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PROJECTION OPERATORS AND SUPPLEMENTARY CONDITIONS FOR SUPERFIELDS WITH AN ARBITRARY SPIN

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I. Introduction

The concept of superfield introduced by Salam and Strathdee[1] has proved to be a useful instrument for realization of supersymmetry (for references see [2]; see also [3] and [4]). Therefore it is interesting to obtain a more detailed information about its structure and properties.

The representations of the supersymmetry algebra in terms of superfields are reducible. A decomposition of the scalar superfield into irreducible parts has been given in [5]. In the present article a general method is proposed for extracting the irreducible multiplets out of a superfield with an arbitrary Lorentz index and with a nonvanishing mass. This method consists in constructing a complete set of projection operators with the help of the Casimir operators. The decomposition of the superfields obtained in this way can also be expressed in terms of simple differential conditions as, for example, the spin one part of a four-vector A_μ is singled out by the equation $\partial^\mu A_\mu = 0$. These results may be useful when constructing Lagrange field theories for superfields with higher spins*.

The plan of this paper is as follows. In Section II some necessary definitions and formulae are collected. In Section III the supertransformation algebra is enlarged by adding the covariant derivative as a new spinor generator.

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*Superfields with spinor and vector indices have already been used in [6] and [7].

Then a superfield with an external (Poincare) spin j appears to form an irreducible unitary representation of the new algebra. Reducing this representation on the initial subalgebra we decompose it into four irreducible representations of the supersymmetry algebra. They can be extracted with the help of projection operators built out of the Casimir operators.

In Sec. IV detailed calculations of the projection operators are carried out. In Sec. V, corresponding supplementary conditions are found.

II. Some Necessary Information

We use the following notations. The γ-matrices satisfy

$$(1/2)\{\gamma_\mu, \gamma_\nu\} = g_{\mu\nu} = \mathrm{diag}\,(+ - - -)\,;$$

$$\gamma_5 = \gamma_0 \gamma_1 \gamma_2 \gamma_3\,; \quad (\gamma^\mu)^+ = g^{\mu\mu} \gamma^\mu\,.$$

The charge conjugation matrix is chosen to be $C = i\gamma^0\gamma^2$. The notation $(\gamma_\mu)_{\alpha\beta}$ means $C_{\beta\rho}(\gamma_\mu)_\alpha{}^\rho$.

The supersymmetry algebra /1/ consists of the Poincare group generators and four anticommuting generators

$$[S_\alpha, P_\mu] = 0$$

$$[S_\alpha, J_{\mu\nu}] = \frac{1}{2}(\sigma_{\mu\nu} S)_\alpha \qquad (1)$$

$$\{S_\alpha, S_\beta\} = \not{P}_{\alpha\beta}.$$

The superfield

$$\Phi_i(p,\theta) = A_i(p) + \bar\theta\psi_i(p) + \bar\theta\theta F_i(p) + \bar\theta\gamma_5\theta G_i(p) +$$
$$+ \bar\theta i\gamma_\nu\gamma_5\theta A^\nu_i(p) + \bar\theta\theta.\bar\theta\chi_i(p) + (\bar\theta\theta)^2 D_i(p) \qquad (2)$$

is a function defined on the mass shell $(p^2 = m^2,\ m \neq 0\,;\ p^0 > 0)$ and on a Grassmann algebra with generators θ_α forming a Majorana spinor. This function transforms as a scalar, spinor, vector and so on under the Poincare group, according to the Lorentz index i. An integer external spin j is described by a symmetric tensor $\Phi_{\mu_1\mu_2\ldots\mu_j}$ satisfying the following supplementary conditions:

$$\Phi^\mu{}_{\mu\mu_3\ldots\mu_j} = 0\,, \qquad p^\mu\Phi_{\mu\mu_2\ldots\mu_j} = 0\,. \qquad (3)$$

A half-integer spin $j + 1/2$ is described by a symmetric spin-tensor $\Phi_{\alpha\mu_1\ldots\mu_j}$ satisfying (3) and

$$(\gamma^\mu)_\alpha{}^\beta\Phi_{\beta\mu\mu_2\ldots\mu_j} = 0\,,$$

$$(1 + i\gamma_5)_\alpha{}^\beta\Phi_{\beta\mu_1\ldots\mu_j} = 0 \qquad (1 - i\gamma_5)_\alpha{}^\beta\Phi_{\beta\mu_1\ldots\mu_j} = 0\,.$$

In what follows we shall use these conditions without reference.

In the space of the superfields (2) the supersymmetry transformations are generated by the operators

$$S_\alpha = i\left(\frac{\partial}{\partial\bar\theta_\alpha} + \frac{1}{2}\not{p}_\alpha{}^\beta\theta_\beta\right). \qquad (4)$$

They form a Majorana spinor if an appropriate scalar product is defined /8,9/.

The so-called "covariant derivative" D_α /5,10/ is defined as a Majorana spinor operator which anticommutes with S_α

$$\{S_\alpha, D_\beta\} = 0 \qquad (5)$$

and obeys relations, similar to (1)

$$[D_\alpha, P_\mu] = 0, \tag{6a}$$

$$[D_\alpha, J_{\mu\nu}] = \frac{1}{2}(\sigma_{\mu\nu} D)_\alpha \tag{6b}$$

$$\{D_\alpha, D_\beta\} = \not{P}_{\alpha\beta}. \tag{6c}$$

The explicit expression for this operator is

$$D_\alpha = \frac{\partial}{\partial \bar{\theta}^\alpha} - \frac{1}{2} \not{p}_\alpha{}^\beta \theta_\beta . \tag{7}$$

A number of useful algebraic identities for D_α has been found in /5/. For convenience some of them are listed in Appendix to our paper.

Algebra (1) has two independent Casimir operators - P^2 and C. C is a generalization of the square of the Pauli-Lubanski vector /1,11/:

$$C = C_{\mu\nu} C^{\mu\nu} . \tag{8a}$$

where

$$C_{\mu\nu} = P_\mu C_\nu - P_\nu C_\mu \tag{8b}$$

and

$$C_\mu = \frac{1}{2} \epsilon_{\mu\nu\kappa\rho} P^\nu J^{\kappa\rho} - \frac{1}{4} \bar{S} i \gamma_\mu \gamma_5 S . \tag{8c}$$

It is shown in /11,12,13/ that all the unitary irreducible representations of algebra (1) with $m \neq 0$ are labelled by the eigenvalues of the Casimir operator C:

$$C = -2m^2 Y(Y+1) ,$$

where Y is an integer of half-integer number called "superspin". Such a representation contains four usual (Poincaré) spins

$$J = Y - \frac{1}{2}, \ Y, Y, Y + \frac{1}{2} . \tag{9}$$

So it becomes clear that the representations realized on superfields (2) are reducible.

III. Superspin Contents of a Superfield

Now we are going to prove the statement made in the introduction that a superfield combines four representations of the supersymmetry algebra into one representation of a larger algebra. This algebra has a new spinor generator D_α, in accordance with (5) and (6). Relations (6) are identical with (1), so to construct the unitary representations of the new algebra we can follow the method used in /12/ for algebra (1).

Let us go to the rest frame ($p_i = 0$, $p_0 = m$). If γ_0 is diagonal, (6c) can be written in the form

$$\{D_a, D_b\} = 0 , \ \{D_a^+, D_b^+\} = 0 , \ \{D_a, D_b^+\} = m\delta_{ab} .$$

Here $a = 1, 2$, $D_1^+ = -D_4$, $D_2^+ = D_3$. Choose an irreducible representation of algebra (1) with superspin Y to play the role of a "vacuum state":

$$D_a^+ | Y \rangle = 0 .$$

Then using the "creation operators" D_a one obtaines a basis of four vectors for a representation of the enlaged algebra:

$$| Y \rangle , \ \frac{1}{\sqrt{m}} D_1 | Y \rangle , \ \frac{1}{\sqrt{m}} D_2 | Y \rangle , \ \frac{1}{m} D_1 D_2 | Y \rangle . \tag{10}$$

In the rest frame the space components C_i of vector (8c) form a $SU(2)$ algebra - the superspin algebra. The operators D_a commute with C_i as "superspinors". Consequently, multiplet (10) contains superspins $Y - \frac{1}{2}$, $Y, Y, Y + \frac{1}{2}$.

So, the irreducible unitary representations of the enlarged algebra (1), (5), (6) are labelled by m^2 and by a "spin" number j. Such a representation combines four representations of algebra (1) with superspins

$$Y = j - \frac{1}{2}, \ j, j, j + \frac{1}{2} .$$

Taking into account (9) and comparing with (2), we see that these representations of the enlarged algebra are realized on superfields with external spin j.

Our aim to extract the four representations of algebra (1) out of the superfield with spin j can be achieved by using the following construction. If in a reducible representation of an algebra its Casimir operator C has eigenvalues $c_1, c_2, \dots, c_n$, then there exists a complete set of projection operators:

$$\Pi_1 = \frac{(C-c_2)\dots(C-c_n)}{(c_1-c_2)\dots(c_1-c_n)}, \dots, \Pi_n = \frac{(C-c_1)\dots(C-c_{n-1})}{(c_n-c_1)\dots(c_n-c_{n-1})} \qquad (11)$$

$$\Pi_i \Pi_j = \delta_{ij} \Pi_j, \quad \Pi_1 + \dots + \Pi_n = 1.$$

Each of them, Π_i, extracts a subspace in which $C = c_i 1$.

Thus, with the help of the Casimir operator (8) one can separate the representations $Y = j - \frac{1}{2}$ and $Y = j + \frac{1}{2}$ but the two representations $Y = j$ cannot be distinguished. This degeneration can be removed by introducing a new symmetry - γ_5 - invariance. Suppose that the superfield transforms as a scalar (in the case of integer external spin) or as a spinor (when a spinor external index is presented) under a transformation which multiplies every spinor by a matrix $\exp(i\alpha\gamma_5)$. The generator Γ of this transformation commutes with all other generators but S_α:

$$[S_\alpha, \Gamma] = (i\gamma_5)_\alpha{}^\beta S_\beta.$$

This larger algebra has a new Casimir operator

$$G = -\bar{S} i \not{P} \gamma_5 S + 2P^2 \Gamma. \qquad (12)$$

The old Casimir operator C (8) is invariant with respect to Γ, too, so the new symmetry conserves the superspin classification. As we shall see further the projection operators of the type (11) constructed out of G distinguishe the two superspins $Y = j$.

IV. Projection Operators

Now one has to insert the explicit expressions for $J_{\mu\nu}$ and S_α into (8), calculate the Casimir operator C and find the projection operators (11). We are going to do this in details in the cases $j = 0, \frac{1}{2}, 1, \frac{3}{2}, 2$, then we shall be able to write out the general formulae for an arbitrary j.

Spin 0. The generators $J_{\mu\nu}$ are realized in the form

$$J_{\mu\nu} = i\left(p_\mu \frac{\partial}{\partial p^\nu} - p_\nu \frac{\partial}{\partial p^\mu}\right) + \frac{1}{2} \bar{\theta} \sigma_{\mu\nu} \frac{\partial}{\partial \bar{\theta}}. \qquad (13)$$

Putting (4) and (13) into (8b) and (8c) and using (7) one obtaines

$$C_{\mu\nu} = \bar{D} X_{\mu\nu} D,$$

where

$$X_{\mu\nu} = \frac{1}{4}(p_\mu g_{\nu\lambda} - p_\nu g_{\mu\lambda}) i \gamma^\lambda \gamma_5.$$

Making use of formula (A.15) in Appendix the Casimir operator is found to be

$$C = (\bar{D} X_{\mu\nu} D)(\bar{D} X^{\mu\nu} D) = -\frac{3}{2} p^4 (1 - A),$$

where

$$A = \frac{1}{4p^2} (\bar{D} D)^2, \quad A^2 = A. \qquad (14)$$

Obviously, C has eigenvalues 0 and $-\frac{3}{2}p^4$ corresponding to superspins $Y = 0, \frac{1}{2}$. The projection operators for these superspins have the form

$$\Pi_{1/2} = 1 - A$$
$$\Pi_0 = A \qquad (15)$$

In fact, there are two representations with superspin 0, i.e., Π_0 is a sum of two orthogonal projection operators. To find them we use the new Casimir operator G (12). Its explicit form in this case is

$$G = \bar{D} i \not{p} \gamma_5 D.$$

In accordance with (A.15), $G^2 = 4p^4 A$ (see (14)), therefore G has eigenvalues 0, $\pm 2p^2$. The corresponding projection operators are

$$\Pi^0 = 1 - A$$

$$\Pi^{\pm 2p^2} = \frac{1}{2} A \pm \frac{1}{4p^2} \bar{D} i \not{p} \gamma_5 D \qquad (16)$$

Combining (15) and (16) we obtain the complete decomposition of the scalar superfield into irreducible representations of the superalgebra:

$$\Pi_{1/2} = 1 - A$$

$$\Pi_0^{\pm} = \frac{1}{2} A \pm \frac{1}{4p^2} \bar{D} i \not{p} \gamma_5 D \qquad (17)$$

This result coincides with the one formulated in /5/.

Spin 1/2. In this case the generator $J_{\mu\nu}$ receives a matrix addition $1/2\,\sigma_{\mu\nu}$ and $C_{\mu\nu}$ (8b) is written as

$(C_{1/2})_{\mu\nu} = \bar{D} X_{\mu\nu} D . \hat{1} + 2 X_{\mu\nu} \not{p}$,

where $\hat{1}$ is the unit spinor matrix. After some calculations one obtains

$$C_{1/2} = \frac{3}{2} p^4 A . \hat{1} + 2p^4 \beta_{1/2} - 3p^4 \hat{1}, \qquad (18)$$

where

$$\beta_{1/2} = \frac{2}{p^4} X_{\mu\nu} \not{p} \bar{D} X^{\mu\nu} D = \frac{1}{4p^2} p_\mu \sigma^{\mu\nu} \gamma_5 \bar{D} i \gamma_\nu \gamma_5 D . \qquad (19)$$

Further, with the help of formulae in Appendix one finds

$$\beta_{1/2}^2 = \beta_{1/2} + \frac{3}{4}(1 - A) . \hat{1}. \qquad (20)$$

A remarkable feature of $\beta_{1/2}$ is the identity

$$\beta_{1/2} . A = A . \beta_{1/2} = 0 . \qquad (21)$$

It allows the degree of p^2 in denominator to be not larger than 1 in all the projection operators (this situation remains unchanged for higher spins too).

Using (20) and (21) one finds the projection operators for superspins $Y = 0, \frac{1}{2}, 1$:

$$\Pi_1 = \frac{1}{2}\left(\frac{3}{2} . \hat{1} - \beta_{1/2}\right)(1 - A)$$

$$\Pi_{1/2} = A . \hat{1}$$

$$\Pi_0 = \frac{1}{2}\left(\frac{1}{2} . \hat{1} + \beta_{1/2}\right)(1 - A) .$$

The two superspins 1/2 are again distinguished by the projection operators

$$\Pi_{1/2}^{\pm} = \left(\frac{1}{2} A \pm \frac{1}{4p^2} \bar{D} i \not{p} \gamma_5 D\right) . \hat{1}$$

$$\Pi_{1/2}^{+} + \Pi_{1/2}^{-} = \Pi_{1/2}$$

constructed out of the Casimir operator G (12). Here γ_5-covariance has another consequence: it splits the spinor superfields into left and right chiral ones but this is a part of our convention in Sec. II.

Spin 1. This case is treated analogously to the previous one. Here $C_{\mu\nu}$ receives a new part $A_{\mu\nu}$:

$(C^1_{\mu\nu})_{\kappa\rho} = (\bar{D} X_{\mu\nu} D) g_{\kappa\rho} + (A_{\mu\nu})_{\kappa\rho}$,

where

$$(A_{\mu\nu})_{\kappa\rho} = i(p_\mu \delta^\lambda_\nu - p_\nu \delta^\lambda_\mu) p^\sigma \epsilon_{\lambda\sigma\kappa\rho} \qquad (22)$$

Further, C_1 has a form similar to $C_{1/2}$ (18):

$C_1 = \frac{3}{2} p^4 A . 1 + 2B_1 - \frac{11}{2} p^4 1$

with $(1)_{\mu\nu} = g_{\mu\nu}$ and

$$(B_1)_{\kappa\rho} = \frac{1}{p^4}(A_{\mu\nu})_{\kappa\rho}\,\bar{D}X^{\mu\nu}D = \frac{i}{2p^2}p^{\lambda}\,\epsilon_{\sigma\lambda\kappa\rho}\,\bar{D}i\gamma^{\sigma}\gamma_5 D. \quad (23)$$

The analogues of (20) and (21) are

$$B_1^2 = B_1 + 2(1-A)\,1$$
$$AB_1 = B_1 A = 0 \quad (24)$$

For the projection operators see the general formula (27).

Spin 3/2. Now

$$C_{3/2} = \frac{3}{2}p^4 A.\,\hat{1}\,1 + 2p^4\beta_{3/2} - 9p^4\,\hat{1}\,1$$

with

$$\beta_{3/2} = \beta_{1/2}\,1 + \hat{1}B_1$$

(see (19) and (23)). The analogue of (20) and (21) reads

$$\beta_{3/2}^2 = \beta_{3/2} + \frac{15}{4}(1-A)\,\hat{1}\,1$$

$$\beta_{3/2}\,A = A\beta_{3/2} = 0\,.$$

The projection operators are given in (28).

Spin 2. Here

$$C_2 = \frac{3}{2}p^4 A.\,1^2 + 2p^4 B_2 - \frac{27}{2}p^4\,,$$

$$B_2 = B_1 1 + 1\,B_1\,.$$

When evaluating B_2^2 the noncommutativity of B_1 acting on different indices must be taken into account:

$$(B_2^2)_{\mu\lambda,\nu\rho} = (B_1^2)_{\mu\lambda}\,g_{\nu\rho} + g_{\mu\lambda}\,(B_1^2)_{\nu\rho} + (B_1)_{\mu\lambda}\,(B_1')_{\nu\rho} + (B_1')_{\nu\rho}(B_1)_{\mu\lambda}. \quad (25)$$

Then B_2^2 becomes (see (24))

$$B_2^2 = B_2 + 6(1-A)\,1^2\,.$$

Arbitrary spin j. We are already able to write out the general formulae. Consider first integer spins. Now (see (22))

$$(C_j) = \bar{D}X_{\mu\nu}D.\,1^j + A_{\mu\nu}1^{j-1} + \ldots + 1^{j-1}A_{\mu\nu}$$

and

$$C_j = \left(\frac{3}{2}A - 2j(j+1) - \frac{3}{2}\right)p^4\,1^j + 2B_j\,,$$

where (see (23))

$$B_j = B_1 1^{j-1} + 1\,B_1 1^{j-2} + \ldots + 1^{j-1}B_1\,.$$

Taking into account (24) and (25) one finds

$$B_j^2 = B_1^2 1^{j-1} + \ldots + 1^{j-1}B_1^2 + \sum_{m,k} 1^m B_1 1^k B_1' 1^{j-m-k-2} + \sum_{m,k} 1^m B_1' 1^k B_1 1^{j-m-k-2} = B_j + j(j+1)(1-A)1^j\,. \quad (26)$$

Formula (26) enables us to establish the projection operators

$$\Pi_{j+1/2} = \frac{1}{2j+1}[(j+1)1^j - B_j](1-A)$$

$$\Pi_j = A.\,1^j \quad (27a)$$

$$\Pi_{j-1/2} = \frac{1}{2j+1}(j\,1^j + B_j)(1-A)$$

In this general case we have the same decomposition of Π_j as in (17)

$$\Pi_j = \Pi_j^+ + \Pi_j^-,\quad \Pi_j^{\pm} = \left(\frac{1}{2}A \pm \frac{1}{4p^2}\bar{D}i\not{p}\gamma_5 D\right)1^j\,. \quad (27b)$$

When j is half-integer, B_j is replaced by β_j:

$$\beta_j = \beta_{1/2}\, 1^{j-1/2} + \hat{1}\, B_{j-1/2} \qquad (28)$$

$$\beta_j^2 = \beta_j + j(j+1)(1-A)\, \hat{1}\, 1^{j-1/2}\, .$$

The projection operators are

$$\Pi_{j+1/2} = \frac{1}{2j+1}\left[(j+1)\,\hat{1}\, 1^{j-1/2} - \beta_j\right](1-A)$$

$$\Pi_j^{\pm} = \left(\frac{1}{2}A \pm \frac{1}{4p^2}\bar{D} i \not{p}\gamma_5 D\right)\hat{1}\, 1^{j-1/2}$$

$$\Pi_{j-1/2} = \frac{1}{2j+1}\left(j\,\hat{1}\, 1^{j-1/2} + \beta_j\right)(1-A)\, .$$

V. Supplementary Conditions

As it was mentioned in Introduction, every projection operator Π can be replaced by an equation (''supplementary condition'') $L\Phi = 0$, where L is some differential operator. This means that if $\Phi\Pi = \Phi$ then $L\Phi = 0$ and vice versa.

Obviously, one can immediately write such an equation:

$$(\Pi - 1)\,\Phi = 0\, . \qquad (29)$$

But it appears that in many cases (29) can be reduced to one or several more simple equations. Take, for example, a scalar superfield with superspin 1/2. Then (29) reads (see (17))

$$(\bar{D}D)^2\,\Phi = 0\, . \qquad (30)$$

This equation can be simplified using the multiplication rules in Appendix. Multiplication by $\bar{D}\gamma_5 D$ gives

$$\bar{D}\gamma_5 D\Phi = 0\, . \qquad (31)$$

Multiplying eq. (30) by $\bar{D} i \not{p}\gamma_5 D$, we obtain another simple equation

$$\bar{D}D\Phi = 0 \qquad (32)$$

and vice versa, the same multiplication turns eq. (31) into eq. (30)*. Finally, eq. (30) is an obvious consequence of eq. (32).

The scalar superfield contains two superspins 0 also. Each of them is singled out by one of the equations (see (17))

$$\Pi_0^{\pm}\,\Phi = \Phi$$

or

$$(\bar{D}D)^2\,\Phi \pm 2\bar{D} i\not{p}\gamma_5 D\Phi = 8p^2\Phi\, . \qquad (33)$$

After multiplication by D_α one obtaines (see (A.5) and (A.16))

$$\bar{D}D\left[2\not{p}(1\mp i\gamma_5)D\right]_\alpha\Phi + 4p^2\left[(1\pm i\gamma_5)D\right]_\alpha\Phi = 8p^2 D_\alpha\Phi$$

The final form of these equations is achieved multiplying by $(1\mp i\gamma_5)_\rho{}^\alpha$:

$$\left[(1\mp i\gamma_5)D\right]_\rho\,\Phi = 0\, . \qquad (34)$$

Inserting (34) into the left-hand side of (33) we see that they are equivalent.

A scalar superfield satisfying one of eqs. (34) is called in /5/ ''chiral superfield''. Our general formulae (27) and (28) show that these ''chirality'' conditions can extract the two superspins j from a superfield with the same external spin j. It is worthwhile to point out that in the two-component formalism of Van der Waerden (see, e.g., [10])

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*It should be mentioned that in [5] both (31) and (32) are required at the same time although, as we see, one of them is sufficient.

these conditions become extremely simple - the superfield should depend only on θ_α or $\bar{\theta}\dot{\alpha}$.

The same method can be applied to the spinor superfield. The initial equation for superspin 1 reads (see (28)):

$$(\Pi_1)_\alpha{}^\beta \Phi_\beta = \Phi_\alpha$$

or

$$\frac{1}{2}\left(\frac{3}{2}\hat{1} - \beta_{1/2}\right)_\alpha{}^\beta (1 - A)\, \Phi_\beta = \Phi_\alpha \,. \tag{35}$$

Using (21) a necessary condition is established

$$A\Phi_\alpha = 0 \,, \qquad \text{i.e.,} \qquad \bar{D}D\Phi_\alpha = 0. \tag{36}$$

Then (35) reduces to (see (19))

$$(p_\mu \sigma^{\mu\nu} \gamma_5)_\alpha{}^\beta \bar{D} i \gamma_\nu \gamma_5 D \Phi_\beta = -2p^2 \Phi_\alpha.$$

Multiplying by $\bar{D}^\alpha$ (with summing), using (A.5) and taking into account (36) one obtaines

$$\bar{D}^\alpha \Phi_\alpha = 0 \,. \tag{37}$$

Eqs. (36) and (37) are not only necessary but also sufficient for (35) to hold. The second condition (37) has clear interpretation: it excludes a scalar superfield $\bar{D}^\alpha \Phi_\alpha$ contained in the spinor superfield Φ_α.

In the case of the lowest superspin 0 the same method leads to equations

$$\bar{D}D\Phi_\alpha = 0 \,, \quad D_\alpha \bar{D}^\beta \Phi_\beta = 2\not{p}_\alpha{}^\beta \Phi_\beta.$$

The vector superfield is treated analogously. For the highest superspin 3/2 the conditions are again simple

$$\bar{D}D\Phi_\mu = 0 \,, \qquad (\gamma^\mu D)_\alpha \Phi_\mu = 0 \,. \tag{38}$$

Clearly, the second excludes a spinor superfield out of the vector one.

In the case of external spin larger than 1/2 the initial equation for the lowest superspin is

$$\Pi_{j-1/2} \Phi_{(j)} = \Phi_{(j)}.$$

It seems there is no way to simplify it.

Finally, spin 3/2. As it can be expected, the initial equation for the highest superspin 2

$$(\Pi_2 - 1)\, \Phi_{\alpha\mu} = 0$$

reduces to the common condition

$$\bar{D}D\Phi_{\alpha\mu} = 0 \,,$$

to the condition (37) for the spinor index and to the condition (38) for the vector index. However, due to the equality $(\gamma^\mu)_\alpha{}^\beta \Phi_{\beta\mu} = 0$ the second condition appears to be a consequence of the third. So only two equations

$$\bar{D}D\Phi_{\alpha\mu} = 0$$

$$(\gamma^\mu)_\alpha{}^\rho D_\rho \Phi_{\beta\mu} = 0 \tag{39}$$

remain.

In the cases of all other integer or halfinteger spins the highest superspin is singled out by condition (38) or (39) imposed on one of the vector indices (due to the symmetry of these indices).

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VI. Appendix

$$\bar{D}\gamma_\mu D = 2p_\mu \tag{A.1}$$

$$\bar{D}\sigma_{\mu\nu} D = 0 \tag{A.2}$$

$$D_\alpha \bar{D}D = \bar{D}DD_\alpha + (2\not{p}D)_\alpha \tag{A.3}$$

$$D_\alpha \bar{D}\gamma_5 D = -\bar{D}D(\gamma_5 D)_\alpha + (2\not{p}\gamma_5 D)_\alpha \tag{A.4}$$

$$D_\alpha \bar{D} i \gamma_\mu \gamma_5 D = -\bar{D}D(i\gamma_\mu\gamma_5 D)_\alpha + 2ip_\mu(\gamma_5 D)_\alpha \tag{A.5}$$

$$\bar D\gamma_5 D D_\alpha = -\bar D D(\gamma_5 D)_\alpha \tag{A.6}$$

$$\bar D i\gamma_\mu\gamma_5 D D_\alpha = -\bar D D(i\gamma_\mu\gamma_5 D)_\alpha - (2p^\lambda\sigma_{\lambda\mu}\gamma_5 D)_\alpha \tag{A.7}$$

$$\bar D D.\bar D\gamma_5 D = -2ip^\lambda\bar D i\gamma_\lambda\gamma_5 D \tag{A.8}$$

$$\bar D\gamma_5 D.\bar D D = 2ip^\lambda\bar D i\gamma_\lambda\gamma_5 D \tag{A.9}$$

$$\bar D D.\bar D i\gamma_\mu\gamma_5 D = 2ip_\mu\bar D\gamma_5 D \tag{A.10}$$

$$\bar D i\gamma_\mu\gamma_5 D.\bar D D = -2ip_\mu\bar D\gamma_5 D \tag{A.11}$$

$$(\bar D\gamma_5 D)^2 = (\bar D D)^2 \tag{A.12}$$

$$\bar D i\gamma_5 D.\bar D i\gamma_\mu\gamma_5 D = -2ip_\mu\bar D D \tag{A.13}$$

$$\bar D i\gamma_\mu\gamma_5 D.\bar D\gamma_5 D = 2ip_\mu\bar D D \tag{A.14}$$

$$\bar D i\gamma_\mu\gamma_5 D.\bar D i\gamma_\nu\gamma_5 D = g_{\mu\nu}(\bar D D)^2 + 2ip^\lambda\epsilon_{\lambda\mu\nu\rho}\bar D i\gamma^\rho\gamma_5 D - 4(g_{\mu\nu}p^2 - p_\mu p_\nu) \tag{A.15}$$

$$D_\alpha(\bar D D)^2 = (2\not p D)_\alpha(\bar D D) \tag{A.16}$$

$$(\bar D D)^3 = 4p^2\bar D D\ . \tag{A.17}$$

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