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Об определении спина $X^0(958)$ -мезона в реакции $K^- p \to X^0 \Lambda$

Показано, что отсутствие анизотропий в распределениях Эдейра в реакции $K^- p \to X^0 \Lambda$ при 1,75 ГэВ/с ($\cos\theta_{c.m.} > 0,6$) не означает псевдоскалярности $X^0(958)$-мезона. Более того, указание на обратное поведение анизотропий при 1,75 ГэВ/с ($\cos\theta_{c.m.} > 0,6$) в сравнении с анизотропиями при 2,18 ГэВ/с ($\cos\theta_{c.m.} > 0,98$) интерпретируется как новый аргумент в пользу спина 2 для X^0-мезона. Показано, что более четкие анизотропии при 1,75 ГэВ/с можно ожидать в интервале $0,4 \leq \cos\theta_{c.m.} \leq 0,8$. Получено совместное распределение по всем распадным характеристикам X^0-мезона и Λ; предлагается использование этого распределения для более достоверного разделения гипотез 0^- и 2^- для спина-четности X^0-мезона, чем в случае использования одномерных распределений Эдейра.

Работа выполнена в Лаборатории высоких энергий ОИЯИ.

Препринт Объединенного института ядерных исследований
Дубна 1975

Lednický R. **E2 - 8652**

About $X^0(958)$ Spin Determination in the Reaction $K^- p \to X^0 \Lambda$

It is shown that the absence of anisotropies in the Adair distributions for the reaction $K^- p \to X^0 \Lambda$ at 1.75 GeV/c) ($\cos\theta_{c.m.} > 0.6$) does not imply pseudoscalarity of the X (958) --meson. Furthermore, an indication of the opposite character of the anisotropies at 1.75 GeV/c ($\cos\theta_{c.m.} > 0.6$) compared to those at 2.18 GeV/c ($\cos\theta_{c.m.} > 0.98$) is interpreted as a new argument in favour of the spin-2 X^0-meson assignment. More pronounced anisotropies at 1.75 GeV/c can be expected in the interval $0.4 \leq \cos\theta_{c.m.} \leq 0.8$. The joint distribution of all the X^0-meson and Λ-decay characteristics has been obtained; the likelihood analysis of this distribution, instead of the one-dimensional Adair analysis, is suggested.

The investigation has been performed at the Laboratory of High Energies, JINR.

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1. At present the ambiguity in the $X^0(958)$ meson spin still exists, $J^P(X^0) = 0^-$ or 2^- /1/ although this question emerged more than seven years ago /2/. However, the majority of physicists prefer spin parity 0^- rather than 2^-; in different kinds of theoretical estimates the X^0 meson is supposed to be the ninth pseudoscalar meson, it is even called the η'-meson. At the same time there exist the symmetry formulae /3/ predicting the η' mass near the mass of another ninth pseudoscalar candidate - E(1420) meson. In addition, the 2^- assignment needs special attention because in this case the X^0-meson Regge trajectory should have the intercept near 1 and can play a serious role in spin forces at high energies /4/.

It is now well-known /2,5/ that the X^0-meson spin can be established only by studying the X^0-meson production and decay correlations *. Such an analysis has been performed for the reaction $K^- p \to X^0 \Lambda$ in several Brookhaven HBC experiments with beam momenta 1.75 GeV/c /6/, 2.18 GeV/c /7/, 2.885 GeV/c /8/, 3.9 and 4.5 GeV/c /9/ and in earlier Berkeley HBC experiments at 2.1, 2.47 and 2.65 GeV/c /10/. In all these data no significant correlations between the X^0 production and decay angles were observed when averaged over all production angles. In refs. /6,7/ the small production angles $\theta_{c.m.}$ were selected ($x = \cos\theta_{c.m.} > 0.6-0.8$) again without revealing

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* The Dalitz plot analysis of the $X^0 \to \eta\pi\pi$ and $X^0 \to \gamma\pi^+\pi^-$ decays cannot distinguish between the 0^- and 2^- hypotheses /1,2/.

any significant deviations from isotropy in the Adair distributions. Since a spin zero particle must decay isotropically, this fact was interpreted as a strong support for the 0^- hypothesis.

However, after Ogievetsky, Tybor and Zaslavsky had remarked that an insufficient $\theta_{c.m.}$ cut could smooth the X^0 meson spin effects, tha data /7,10/ were reanalyzed in refs. /11,12/. The Adair distributions critical for solving the X^0 meson spin alternative /5/ were obtained for very small X^0 production angles ($x > 0.98$). The anisotropies in the angular ($\cos\theta$) distributions were observed at 2.18 GeV/c between the K^- beam momentum ($\vec{K}^*$) in the X^0 rest frame and the decay analyzers ($\hat{v}$) chosen along a) normal ($\hat{n}$) to the $X^0 \to \eta\pi\pi$ decay plane, b) η - meson momentum ($\vec{k}$) in the $X^0 \to \eta\pi\pi$ decay, c) γ - momentum ($\vec{k}$) in the $X^0 \to \gamma\pi\pi$ decay. The corresponding polar-equatorial ratios $\frac{P}{E} = \frac{N(|\cos\theta| > 0.5)}{N(|\cos\theta| < 0.5)}$ shown in Table 1 have a probability (in a χ^2 sence) of a small fraction of a percent to be in agreement with isotropy /12/. Therefore, based on the angular momentum conservation only, these anisotropies essentially weaken an evidence for the possibility of the 0^- spin parity X^0 assignment coming from the Dalitz plot analysis.

As we have pointed out in ref. /13/, the absence of the anisotropies in the LBL data /14/ is possibly connected with the increase of energy (LBL data at 2.1 GeV/c reveal some anisotropy /12/). We have also stressed /13/ a significance of the near threshold $K^- p \to X^0 \Lambda$ experiment. In such an experiment the X^0 -meson spin projections ± 2 on the c.m.s. beam direction ($\vec{K}$) should be damped, i.e., the X^0 spin alignment and corresponding anisotropies should appear at not too small production angles. In this context it has been pointed out in ref. /6/ that the cut $x > 0.6$ should reveal the anisotropies if only s- and p-waves essentially contribute to the $X^0\Lambda$ final state. The absence of higher waves can be expected in the near threshold experiment at 1.75 GeV/c /6/. However, this experiment reveals no significant anisotropies in the Adair dis-

Table 1

Number of polar events (P) and number of equatorial events (E) for the Adair distributions discussed in the text; N_σ is the number of standard deviations, the respective entries differ from equal numbers of P and E (isotropic distribution)

Experiment	1.75 GeV/c /6/ $x > 0.6$ $p_T \lesssim 200$ MeV/c			2.18 GeV/c /11,12/ $x > 0.98$ $p_T \lesssim 100$ MeV/c			Prediction for the case $\rho_{22} = 0$
Decay analyzer	P	E	N_σ	P	E	N_σ	P/E
$\hat{n}$	34	24	1.3	23	43	2.6	<1
$\eta(\vec{k})$	24	34	1.3	39	27	1.5	>1
$\pi\pi(\vec{q})$							$>(\frac{P}{E})_\eta$
$\hat{n}$							>1
$\gamma(\vec{k})$	22	20	0.3	7 ± 4	20 ± 4	$1.4^{x/}$	?
$\pi\pi(\vec{q})$							?

x/ These are background-subtracted numbers.

tributions ($x>0.6$) similar to those found at 2.18 GeV/c ($x>0.98$)/11/. Furthermore, from Table 1 we see that the Polar-equatorial ratios at 1.75 GeV/c ($x>0.6$) have a rather opposite character than the corresponding ones at 2.18 GeV/c ($x>0.98$).

Later on we'll show that such a behavior of the P/E ratios at 1.75 GeV/c ($x>0.6$) can naturally be understood and, in fact, it is another argument in favour of the 2^- X^0 -meson spin-parity assignment.

2. The distribution over the angle θ between the production and decay X^0-meson spin analyzers for the zero X^0 spin is isotropic. If the X^0-meson spin is 2, this distribution depends on the Legendre polynomials $P_L = d^L_{00}(\theta)$, $L = 0, 2, 4$, it having the following general form (see Appendix):

$$W(\cos\theta) = \frac{1}{2}\left[1 + \frac{10}{7} c_2 d_2^{(v)} d^2_{00}(\theta) + \frac{18}{7} c_4 d_4^{(v)} d^4_{00}(\theta)\right], \quad (1)$$

where the quantities $c_{2,4}$ are determined by the production mechanism only. Choosing the production analyzer (z -axis) in the X^0 production plane (say, along the c.m.s. beam momentum $\vec{K}$) and supposing parity conservation in the production process, these quantities can be written through the normalized X^0-meson spin density matrix elements in the form

$$c_2 = \rho_{00} + \rho_{11} - 2\rho_{22}, \quad c_4 = \rho_{00} - \frac{4}{3}\rho_{11} + \frac{1}{3}\rho_{22}. \quad (2)$$

The quantities $d^{(v)}_{2,4}$ depend on the X^0 decay mechanism only. They are determined by the formulae (A.13) similar to eqs. (2), where the X^0 -meson spin density matrix elements (with quantization ζ - axis directed along decay analyzer $\vec{v}$) should be averaged over the decay phase space and then normalized.

The X^0-meson spin will most clearly manifest itself in the distribution (1) if the X^0-meson production and decay analyzers are chosen in such a way that the corresponding quantities c_L and d_L achieve maximal absolute values. Note that these quantities are limited by the definition

$$-1 \le c_2, d_2 \le 1, \quad -\frac{2}{3} \le c_4, d_4 \le 1. \quad (3)$$

We have calculated the decay elements $d^{(v)}_L$ in ref./13/ and analyzed the question on the best decay analyzer in ref./14/. Here we briefly summarize the results:

$X^0 \to \gamma\gamma$ *decay*. The only natural decay analyzer is the γ momentum in the X^0 -meson rest frame. Bose symmetry and γ-quantum transversality unambiguously determine the decay matrix element (A.14) which leads to the maximal possible d_L values $d_2 = d_4 = 1$ thus making the $X^0 \to \gamma\gamma$ decay especially attractive.

In the three-particle $X^0 \to \eta\pi\pi$ and $X^0 \to \gamma\pi^+\pi^-$ decays there are three natural decay analyzers: normal $\hat{n}$ to the X^0 decay plane, η-meson (γ -quantum) momentum $\vec{k}$ in the X^0 rest frame and π-meson momentum $\vec{q}$ is the dipion rest frame. The matrix elements of these decays cannot be determined unambiguously; even in the lowest orbital momentum approximation they depend on free parameters. However, using the experimental X^0-decay information, the following estimates can be done (see Appendix):

$X^0 \to \eta\pi\pi$ *decay:*

$$d_2^{(n)} = -0.5 \div -0.8, \quad d_4^{(n)} = 0.4 \div 0.25, \quad d_2^{(k)} = d_4^{(k)} \simeq 0.4,$$

$$d_2^{(q)} = d_4^{(q)} \simeq 0.6. \quad (4)$$

$X^0 \to \gamma\pi^+\pi^-$ *decay*

$$d_2^{(n)} = 0.3 \div 0.8, \quad d_2^{(k)} = +0.3 \div -0.8, \quad d_2^{(q)} = -0.7 \div 0.5$$

$$d_4^{(n)} = 0 \div 0.3, \quad d_4^{(k)} = 0.4 \div 0.1, \quad d_4^{(q)} = 0. \quad (5)$$

The quantities c_L can vanish in the case when there is no diagonal X^0-meson spin alignment, i.e., if $\rho_{mm} = 1/5$,

$m = 0, \pm 1, \pm 2$. But in the X^0-meson forward production $K^- p \to X^0 \Lambda$ or at threshold of this reaction, the X^0-meson spin projections ± 2 on the c.m.s. beam momentum $\vec{K}$ (z-axis) are forbidden, $\rho_{22} = 0$. Consequently,

$$c_2 = \frac{1}{2}(1 + \rho_{00}), \quad c_4 = \frac{1}{3}(5\rho_{00} - 2) \qquad (6)$$

so that the anisotropies should be presented in the distributions (1) for an arbitrary ρ_{00} value ($c_2 \geq 1/2$). Using the d_L estimates (4) and (5) and the inequality $c_2 \geq 0.5$ the qualitative predictions given in Table 1 for the $\frac{P}{E}$ ratios

$$\frac{P}{E} = \frac{1 + \frac{15}{8}\langle P_2 \rangle - \frac{135}{128}\langle P_4 \rangle}{1 - \frac{15}{8}\langle P_2 \rangle + \frac{135}{128}\langle P_4 \rangle}, \quad \langle P_L \rangle = \frac{2}{7} c_L d_L^{(v)}, \qquad (7)$$

can be obtained. They are in agreement with the Brookhaven-Michigan data /12/ at 2.18 GeV/c ($x > 0.98$).

Let us now discuss the disagreement between the P/E ratios for 2.18 GeV/c ($x > 0.98$) /12/ and for 1.75 GeV/c ($x > 0.6$) /6/. It has been pointed out in ref. /6/ that the cut ($x > 0.6$) should be sufficient to essentially damp the ρ_{22} value assuming that only s- and p-waves are present in the final state of the reaction $K^- p \to X^0 \Lambda$. Such an assumption is quite natural in the near threshold experiment at 1.75 GeV/c and is also supported by the $\cos\theta_{c.m.}$ distribution. This distribution

$$W(x) = \rho_{00}(x) + 2\rho_{11}(x) + 2\rho_{22}(x), \qquad (8)$$

shown in Fig. 1, can be well described by the solid curve $W(x)$ drawn in this figure; $W(x)$ was fitted by the Legendre polynomials $P_L(x)$ with $L \leq 2$ /6/ (higher moments $\langle P_L \rangle$, $L \geq 3$ are consistent with zero within two standard deviations) thus indicating the absence of the orbital angular momentum waves with $\ell \geq 2$.

However, the disagreement between the P/E ratio predictions, obtained, if $\rho_{22} = 0$, and those P/E ratios obtained at 1.75 GeV/c ($x > 0.6$) leads us to another conclu-

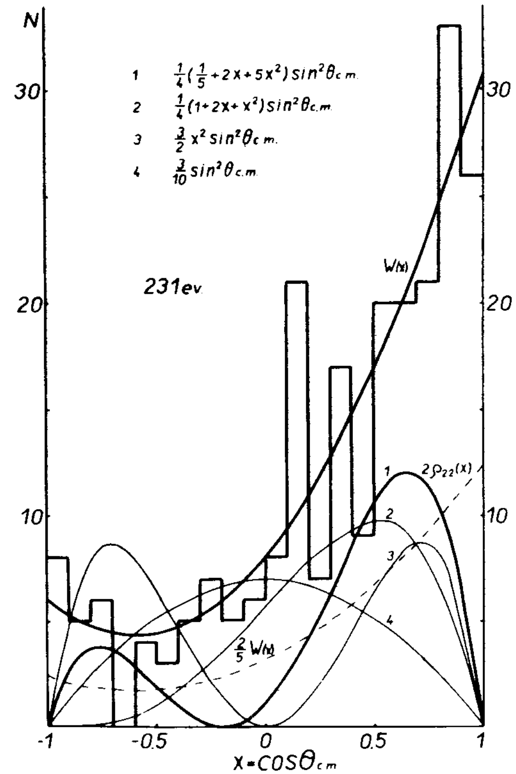

Fig. 1. The $\cos\theta_{c.m}$ distribution for the reaction $K^- p \to X^0 \Lambda$ at 1.75 GeV/c /6/ for both the $\pi^+\pi^-\eta$ and $\pi^+\pi^-\gamma$ decay modes. The solid curve $W(x)$ is the lowest Legendre polynomial fit to this distribution: $W(x) = \sum_{L=0,1,2} \frac{2L+1}{2} A_L P_L(x)$; the dashed curve is $\frac{2}{5} W(x)$. Curves 1,2,3 and 4 describe possible $2\rho_{22}(x)$ elements of the X^0-meson spin density matrix in the case when only orbital momentum waves with $\ell \leq 2$ contribute to the $X^0 \Lambda$ final state. The normalization $\langle \rho_{22}(x) \rangle = \frac{1}{5} \langle W(x) \rangle$ is used.

sion. Namely, the waves with $\ell \geq 2$ should essentially contribute to the ρ_{22} element thus making the cut $x > 0.6$ insufficient for suppressing this element to the value much more smaller than 1/5. There are several facts supporting such a conclusion.

(a) Because the ± 2 X^0-meson spin projections cannot be constructed from the Λ and proton spin projections only, the ρ_{22} value essentially depends on the ℓ_z-component of the orbital angular momentum. Roughly speaking, the averaged ℓ_z-component is proportional to the maximal transversal momentum p_T allowed by the x-cut. Therefore claiming the same p_T-cut for 1.75 GeV/c, as well as for 2.18 GeV/c ($x > 0.98$, $p_T \lesssim 100$ MeV/c), we should require $x > 0.92$ which is a much more stronger cut than $x > 0.6$.

(b) The beam momentum 1.75 GeV/c corresponds to the c.m.s. energy $\sqrt{s} = 2130$ MeV just near the strong $K^- p$ resonance $\Lambda(2100)\ 7/2^-$ which can decay into the $X^0\Lambda$ state with the orbital momenta $\ell = 2, 4, 6$. Therefore, at least, a d-wave contribution should be expected in the final state of the reaction $K^- p \to X^0 \Lambda$ at 1.75 GeV/c.

(c) Despite the fact that the $\cos\theta_{c.m.}$ distribution is well described by the lowest Legendre polynomials $P_L(x)$, $L \le 2$, there are $\approx 2\sigma$ effects in the moments $\langle P_L(x)\rangle$ for $L = 6,7,8$. A rather strong p-wave contribution also indicates that essential higher waves could be present. Besides, the waves with $\ell \ge 2$ can contribute to the elements ρ_{mm} and cancel in their sum $W(x) = \sum_m \rho_{mm}(x)$.

In fact, we do not need many additional waves in order to explain the P/E ratios in the near threshold experiment at 1.75 GeV/c. Below we'll show that even the only additional d-wave contribution is enough to obtain the ρ_{22} value near 1/5 in the interval $x > 0.6$ and thus to explain the absence of the anisotropies expected in the case of the near zero ρ_{22} value. First we note that the ρ_{22} element should contain the amplitudes with $\ell_z \ge 1$, i.e., $\rho_{22}(x) = 0$ if $\ell^{max} = 0$ and generally

$$\rho_{22}(x) = F_n(x) \sin^2\theta_{c.m.}, \qquad (9)$$

where $F_n(x)$ is a polynomial in x of the order of $n = 2\ell^{max} - 2$. The ρ_{22} value can be fixed from the fact that no anisotropies were seen in the overall decay angular distributions, i.e.,

$$\langle \rho_{mm} \rangle \cong \frac{1}{5} \langle W \rangle, \qquad (10)$$

where $\langle\rho\rangle = \int_{-1}^{1} \rho(x)dx$. For $\ell^{max} = 1$ we then have $2\rho_{22}(x) = \frac{3}{10}\sin^2\theta_{c.m.}$, curve 4 in Fig. 1, yielding the averaged value $\langle\rho_{22}\rangle = 0.05 \ll \frac{1}{5}$ in the interval $x > 0.6$. For $\ell^{max} = 2$ the ρ_{22} element cannot be determined unambiguously. In Fig. 1. we show several functions $2\rho_{22}(x)$ normalized by the condition (10), curves 1, 2, and 3, leading to large ρ_{22} values in the interval $x > 0.6$. Curve 1 yields the maximal possible ρ_{22} values for x close to 1, $\langle\rho_{22}\rangle = 0.18 \cong 1/5$ for $x > 0.6$ ($\langle\rho_{22}\rangle = 0.11$ for $x > 0.8$); this curve also satisfies the positivity condition $2\rho_{22}(x) \le W(x)$. Of course, the only additional d-wave contribution cannot explain the opposite character of the anisotropies for 2.18 GeV/c ($x > 0.98$) and those for 1.75 GeV/c ($x > 0.6$). Supposing the P/E ratios at 1.75 GeV/c to be statistically meaningful, the averaged ρ_{22} value in the interval $x > 0.6$ should be larger than 1/5 ($c_2 < 0$), i.e., the waves with $\ell \ge 3$ must contribute. However, near threshold these waves cannot essentially change the qualitative $\rho_{22}(x)$ behaviour as expected from the considerations of the waves with $\ell \le 2$. The true x-dependence of the function $2\rho_{22}(x)$ should then be close to curve 1 thus indicating $\rho_{22} > 1/5$ ($c_2 < 0$) in the interval $0.4 \lesssim x \lesssim 0.8$. Therefore, in this interval we can expect more pronounced anisotropies of the same character as those found for $x > 0.6$.

3. Based on the analysis demonstrated in the previous section, we thus come to the conclusion different from Baltay's et al in their paper /6/. Namely, an indication of rather opposite anisotropies in the Adair distributions at 1.75 GeV/c ($x > 0.6$) /6/ in comparison with those found at 2.18 GeV/c ($x > 0.98$) /11/ is naturally explained and supports an evidence for the spin-2 assignment for the X^0-meson coming from the Adair analysis carried out in refs. /11,12/.

It should be stressed that not all experimental information available has been analyzed. From the 6 natural analyzers in the decays $X^0 \to \eta\pi\pi$ and $X^0 \to \gamma\pi^+\pi^-$ only the 3 analyzers (see Table 1) were used in the Adair analysis even despite the fact that some time ago we pointed out to a great importance of three other analyzers /13/: the

π -meson momentum $\vec{q}$ in the dipion rest frame for the decays $X^0 \to \eta\pi\pi\ (\gamma\pi^+\pi^-)$ and the normal $\hat{n}$ to the $X^0 \to \gamma\pi^+\pi^-$ decay plane. The use of the best decay analyzers /14/ could also be very helpful. However, present knowledge of the X^0 -meson decay amplitudes may be insufficient for the determination of the best analyzer.

The way for increasing the confidence level of the arguments in favour of or against the 2^- X^0-meson spin-parity assignment, even without increasing the statistics available, is the likelihood analysis of the distribution (A.12) containing all the $X^0\Lambda$ -production and decay information. We show in Appendix that the production information is described by 30 real x -functions, being the bilinear products of 10 complex amplitudes. In a given narrow x -interval 29 normalized production (multipole) parameters could then be determined. Note that in the case of the pseudoscalar X^0-meson there is only 1 nonzero parameter describing the Λ -production. All other 28 production parameters should be equal to zero. This fact can be used in a preliminary simple analysis using the method of moments.

The X^0 -decay information alone was actually analyzed by the likelihood fit in ref. /9/. What we suggest here is just an extension of this analysis including all the information available. Such a joint analysis should also improve the knowledge of the X^0-meson decay parameters. For the 0^- and 2^- likelihood ratio we expect (based on the data in Table 1) a value less than 10^{-3}.

The problem of the X^0 -meson spin parity is of so great importance that further experiments are necessary for its final solution /13/. Probably, the most simple experiment is a study of the Adair distribution with the aid of electronics in the reaction $\pi^- p \to X^0 n$ /15/.
$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad \hookrightarrow \gamma\gamma$

The author is much grateful to V.I.Ogievetsky, W.Tybor and A.N.Zaslavsky for very useful discussions.

Appendix

The differential cross section of the reaction $K^- p \to X^0 \Lambda$ ($X^0 \to 1..a$, $\Lambda \to p\pi^-$) can be expressed through the joint spin density matrix elements in the production $X^0\Lambda$ process ($\rho^{nn'}_{mm'}$) and spin density matrix elements in the X^0 and Λ decays ($r^1_{m'm}$ and $r^2_{n'n}$) determined in coordinate systems $x_1 y_1 z_1$ and $x_2 y_2 z_2$ in the X^0 and Λ rest frames, respectively,

$$d\sigma = \sum r^1_{m'm}\, r^2_{n'n}\, \rho^{nn'}_{mm'}(x)\, dx\, d_a(X; 1\ldots a)\, d_2(\Lambda; p\pi^-), \quad (A.1)$$

where $x = \cos\theta_{c.m.}$ and the decay phase space elements are of the form

$$d_a(a; 1\ldots a) = \prod_{j=1}^{a} \frac{d\vec{p}_j}{2\omega_j}\, \delta^{(4)}(p_a - \sum_{i=1}^{a} p_i), \quad (A.2)$$

$p_j = (\vec{p}_j, i\omega_j)$ is the 4-momentum of the particle j. With the aid of the vectors in the X^0 and Λ decays, the coordinate systems $\xi_1 \eta_1 \zeta_1$ and $\xi_2 \eta_2 \zeta_2$ can be fixed. Let us denote the Euler angles of rotations $x_i y_i z_i \to \xi_i \eta_i \zeta_i$ by $\Omega_i = (\phi_i, \theta_i, \psi_i)$, $i=1,2$. The phase space elements in the two- and three-particle decays $a \to 12$ and $a \to 123$ can then be written in the form

$$d_2(a;12) = \frac{k}{4m_a} d\phi\, d\cos\theta,$$

$$d_3(a;123) = \frac{kq}{8m_a} dm_{23}\, d\cos\delta\, d\phi\, d\cos\theta\, d\psi, \quad (A.3)$$

where $\vec{k} = \vec{p}_1^{(a)}$ is the momentum of particle 1 in the a -rest frame; $\vec{q} = \vec{p}_2^{(23)}$ is the momentum of particle 2 in the c.m.s. of particles 2,3; m_{23} is the effective mass of particles 2,3, and δ is the angle between the vectors $\vec{k}$ and $\vec{q}$.

Note that the decay density matrix elements are determined through the X^0 and Λ decay amplitudes $A^i_{\{\lambda\}}(m_i)$, $i = 1,2$:

$$r^i_{m'_i m_i} = \sum A^{i*}_{\{\lambda\}}(m'_i)\, A^i_{\{\lambda\}}(m_i) \quad (A.4)$$

where $\{\lambda\}$ are the helicities of the decay particles. Under the rotation $x_i y_i z_i \to \xi_i \eta_i \zeta_i$ the decay amplitudes are transformed with the aid of the D -functions according to the law /16/

$$A^{i\,\prime}_{\{\lambda\}}(\mu_i) = \sum_{m_i} A^{i}_{\{\lambda\}}(m_i) D^{J_i}_{m_i \mu_i}(\phi_i, \theta_i, \psi_i), \quad (A.5)$$

$J_1 = 2$ and $J_2 = 1/2$. Using this transformation, the Ω_i dependence of the distribution (A.1) can explicitly be calculated

$$d\sigma = \sum (2L_1+1)(2L_2+1) t^{L_2 M_2 *}_{L_1 M_1}(x) \; T^1_{L_1 N_1} T^2_{L_2 N_2} D^{L_1 *}_{M_1 N_1}(\Omega_1) \times$$

$$\times D^{L_2 *}_{M_2 N_2}(\Omega_2)\, dx\, d_\alpha(X; 1 \ldots \alpha)\, d_2(\Lambda; p\pi^-), \quad (A.6)$$

where the multipole parameters in the production and decay are expressed through the density matrix elements by means of the Clebsch-Gordan coefficients:

$$t^{L_2 M_2 *}_{L_1 M_1}(x) = \sum_{\{m\}} \rho^{m_2 m_2'}_{m_1 m_1'}(x)(2m_1' L_1 M_1 | 2m_1)(\tfrac{1}{2} m_2' L_2 M_2 | \tfrac{1}{2} m_2), \quad (A.7)$$

$$T^i_{L_i N_i} = \sum_{\{\mu\}} r^i_{\mu_i' \mu_i} (J_i \mu_i' L_i N_i | J_i \mu_i). \quad (A.8)$$

Hermiticity of the ρ- and r-matrices implies

$$t^{L_2 - M_2 *}_{L_1 - M_1} = (-)^{M_1 + M_2} t^{L_2 M_2}_{L_1 M_1}, \quad T^{i*}_{L_i N_i} = (-)^{N_i} T^i_{L_i - N_i} \quad (A.9)$$

From parity conservation in the production process (assuming that the z_i -axes are chosen in the production plane) it follows

$$t^{L_2 - M_2}_{L_1 - M_1} = (-)^{L_1 + M_1 + L_2 + M_2} t^{L_2 M_2}_{L_1 M_1}. \quad (A.10)$$

Note that for the Adair analysis it is useful to choose the

z_1 -axis along the beam momentum in the overall c.m.s. and $z_2 = -z_1$ and $y_1 = y_2 = \vec{K} \times \vec{p}_X$.

Let us further fix the coordinate systems $\xi_i \eta_i \zeta_i$. In a two-particle decay $a \to 12(\Lambda \to p\pi^-, X^0 \to \gamma\gamma)$ it is natural to direct the ζ -axis along the momentum $\vec{p}_1^{(a)}$. Since the decay amplitudes cannot depend on the rotation around this axis (assuming the final spins are not measured), all the nondiagonal r -matrix elements should be equal to zero $(N_i = 0)$, and we can put $\psi_i = 0$. In the three-particle decays $X^0 \to \eta\pi\pi$ and $X^0 \to \gamma\pi^+\pi^-$ we choose the ζ -axis along the normal $\hat{n}$ to the decay plane and the ξ -axis along the η -meson (photon) momentum $\vec{k}$ in the X^0 rest frame. Note that parity conservation in the X^0 decay, in these coordinate systems, yields particularly simple relations

$$T^1_{L_1 N_1} = 0 \quad \text{for odd } L_1 \text{ or } N_1. \quad (A.11)$$

In the $\Lambda \to p\pi^-$ decay, parity is not conserved implying the asymmetric Λ decay. The Λ asymmetry parameter $\alpha_\Lambda = 0.646$; $\alpha_\Lambda = \sqrt{3}\, T^2_{10}$ providing that $T^2_{00} = 1$.

Using the relations (A.9-11), the distribution (A.6) can be rewritten in the form useful for calculations

$$d\sigma = \sum_{L=0,2,4} (2L+1) \{ t^{00}_{L0}(x) [T^1_{L0} d^L_{00}(\theta_1) + 2 \sum_{N=2,\ldots L} \mathrm{Re}\, B_{LN} d^L_{0N}(\theta_1)] +$$

$$+2 \sum_{M=1,\ldots L} t^{00}_{LM}(x) [T^1_{L0} \cos M\phi_1\, d^L_{M0}(\theta_1) + \sum_{N=2,\ldots L} (\mathrm{Re}\, B_{LN} \cos M\phi_1 \times$$

$$\times d^{L+}_{MN}(\theta_1) - \mathrm{Im}\, B_{LN} \sin M\phi_1\, d^{L-}_{MN}(\theta_1))] +$$

$$+2\sqrt{3}\,\alpha_\Lambda \sum_{M=1,\ldots L} \mathrm{Im}\, t^{10}_{LM}(x) \cos\theta_2 [T^1_{L0} \sin M\phi_1\, d^L_{M0}(\theta_1) +$$

$$+ \sum_{N=2,\ldots L} (\mathrm{Re}\, B_{LN} \sin M\phi_1\, d^{L+}_{MN}(\theta_1) + \mathrm{Im}\, B_{LN} \cos M\phi_1\, d^{L-}_{MN}(\theta_1))] -$$

$$-\sqrt{6}\,a_\Lambda \sum_{M=0,\pm 1,\ldots \pm L} \mathrm{Im}\, t^{11}_{LM}(x) \sin\theta_2 [T^{1}_{L0} \sin(M\phi_1+\phi_2) d^{L}_{M0}(\theta_1) +$$

$$+ \sum_{N=2,\ldots L} (\mathrm{Re}\, B_{LN} \sin(M\phi_1+\phi_2) d^{L+}_{MN}(\theta_1) + \mathrm{Im}\, B_{LN} \cos(M\phi_1+\phi_2) d^{L-}_{MN}(\theta_1))]\}\cdot$$

$$\times dx\, d\Omega_2\, d_a(X;1\ldots a), \qquad (A.12)$$

where $B_{LN} = T^{1}_{LN} e^{iN\psi_1}$ and $d^{L\pm}_{MN} = \frac{1}{2}(d^{L}_{MN} \pm d^{L}_{M-N})$. Production characteristics are described by 30 real functions $t^{00}_{LM}(x)$, $\mathrm{i}t^{10}_{LM}(x)$ and $\mathrm{i}t^{11}_{LM}(x)$; more precisely, we have 9 elements t^{00}_{LM}, $L = 0,2,4$, $M = 0,1 \ldots L$, 6 elements $\mathrm{Im}\, t^{10}_{LM}$, $L = 2,4$, $M = 1,\ldots L$ and 15 elements $\mathrm{Im}\, t^{11}_{LM}$, $L = 0,2,4$, $M = 0, \pm 1, \ldots \pm L$.* Note that in the case of the zero X^0-meson spin there are only two independent elements $t^{00}_{00} = \rho^{++} + \rho^{--}$ and $\mathrm{Im}\, t^{11}_{00} = -\sqrt{\frac{2}{3}}\rho^{+-}$; all other 28 elements are equal to zero.

Integrating (A.12) over the phase space and introducing the quantities d_L and c_L

$$d_L = \pm\sqrt{\frac{7}{2}} \frac{\int T^{1}_{L0}\, d_a(X;1\ldots a)}{\int T^{1}_{00}\, d_a(X;1\ldots a)}, \quad c_L = \mp\sqrt{\frac{7}{2}} \frac{\int t^{00}_{L0}(x)\,dx}{\int t^{00}_{00}(x)\,dx},$$

$$L = 2,4, \qquad (A.13)$$

we get formula (1) for the $W(\cos\theta)$ distribution. The X^0-meson decay multipole parameters T^{1}_{LN} have been

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* The multipole parameters, being the bilinear products of 10 independent complex amplitudes, depend on 18 real parameters (common phase and normalization factor not included). In the collinear case $|x| = 1$, only the 2 independent amplitudes are left and from the 30 multipole parameters only the 5 ones, i.e., t^{00}_{00}, t^{00}_{L0}, $\mathrm{Im}\, t^{11}_{L1}$, $L = 2,4$ can be different from zero.

calculated in ref. /14/ using, however, different normalization factors. Let us briefly reproduce the calculation here.

The $X^0 \to \gamma\gamma$ decay. The amplitude of this decay is unambiguously determined by the Bose-symmetry and by the γ-quantum transversality

$$A_{ij} = k_i [\vec{e}^{(1)} \times \vec{e}^{(2)}]_j, \qquad (A.14)$$

where $\vec{e}^{(1,2)}$ are the γ polarization vectors and $\vec{k} = k(0,0,1)$, The tensor representation is connected with the representation of the X^0-meson spin projections on the ζ-axis by the well-known relations:

$$A(\pm 2) = \frac{1}{2}(A_{11} - A_{22}) \pm \frac{i}{2}(A_{12} + A_{21})$$

$$A(\pm 1) = \mp\frac{1}{2}(A_{13} + A_{31}) - \frac{i}{2}(A_{23} + A_{32}) \qquad (A.15)$$

$$A(0) = \frac{1}{\sqrt{6}}(2A_{33} - A_{11} - A_{22}).$$

These relations automatically pick out the symmetric and zero trace parts of the amplitudes A_{ij}. Among the amplitudes (A.14) only $A_{33} \neq 0$, i.e., only the r_{00} element is different from zero and according to (A.8) and (A.13), we get

$$T^{1}_{00} = r_{00}, \quad T^{1}_{20} = -T^{1}_{40} = -\sqrt{\frac{2}{7}}\, r_{00}, \quad d_2 = d_4 = 1. \qquad (A.16)$$

The $X^0 \to \eta\pi\pi$ decay. In the lowest orbital momentum approximation $\ell_\eta = 2, \ell_{\pi\pi} = 0$ and $\ell_\eta = 0$, $\ell_{\pi\pi} = 2$, the decay amplitudes are of the form

$$A_{ij} = w_0 k_i k_j + w_2 q_i q_j, \qquad (A.17)$$

where $\vec{k} = k(0,0,1)$, $\vec{q} = q(\cos\delta, \sin\delta, 0)$ in the $\xi_1\eta_1\zeta_1$ system. Using again (A.15) and (A.8), we obtain the decay multipole parameters

$$T_{00}=\frac{2}{3}[\,|w_0|^2k^4+|w_2|^2q^4+2\mathrm{Re}\,w_0w_2^*k^2q^2d^2_{00}(\delta)\,]$$

$$T_{20}=\frac{1}{3}\sqrt{\frac{2}{7}}[\,|w_0|^2k^4+|w_2|^2q^4-2\mathrm{Re}\,w_0w_2^*k^2q^2(1-2d^2_{00}(\delta))]$$

$$T_{22}=-\frac{1}{\sqrt{21}}[\,|w_0|^2k^4+\mathrm{Re}\,w_0\,w_2^*k^2q^2+(\mathrm{Re}\,w_0w_2^*k^2q^2+|w_2|^2q^4)e^{2i\delta}]$$

$$T_{40}=\frac{1}{\sqrt{14}}[\frac{1}{2}(|w_0|^2k^4+|w_2|^2q^4)+\frac{1}{9}\mathrm{Re}\,w_0\,w_2^*k^2q^2(5+4d^2_{00}(\delta))]$$

$$T_{42}=\frac{1}{2}\sqrt{\frac{5}{3}}T_{22}$$

$$T_{44}=\frac{\sqrt{5}}{12}[\,|w_0|^2k^4+2\mathrm{Re}\,w_0w_2^*k^2q^2e^{2i\delta}+|w_2|^2q^4e^{4i\delta}]. \qquad (A.18)$$

The parameters w_ℓ can depend on the $m_{\pi\pi}$ mass. Supposing this dependence negligible, the only complex parameter $w=w_2/w_0$, is left. For the quantities d_L we then get

$$d_2^{(n)}=-\frac{1}{2}+\frac{\mathrm{Re}\,w\ \alpha_3}{\alpha_1+|w|^2\alpha_2},\quad d_4^{(n)}=\frac{3}{8}+\frac{5}{12}\frac{\mathrm{Re}\,w\ \alpha_3}{\alpha_1+|w|^2\alpha_2}, \qquad (A.19)$$

where α_1, α_2 and α_3 are phase space integrals over the quantities k^4, q^4 and k^2q^2; $\alpha_1:\alpha_2:\alpha_3=6.6:1:1.5$. The d_L values for the decay analyzers $\vec{k}$ and $\vec{q}$ can be obtained by means of the rotation (A.5) /13,14/:

$$d_2^{(k)}=d_4^{(k)}=\frac{\alpha_1}{\alpha_1+|w|^2\alpha_2},\quad d_2^{(q)}=d_4^{(q)}=1-d_2^{(k)}, \qquad (A.20)$$

If $\mathrm{Re}\,w\le 0$, the extreme d_2 values are equal to /14/

$$d_2^{min}=d_2^{(n)}\simeq-0.5,\qquad d_2^{max}=d_2^{(v_0)}\simeq 0.86, \qquad (A.21)$$

where the vector $\vec{v}_0$ lies in the X^0 decay plane: $\vec{v}_0=(\cos\alpha,\ \sin\alpha,0)$; the angle α is determined by the condition /14/

$$e^{2i\alpha}=\frac{T^1_{22}}{|T^1_{22}|}. \qquad (A.22)$$

The numerical estimates are given for an almost purely imaginary Brookhaven experimental value of w/11/ : $w^{-1}=-0.02\pm0.05+(0.35\pm0.02)i$. The real value $w=-4$ is however predicted by the Adler selfconsistency condition /2/. Such a discrepancy can probably be explained by an essential $m_{\pi\pi}$ dependence of the parameter w_2 resulting from the final state $\pi\pi$ -interaction /17/.

The $X^0\to\gamma\pi^+\pi^-$ *decay.* Taking into account only the Ml and E2 transition amplitudes in the dominating $X^0\to\gamma\rho^0$ decay channel, we can write

$$A_{ij}=\{g_1q_i[\vec{k}\times\vec{e}\,]_j+g_2e_i[\vec{k}\times\vec{q}\,]_j\}f(m_{\pi\pi}), \qquad (A.23)$$

where $f(m_{\pi\pi})$ is the ρ^0 -meson propagator. Here the parameters $g_{1,2}$ are expected to be real and independent of the $m_{\pi\pi}$ mass. Introducing a real parameter $g=g_2/g_1$ and omitting the inessential factor $g_1kqf(m_{\pi\pi})$, we get the following formulae for the decay multipole parameters

$$T_{00}=\frac{1}{9}[10+10g+7g^2-(1+10g+7g^2)d^2_{00}(\delta)]$$

$$T_{20}=\frac{1}{18}\sqrt{\frac{2}{7}}[\frac{5}{2}-14g-11g^2+(2+14g+11g^2)d^2_{00}(\delta)]$$

$$T_{22}=\frac{1}{4\sqrt{21}}[2+7g+3g^2\sin^2\delta-(5+7g)e^{2i\delta}]$$

$$T_{40}=\frac{1}{9}\sqrt{\frac{2}{7}}[-\frac{5}{4}+2g^2-(1+2g^2)d^2_{00}(\delta)]$$

$$T_{42} = \frac{1}{12}\sqrt{\frac{5}{7}}\,[\,1 - 2g^2\sin^2\delta + e^{2i\delta}\,]$$

$$T_{44} = -\frac{\sqrt{5}}{12}\,e^{2i\delta} \quad . \qquad \text{(A.24)}$$

For the quantities d_L corresponding to the analyzers $\hat{n}, \vec{k}$ and $\vec{q}$, we have the following expressions:

$$d_2^{(n)} = \frac{1}{4}\,\frac{-0.5 + 2.8g + 2.2g^2}{1 + g + 0.7g^2}, \quad d_4^{(n)} = \frac{1}{4}\,\frac{-0.5 + 0.8g^2}{1 + g + 0.7g^2}$$

$$d_2^{(k)} = -\frac{1}{2}\,\frac{0.7 + 2.8g + g^2}{1 + g + 0.7g^2}, \quad d_4^{(k)} = \frac{0.2g^2}{1 + g + 0.7g^2}$$

$$d_2^{(q)} = 0.7\,\frac{1 + g - 0.2g^2}{1 + g + 0.7g^2}, \qquad d_4^{(q)} = 0 . \qquad \text{(A.25)}$$

The g -dependence of the quantities d_2 is shown in Fig. 2 together with the extreme d_2 values. In Fig. 2 we also present the ρ_{00} spin density matrix element of the ρ^0 - meson produced in the $X^0 \to \gamma\rho^0$ decay which (in the helicity frame) takes the form

$$\rho_{00}^{H} = \frac{0.3}{1 + g + 0.7g^2} . \qquad \text{(A.26)}$$

The experimental ρ_{00} value, shown also in Fig. 2, yields then the following estimates for the parameter g: $g = -3.5^{+1.4}_{-\infty}$ and $g = 2.0^{+\infty}_{-1.3}$. Besides, the small negative g values are probably excluded by the anisotropy observed in the $K\bar{K}$ distribution /11/ ($P/E < 1$ implies $d_2^{(k)} < 0$).

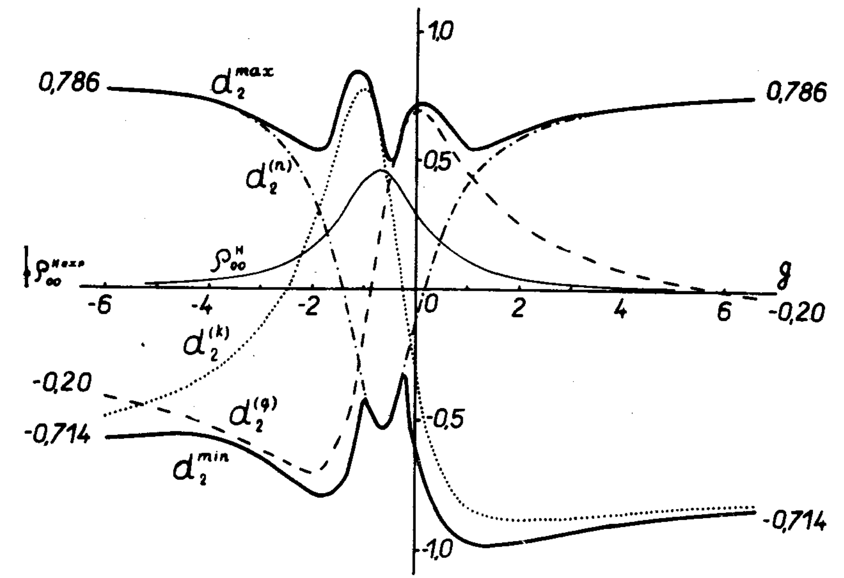

Fig. 2. The decay coefficients d_2 vs the mixing parameter g of the E2 and M1 transition amplitudes in the $X^0 \to \gamma\rho^0$ decay. The g-dependence of the ρ_{00}-density matrix element (helicity frame) of the ρ^0 -meson produced in the $X^0 \to \gamma\rho^0$ decay is presented as well.

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