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ASYMPTOTIC RELATIONS BETWEEN THE PROTON FORM FACTORS AND ELASTIC SCATTERING OF POLARIZED LEPTONS ON POLARIZED PROTONS

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Numerous theoretical papers have been devoted to study of the asymptotic behaviour of the nucleon form factors [1-3]. If one regards a nucleon as a two-body bound state composed of spin 0 and spin 1/2 constituents, whose wave function is given by the solution of the Bethe-Salpeter equation (in the ladder approximation) an equal asymptotic ($q^2 \gg M^2$) behaviour of the Dirac F_1 and Pauli F_2 nucleon form factors is obtained [1] (up to $\ell n \frac{q^2}{M^2}$) .). On the other hand, in the framework of the parton model [2] an equal asymptotic behaviour is obtained for the electric G_E and magnetic G_M nucleon form factors. The same result (up to $\ell n \frac{q^2}{M^2}$) has been obtained in asymptotically free theories, too [3].

In the following we shall show that elastic scattering of polarized leptons on a polarized proton target makes it possible to distinguish between the two possible asymptotic relations for the proton form factors:

$$F_1(q^2) \sim F_2(q^2) \tag{1}$$

and

$$G_E(q^2) \sim G_M(q^2) \; . \tag{2}$$

We shall give also the results of analysis of the data on elastic $e-p$ scattering with unpolarized leptons and protons. The purpose of the analysis was to distinguish between (1) and (2).

Consider at first the elastic scattering of polarized leptons on a polarized proton target*:

$$\ell + p \to \ell + p . \tag{3}$$

Polarization effects in elastic e-p scattering have been considered first in /4/ and later on in /5,6/ In paper /5/ it was pointed out that the asymmetry which appears in scattering of longitudinally polarized leptons off a target whose polarization in perpendicular to the direction of the recoil nucleon in the lab. frame is proportional to $G_M G_E$. Thus measuring it one may get information about G_E.

The matrix element for process (3) is (one-photon approximation):

$$<f|S|i> = i e^2 N \bar{u}(k') \gamma_\alpha u(k) \frac{1}{q^2} \times \tag{4}$$

$$\times \bar{u}(p') [\gamma_\alpha G_M + i(p+p') \frac{F_2}{2M}] u(p) (2\pi)^4 \delta^4(p'-p-q) :$$

Here $k, p (k', p')$ denote the four-momentum vectors of the incident (final) lepton and proton, $q = k - k'$; M is the proton mass, N is a standard normalizing factor.

The form factor F_2 is expressed in terms of G_M and G_E as follows:

$$F_2 = \frac{G_M - G_E}{1 + q^2/4M^2} . \tag{5}$$

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* Such experiments are carried out at present in SLAC.

As a consequence of conservation of helicity in the lepton vertex only the longitudinal polarization of the leptons survives at $k_0 \gg m$, $k_0' \gg m$ (m is the lepton mass). The cross section of the scattering of longitudinally polarized leptons off a polarized proton target has the form (in the lab. frame):

$$\left(\frac{d\sigma}{d\Omega}\right)_{\vec{P}} = \left(\frac{d\sigma}{d\Omega}\right)_0 (1 + \lambda \vec{P}\vec{A}) : \tag{6}$$

Here λ is the degree of the longitudinal polarization of the leptons, $\vec{P}$ is the polarization vector of the initial proton, $\left(\frac{d\sigma}{d\Omega}\right)_0$ is the unpolarized cross section, $\vec{A}$ is the asymmetry vector. For the vector $\vec{A}$ which lies in the scattering plane we can write:

$$\vec{A} = A_\parallel \vec{\kappa} + A_\perp \vec{s} , \tag{7}$$

where $\vec{\kappa}$ and $\vec{s}$ are two orthogonal unit vectors in this plane:

$$\vec{\kappa} = \frac{\vec{k}}{|\vec{k}|} , \quad \vec{s} = \vec{n} \times \vec{\kappa}$$

and $\vec{n}$ is a unit vector perpendicular to the scattering plane $\left(\vec{n} = \frac{\vec{k} \times \vec{k}'}{|\vec{k} \times \vec{k}'|}\right)$. From (4) we obtain the following expressions for the longitudinal $A_\parallel$ and transverse $A_\perp$ asymmetries * :

$$A_\parallel = \frac{-q^2}{2Mk_0} \frac{1}{W} [(1 + \frac{k_0}{M} tg^2 \frac{\theta}{2}) G_M^2 - F_2 G_M] , \tag{8}$$

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* These expressions in a somewhat different form are contained in ref./4/.

$$A_{\perp} = \frac{-q^2}{2Mk_0}\frac{1}{W}[G_M^2 - (1+\frac{k_0}{M}) F_2 G_M] . \qquad (9)$$

Here θ is the scattering angle in the lab.system, k_0 is the energy of the initial lepton in this system and

$$W = \frac{(\frac{d\sigma}{d\Omega})_0}{(\frac{d\sigma}{d\Omega})_M} = \frac{G_E^2 + \frac{q^2}{4M^2}G_M^2}{1+\frac{q^2}{4M^2}} + 2\frac{q^2}{4M^2}G_M^2 \, tg^2\frac{\theta}{2} ,$$

$$(\frac{d\sigma}{d\Omega})_M = \frac{\alpha^2\cos^2\frac{\theta}{2}}{4k_0^2\sin^4\frac{\theta}{2}} . \qquad (10)$$

We are interested in the high q^2-region. If we assume relation (2) between the form factors, the second term in the matrix element (4) in the considered q^2-region becomes smaller that the first one. Thus we may expect that the longitudinal asymmetry will turn out to be greater (in modulus) than the transverse one (approximate helicity conservation). If relation (3) holds then we should expect that $A_{\parallel}$ and $A_{\perp}$ will be of the same order. Numerical calculations confirm these expectations.

In fig. 1 the longitudinal and transverse asymmetries are plotted versus q^2 by assuming the "form factor scaling":

$$G_M(q^2) = \mu_p G_E(q^2) . \qquad (11)$$

In (11) μ_p is the magnetic moment of the proton ($\mu_p = 2.79$). We have taken the energies of the initial leptons equal 20, 50 GeV.

In figs. 2 and 3 the asymmetries $A_{\parallel}$ and $A_{\perp}$ provided an equal q^2-asymptotic behaviour for F_1 and F_2 are presented. The asymmetries plotted in fig. 2 have been calculated by assuming that

$$F_2(q^2) = (\mu_p - 1) F_1(q^2) , \qquad (12)$$

while those in fig. 3 by assuming that

$$F_2(q^2) = 0{,}24 F_1(q^2) . \qquad (13)$$

The numerical coefficient in (13) has been obtained by analysing elastic e-p scattering data (see below).

As one can see from figs. 1-3, if an equal asymptotic behaviour for G_M and G_E is accepted, the asymmetry $A_{\parallel}$ is considerably greater (in modulus) than $A_{\perp}$. If in the high q^2-region the form factors F_1 and F_2 exhibit an equal q^2-behaviour, then $A_{\parallel}$ and $A_{\perp}$ are of the same order.

Therefore measurement of $A_{\parallel}$ and $A_{\perp}$ will make it possible to solve the important problem on the relation between the proton form factors at $q^2 \gg M^2$.

In fig. 4 we have plotted the asymmetry A_D, calculated in ref. /5/. We have assumed that the form factors are connected by the "form factor scaling law" (11). As it is seen from fig. 4 the asymmetry A_D does not exceed 10% in the region $10 < q^2 < 60$ (GeV/c)^2.

Now we shall give the results of analysis of the data of ref. /7/ on elastic e-p scattering. If we assume the following expression for the magnetic form factor G_M:

$$G_M(q^2) = a\frac{M^4}{q^4} \qquad (14)$$

and suppose that the electric form factor G_E is connected with G_M through eq. (11)*, then a satisfactory descrip-

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*If relation (11) takes place, as is well known, the contribution of G_E to the cross section of elastic e-p scattering for high values of q^2 is much smaller than the contribution of G_M. (for example at $q^2 = 15$ (GeV/c)^2 the contribution of G_E is ~2% the contribution of G_M).

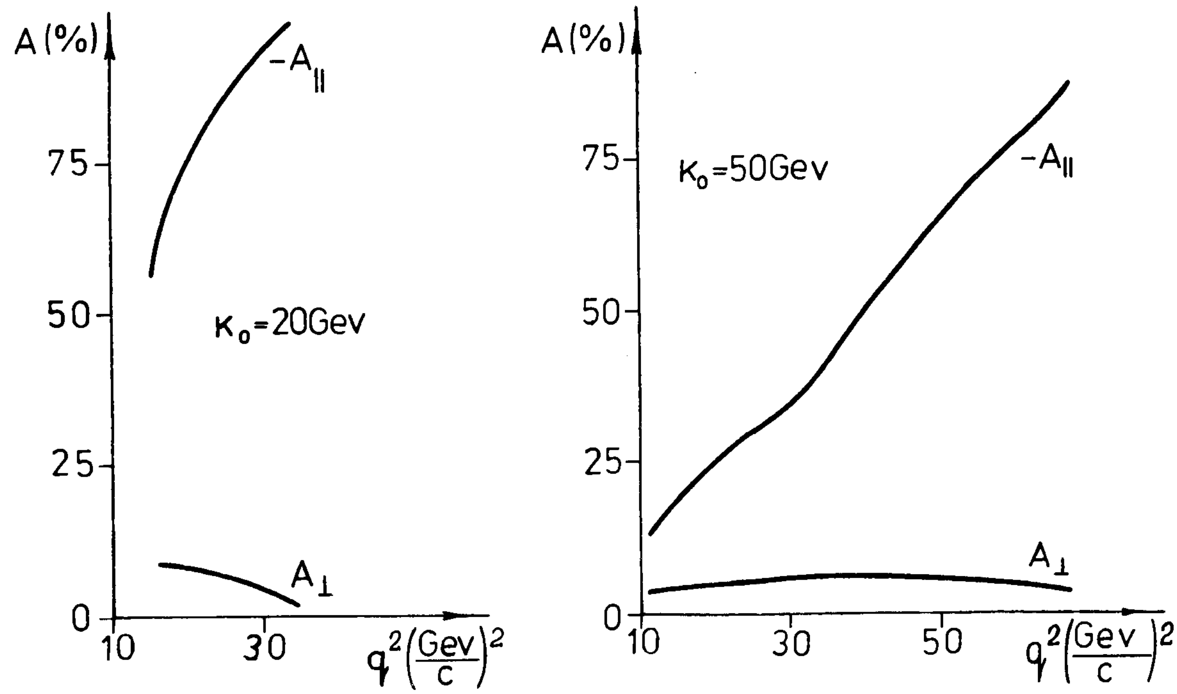

Fig. 1. The longitudinal $A_{\parallel}$ and transverse $A_{\perp}$ asymmetries calculated by assuming $G_M = \mu_p G_E$; ; k_0 is the energy of the incident lepton.

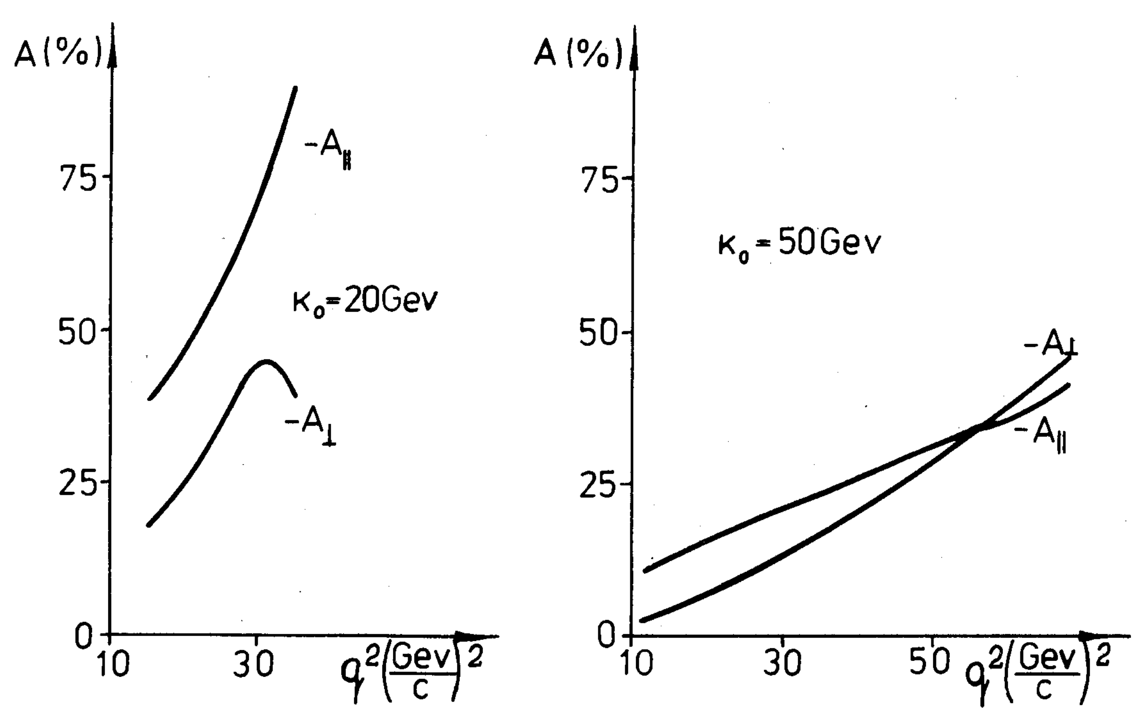

Fig. 2. The longitudinal $A_{\parallel}$ and transverse $A_{\perp}$ asymmetries calculated by assuming $F_2 = (\mu_p - 1)F_1$; k_0 is the energy of the incident lepton.

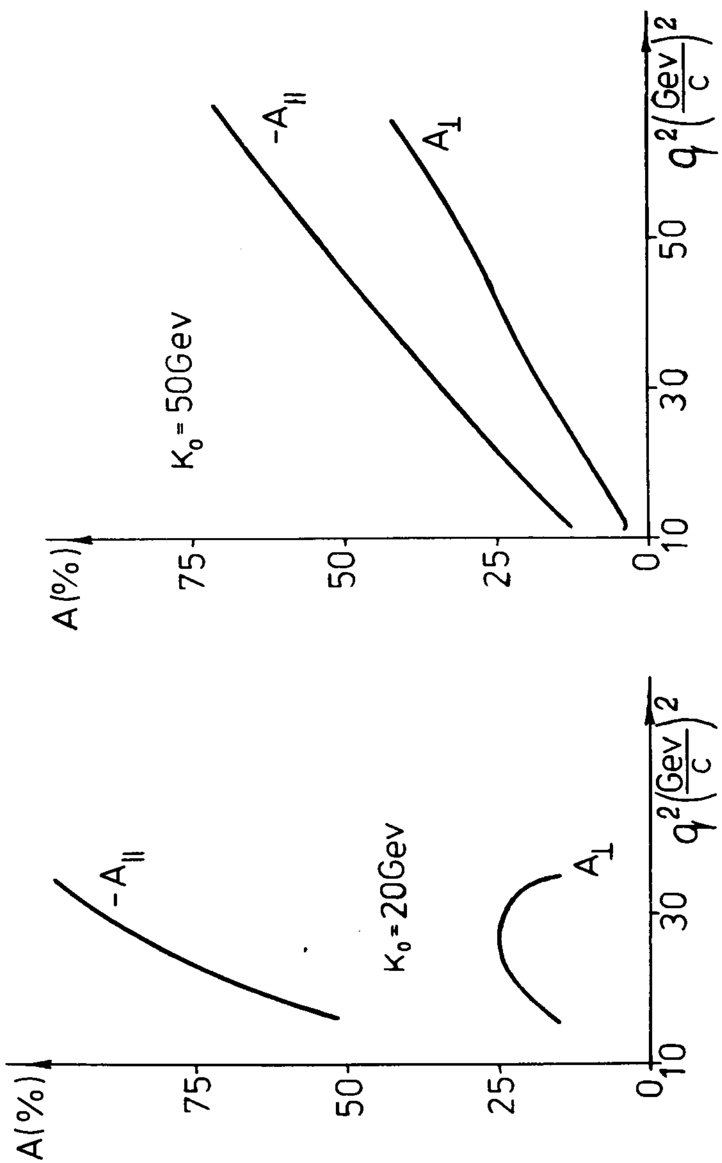

Fig. 3. The longitudinal $A_{\parallel}$ and transverse $A_{\perp}$ asymmetries calculated by assuming $F_2 = 0.27 F_1$; k_0 is the energy of the incident lepton.

tion of the data at $q^2 > 6\,(\mathrm{GeV/c})^2$ can be obtained. The parameter a is determined to be

$$a = 1{,}44 \pm 0{,}02$$

$$(\chi^2/\bar{\chi}^2 = 7{,}4/7)\ .$$

Finally, we shall note that we analysed the data on e–p scattering trying to answer the question whether we can eliminate rel. (1) using the available data [7]. It turned out that contrary to the suggestions made in ref. [8] present data cannot exclude the possibility for an equal q^2 -asymptotic behaviour for F_1 and F_2. The results of ref. [7] have been analysed by making the following assumptions for the form factors:

$$F_1(q^2) = a_1 \frac{M^4}{q^4}\ , \tag{15}$$

$$F_2(q^2) = a_2 \frac{M^4}{q^4}\ . \tag{16}$$

By means of (15) and (16) we obtain a satisfactiry description of the data at $q^2 > 6\,(\mathrm{GeV/c})^2$ and the values of the parameters are $(\chi^2/\bar{\chi}^2 = 8/6)$:

$$a_1 = 1.16 \pm 0.04, \quad a_2 = 0.27 \pm 0.08\ ,$$

Let us note that if the right-hand side of (15) is multiplied by $\ln \frac{q^2}{M^2}$ (such multipliers appear in the bound-state models of the nucleon [1]) the data in the same q^2 -region cannot be described.

The analysis carried out makes it evident that experiments of scattering of longitudinally polarized leptons on a polarized proton target are of current interest.

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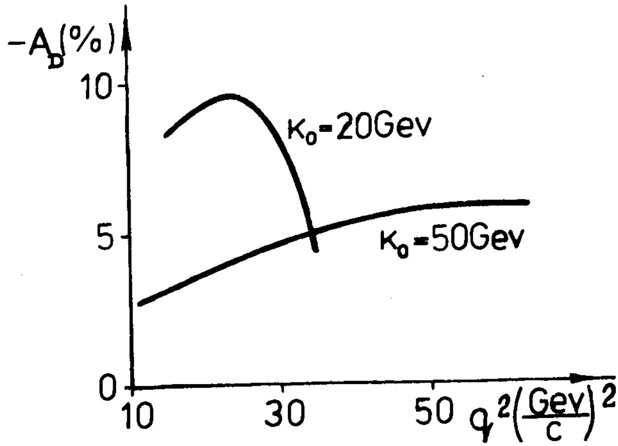

Fig. 4. The asymmetry $(-A_D)$ calculated by assuming $G_M = \mu\ G_E$; ; k_0 is the energy of the incident lepton.

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