.a}$$

$$\sum_{n \geq 2} \frac{\partial^2 W(\Phi_n)}{\partial \phi^2_{\alpha\beta}} \Big|_{\tilde{\phi}=0} \geq 0. \tag{8.b}$$

The eq. (8.a) can be rewritten into the following form

$$\sum \frac{\partial W(\Phi_n)}{\partial \Phi_n} \Big|_{\tilde{\phi}=0} \cdot \frac{n}{v} \cdot a_n = 0, \tag{9}$$

where

$$a_n = \Phi_n \Big|_{\tilde{\phi}=0} = \left(\frac{1}{2\sqrt{6}}\right)^n v^n \left(3 + (-3)^n\right).$$

Now inserting the explicit form of the potential from (4.a) into (9) and (8.b) one gets:

1. the equation for v

$$v\left\{\mu^2 + \frac{1}{2}\left(\lambda + \frac{7}{6} f_2\right) v^2 - \frac{1}{2}\sqrt{\frac{3}{2}}\, f_1 v\right\} = 0 \tag{9.a}$$

solutions of which are

a, $v = 0$

$$\text{b,}\quad v_{\pm} = \frac{\frac{1}{2}\sqrt{\frac{3}{2}}\, f_1 \pm \sqrt{\frac{3}{8} f_1^2 - 2\mu^2\left(\lambda + \frac{7}{6} f_2\right)}}{\lambda + \frac{7}{6} f_2}. \tag{10}$$

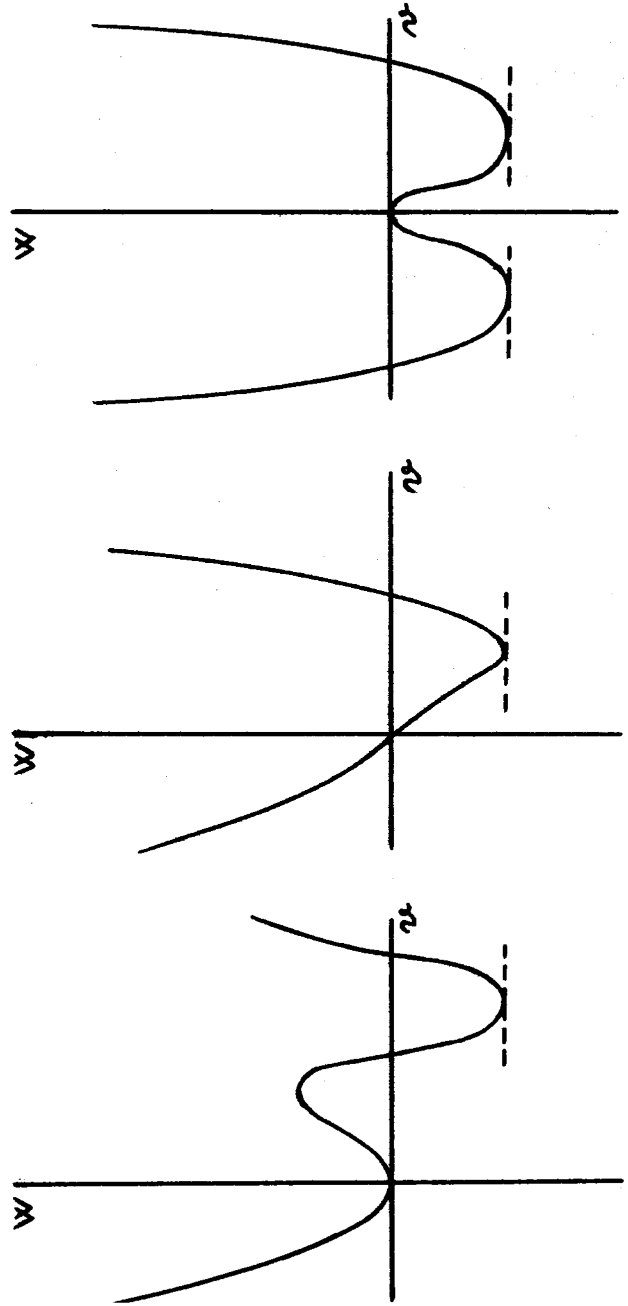
2. conditions for minima

$$(\pm)\, v_{\pm} \sqrt{\frac{3}{8} f_1^2 - 2\mu^2\left(\lambda + \frac{7}{6} f_2\right)} \geq 0. \tag{11}$$

where we suppose that $\lambda + \frac{7}{6} f_2 > 0$ and $f_1 > 0$. The second condition is necessary in order to have the physical masses of the scalar particles non-negative.

Now we are ready to show the shape of $W(v) = W(v,0,0)$ for $\mu^2 > 0$, $\mu^2 = 0$ and $\mu^2 < 0$ (see fig.1).

Thus, we can see that Goldstone mode of Σ-model is realized in all cases ($\mu^2 > 0$, $\mu^2 = 0$, $\mu^2 < 0$) that is the first moment which differs from the σ-model in the



The shape of $W(v,0,0)$; a) corresponds to $\mu^2 > 0$, b) to $\mu^2 = 0$ and c) to $\mu^2 < 0$.

case of $[1/2, 1/2]$ representation, where the Goldstone mode is realized in the special case $\mu^2 < 0$ only /1/.

Now comparing the results obtained in $[1/2, 1/2]$ and $[1,1]$ representations one can see immediately the important role of the trilinear interaction term in $[1,1]$ representation, which will appear in all $[N/2, N/2]$ representations with N-even. From here the conclusion can be drawn, that the Goldstone mode generally in $[N/2, N/2]$ representation of $SU(2)\times SU(2)$ group for N-even in the case of renormalizable Lagrangians (we restrict ourselves maximally to quartic terms in meson fields) is realized like in the extended Σ-model and for N odd is realized like in the case of σ-model.

Finally we shall find the expressions for the masses of particles under consideration as functions of coupling constants appeared in the potential (4.a) and show that the mass of the pion is equal to zero.

The masses of Σ and T_{ik} are obtained from Lagrangian (4)

$$m^2_\Sigma = \sum_{n,m\geq 2} \frac{\partial^2 W(\Phi_n)}{\partial\Phi_n\,\partial\Phi_m}\Big|_{\tilde\phi=0} \cdot \frac{n}{v}\cdot\frac{m}{v}\, a_n a_m + \sum_{n\geq 2}\frac{\partial W}{\partial\Phi_n}\Big|_{\tilde\phi=0}\frac{n(n-1)}{v^2}\, a_n \qquad (12)$$

$$m^2_{T_{ik}} = \sum_{n\geq 2}\frac{\partial W(\Phi_n)}{\partial\Phi_n}\Big|_{\tilde\phi=0}\cdot n(n-1)\,v^{n-1} \qquad (13)$$

and using eq. (4.a) explicitly one gets

$$m^2_\Sigma = -2\mu^2 + \frac{1}{2}\sqrt{\frac{3}{2}}\, f_1 v \qquad (14.a)$$

and

$$m^2_{T_{ik}} = \sqrt{\frac{3}{2}}\, f_1 v - \frac{1}{6} f_2 v\ . \qquad (14.b)$$

The last question we would like to discuss is the manifestation that the pion is Goldstone.

The mass of the pion is given by the following relation

$$m^2_{\pi_i} = \frac{1}{v}\sum_{n\geq 2}\frac{\partial W(\Phi_n)}{\partial\Phi_n}\Big|_{\tilde\phi=0}\cdot\frac{n}{v}\cdot a_n$$

From the condition for minima (eq. (9)) one can see immediately that

$$m^2_{\pi_i} = 0\,. \qquad (15)$$

3. *Nonlinear Realization of the Σ-Model*

The nonlinear realization /6/ for pions is possible to achieve by means of several equivalent ways.

One of them is to perform nonlinear /5/ canonical transformations of the Σ, π_i and T_{ik} fields. This is allowed because $(V)_{\alpha\beta} = v\,J_{\alpha\beta}$ is not equal to zero identically. Then we can write the canonical transformation $(\Sigma, \pi_i, T_{ik}) \to (\Phi, \xi_i, F_{ik})$ in the following form* :

$$M = (\tilde\Sigma + v)P_4 + \pi_i P_i + T_{ik}P_{ik} = e^{i\xi_s A_s}\{(\tilde\Phi + v)P_4 + F_{ik}P_{ik}\}e^{-i\xi_s A_s}, \qquad (16)$$

where A_s are axial-vector generators of $SU(2)\times SU(2)$ group in $[1,1]$ representation and P_{ab} $(a,b=1,\ldots 4)$ $(s=1,2,3)$ are projection operators for extracting the states with isospin 0,1,2 from the basis of $[1,1]$ representation.

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* We would like to note that M in (16) is given in the antisymmetric basis of $[1,1]$ representation and the relation between M and Φ_n used in Lagrangian (4) looks as follows:

$$\Phi_2 = \mathrm{Tr}\, M.M,\quad \Phi_3 = \frac{1}{3^{1/3}}\sqrt{\det M},\quad \Phi_4 = \mathrm{Tr}(M)^4$$

The expressions of Σ, π_i and T_{ik} in terms of Φ, ξ_i and F_{ik} and the explicit form of matrices A_s and P_{ab} are shown in Appendix.

Substituting the expression (16) in eq. (4) we obtain a new form of the Lagrangian in terms of Φ, ξ_i and F_{ik}.

$$\mathcal{L} = \frac{1}{2}\mathfrak{D}_\mu \Phi \mathfrak{D}_\mu \Phi + \frac{1}{2}\mathfrak{D}_\mu F_{ik}\mathfrak{D}_\mu F_{ki} + \tag{17}$$

$$+\frac{1}{2}\mathfrak{D}_\mu \xi_i \mathfrak{D}_\mu \xi_j \{\delta_{ij}(1+\Phi)^2 - 2(1+\Phi)F_{ij} + F_{ik}F_{kj}\} - W(M),$$

where

$$W(M) = \mu^2 \mathrm{Tr}(M)^2 + \frac{\lambda}{2}(\mathrm{Tr}(M)^2)^2 + \frac{f_1}{3^{1/3}}\sqrt{\det M} + f_2 \mathrm{Tr}(M)^4 \tag{17.a}$$

and

$$\mathfrak{D}_\mu \Phi = \partial_\mu \Phi$$

$$\mathfrak{D}_\mu \vec{\xi} = \partial_\mu \vec{\xi} \cdot \frac{\sin\sqrt{\xi^2}}{\sqrt{\xi^2}} + \frac{(\xi_\ell \partial_\mu \xi_\ell)\vec{\xi}}{\xi^2}\left(1 - \frac{\sin\sqrt{\xi^2}}{\sqrt{\xi^2}}\right)$$

$$\mathfrak{D}_\mu F_{ik} = \partial_\mu F_{ik} + i v_\mu^j (T^j)_{is} F_{sk} \tag{18}$$

with $v_\mu^j = -\epsilon_{ikj}\xi_i \partial_\mu \xi_k \cos\sqrt{\xi^2}/\xi^2$
and $(T^j)_{m\ell,ik} = \epsilon^{jmi}\delta_{\ell k} + \epsilon^{jmk}\delta_{\ell i} + \epsilon^{j\ell k}\delta_{mi}$.

A remarkable fact is that eq. (17.a) depends only on Φ and F_{ik} consequently, the particles represented by $\vec{\xi}$ have the zero masses. Now, taking the limit [5] $m_{F_{ik}}$, $m_\Phi \to \infty$ the corresponding fields may be dropped out of the Lagrangian and the matrix M given by eq. (16) reduces to

$$M = e^{i\xi_s A_s} v P_4 e^{-i\xi_s A_s}. \tag{19}$$

The nonlinear Lagrangian obtained in this way is

$$\mathcal{L} = \frac{1}{2}\mathfrak{D}_\mu \xi_i \mathfrak{D}_\mu \xi_i . \tag{20}$$

The terms quartic in the meson field may be used to calculate the T-matrix of the pion-pion scattering [5,9] $(q_1,\alpha)+(q_2,\beta)\to(q_3,\gamma)+(q_4,\delta)$:

$$T_{\alpha\beta,\gamma\delta} = -\frac{1}{v^2}\{\delta_{\alpha\beta}\delta_{\gamma\delta}\cdot s + \delta_{\alpha\gamma}\delta_{\beta\delta}\, t + \delta_{\alpha\delta}\delta_{\beta\gamma}\cdot u\}, \tag{21}$$

where

$$s = (q_1+q_2)^2,\ t = (q_1-q_3)^2,\ u = (q_1-q_4)^2.$$

It has been observed [5] that a transition from the linear σ-model to the nonlinear realization can be performed by taking the infinite mass of σ-particle [4] ($m_\sigma \to \infty$). To carry out this program in the Σ-model one has to eliminate the Σ and T_{ik} particles from the theory *. Now using the standard procedure proposed by Weinberg [5] one can see immediately that in the case of $\pi(q_1,\alpha) + \pi(q_2\beta)\to\pi(q_3,\gamma)+\pi(q_4,\delta)$ the T_{ik} and Σ pole terms give

$$(-i)^3[\delta_{\alpha\beta}\delta_{\gamma\delta}\{\frac{4}{3}\left(\frac{m_{T_{ik}}^2}{2v}\right)^2\left(\frac{1}{m_{T_{ik}}^2} + \frac{s}{m_{T_{ik}}^4} + \dots\right) +$$

$$+\left(\frac{m_\Sigma^2}{v}\right)^2\left(\frac{1}{m_\Sigma^2} + \frac{s}{m_\Sigma^4} + \dots\right) + \text{permutations} \tag{22}$$

while the constant terms give

$$(-i)[\delta_{\alpha\beta}\delta_{\gamma\delta}\{\frac{m_\Sigma^2}{v^2} + \frac{m_{T_{ik}}^2}{3v^2}\} + \text{permutations} \tag{23}$$

The sum of (22) and (23) takes the form

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* I am indebted to Dr. E.A.Ivanov for the drawing my attention to the possibility of a solution of the problem in this way and also for a fruitful discussion about it.

$$\frac{i}{\sqrt{2}}[\delta_{\alpha\beta}\delta_{\gamma\delta}\, .s+\delta_{\alpha\gamma}\delta_{\beta\delta}\, .t+\delta_{\alpha\sigma}\delta_{\beta\gamma}\, .u] \tag{24}$$

in the limit $m_\Sigma \to \infty$, $m_{T_{ik}} \to \infty$ and is consistent with the T-matrix obtained from (20).

It is possible to show that in the limit in which masses of Goldstone partners become very large, Lagrangian (4) turns into (20). If in the equations of motion [5] for T_{ik} and Σ fields, respectively

$$\Box\, \Phi^{(2)}_{ik} + (\mu^2 + \lambda\Phi_2)\, \Phi^{(2)}_{ik} + f_1\Phi^{(3)}_{ik} + f_2\Phi^{(4)}_{ik} = 0 \tag{25}$$

$$\Box\, \Phi^{(2)}_{44} + (\mu^2 + \lambda\Phi_2)\, \Phi^{(2)}_{44} + f_1\Phi^{(3)}_{44} + f_2\Phi^{(4)}_{44} = 0, \tag{26}$$

where $\Phi^{(n)}_{ab} = \dfrac{\delta\Phi_n}{\delta\phi_{ab}}$

one takes the limit $m_\Sigma \to \infty$, $m_{T_{ik}} \to \infty$ then the "kinetic energy" terms may be neglected and one gets the solutions consistent with (19)

$$T_{ik} = \frac{3(v-\Sigma)}{4\pi^2}\{\pi_i\pi_k - \frac{1}{3}\delta_{ik}\pi^2\}. \tag{27}$$

Conclusion

In this section we summarize the main results which have been obtained in the analysis of the spontaneous breakdown of the extended Σ-model:

1. The Goldstone mode manifests itself in all cases ($\mu^2 > 0$, $\mu^2 = 0$, $\mu^2 < 0$).

2. In the limit in which the partners of Goldstone bosons acquire infinite mass, the standard form of the nonlinear realization of σ-model is obtained. There are indications for the validity of this result in all [N/2,N/2] representations of SU(2)×SU(2) group.

3. There is another generalization for all [N/2,N/2] representations. In the case of renormalizable Lagrangians the Goldstone mode for N-odd is realized like in the case of σ-model ($\mu^2 < 0$) and for N-even it is realized in all three cases ($\mu^2 > 0$, $\mu^2 = 0$, $\mu^2 < 0$).

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Appendix

To find the expressions of Σ, π_i, T_{ik} in terms of Φ, ξ_i, F_{ik} from (16) we are in need of an explicit form of axial A_s and vector V_s generators of SU(2)×SU(2) group and matrices P_4, P_i, P_{ik} (i,k = 1,2,3) in [1,1] representation.

They can be taken [6,7] in the following forms

$$A_s = \frac{i}{2}\tau_3 \otimes \rho_s$$

$$V_s = \frac{i}{2} I \otimes \rho_s \qquad a,b = 1,2,3$$

$$P_4 = \frac{1}{2} I \otimes \tau_1 \qquad I = \delta_{ab}$$

$$P_i = -i\rho_i \otimes \tau_2 \qquad Z_{ik} = \frac{1}{2}\{\delta_{ia}\delta_{kb} + \delta_{ib}\delta_{ka} - \frac{2}{3}\delta_{ik}\delta_{ab}\},$$

$$P_{ik} = Z_{ik} \otimes \tau_1 \tag{A.1}$$

where $(\rho_s)_{ab} = \epsilon_{asb}$ and τ_i are Pauli matrices.

Now inserting these into the eq. (16) and using the following comutation relations

$$[A_s, P_4] = -\frac{i}{2} P_s$$

$$[A_s, P_{kd}] = \frac{i}{4}\{\delta_{ks} P_d + \delta_{sd} P_k + \delta_{kd} P_s\}$$

$$[A_s, P_k] = i\{\frac{4}{3} P_4 \delta_{sk} - P_{sk}\}$$

$$[V_s, P_4] = 0$$

$$[V_s, P_i] = i\epsilon^{sim} P_m$$

$$[V_s, P_{ik}] = i\epsilon^{sk\ell} P_{\ell i} + i\epsilon^{si\ell} P_{i\ell} \qquad (A.2)$$

with the properties of traces of matrices in (A.1) we get the final results

$$\Sigma = v + \tilde{\Phi} - \frac{4}{3}\Pi^2(v + \tilde{\Phi}) + \frac{4}{3} F_{ik}\Pi_i \Pi_k$$

$$\tilde{\pi}_i = \Pi_i [\sigma_\Pi (v + \tilde{\Phi}) - \frac{1}{\sigma_\Pi + 1} F_{jk} \Pi_j \Pi_k] + \Pi_j F_{ji}$$

and

$$T_{ik} = F_{ik} - \frac{1}{3}[(v + \tilde{\Phi})\Pi^2 - \Pi_j \Pi_\ell F_{j\ell}]\delta_{ik} +$$

$$+ \Pi_i \Pi_k [v + \tilde{\Phi} + \frac{1}{(\sigma_\Pi + 1)^2}\Pi_j \Pi_\ell F_{j\ell}] -$$

$$- \frac{1}{\sigma_\Pi + 1}[F_{\ell i} \Pi_k \Pi_\ell + F_{\ell k} \Pi_i \Pi_\ell]$$

with

$$\vec{\Pi} = \vec{\xi} \cdot \frac{\sin\sqrt{\xi^2}}{\sqrt{\xi^2}} \text{ and } \sigma_\Pi = 1 - \Pi^2$$

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