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APPLICATION OF 'T HOOFT'S RENORMALIZATION SCHEME TO TWO-LOOP CALCULATIONS







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APPLICATION OF 'T HOOFT'S RENORMALIZATION SCHEME TO TWO-LOOP CALCULATIONS

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Применение перенормировочной схемы 'т Хоофта для двухпетлевых вычислений

Продемонстрированы несомненные преимущества схемы 'т Хоофта для асимптотических расчетов в ренормализационной группе. Выполнены двухпетлевые вычисления в трех ренормируемых моделях: в скалярной электродинамике, псевдоскалярной юкавской теории и суперсимметричной модели Весса и Зумино.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Application of 't Hooft's Renormalization Scheme to Two-Loop Calculations

The manifest advantages of 't Hooft's scheme for the asymptotic calculations are demonstrated. The two-loop computations are carried out in three particular models: scalar electrodynamics, pseudoscalar Yukawa theory and supersymmetric model by Wess and Zumino.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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I. Introduction

In the previous paper^{/1/}the connection between the different renormalization approaches was studied and the conversion formulas were derived. These formulas allow us to reconstruct renormalization group functions of any renormalization scheme from those of 't Hooft's scheme with the use of only lower-order extra information. A special role of 't Hooft's scheme is based on its remarkable features, which are discussed in Sec.II. The necessary relations of this scheme are presented in Sec.III, where also the most convenient way to perform the R-operation is described. Then, in Sec. IV the 't Hooft scheme is used for two-loop calculations in three renormalizable theories: scalar electrodynamics, pseudoscalar theory with Yukawa-type coupling and supersymmetric Wess-Zumino model.

Note that this paper is a sequel to Ref. 1 and should be read in conjunction with it, especially when the properties of renormalization group equations are concerned. Hereafter the prefix I will refer to equations of Ref. 1.

I wish to express my gratitude to D.V.Shirkov for interest in this work and helpful discussions.

II. 't Hoort's scheme of renormalization

The dimensional regularization method /2/ has generally been recognized mainly because of its remarkable property to maintain the initial symmetry of the Lagrangian in the regularized expressions. The proof of this property, though not for the most general case, can be found in /2, 3/. The different kinds of subtraction procedure can be used to obtain the finite results. Probably the most natural and convenient scheme was proposed by 't Hooft/4/. It is manifestly gauge invariant and leads to a new form of renormalization group equations /5, 6, 7/. Some interesting properties of these equations will be considered later.

The proof of Bogoliubov-Parasiuk's theorem /8/ for 't Hooft's scheme is given in /3,9/, where also the following important fact (which is the necessary condition of renormalizability) is established: all R-operation counterterms have only the polynomial dependence on the external momenta. Consequently, the renormalization constants do not depend on the ratios of external momenta at all $^{/6/}$, and we can simplify the calculations by setting some of the external momenta to be zero $^{/10/}$. It is shown in paper/11/ that the counterterms are also polynomials in masses if there is no normal ordering in the theory. It results in the mass-independence of all renormalization constants, all the masses being renormalized multiplicatively. However in the present paper only the asymptotic form of renormalization group equations is studied so that all masses can be put equal to zero from the very beginning. III. Renormalization group equation in 't Hooft's scheme.

It is necessary now to write down the basic formulas of

't Hoort's approach to clarify the definitions which will be used in the following section. Consider the two-charge theory with a gauge field. The renormalizations look as follows:

$$\begin{split} h_{B} &= \left(\mu^{2}\right)^{\mathcal{E}} \left(h + \sum_{\substack{\nu=1 \\ \nu=1}}^{\infty} \frac{\alpha_{\nu}(h, g)}{\varepsilon^{\nu}}\right) , \\ g_{B} &= \left(\mu^{2}\right)^{\mathcal{E}} \left(g + \sum_{\substack{\nu=1 \\ \nu=1}}^{\infty} \frac{\beta_{\nu}(h, g)}{\varepsilon^{\nu}}\right) , \\ \alpha_{B} &= \alpha \left(1 + \sum_{\substack{\nu=1 \\ \nu=1}}^{\infty} \frac{\alpha_{\nu}(h, g, x)}{\varepsilon^{\nu}}\right) , \\ \mathcal{Z} &= 1 + \sum_{\substack{\nu=1 \\ \nu=1}}^{\infty} \frac{\alpha_{\nu}(h, g, x)}{\varepsilon^{\nu}} , \end{split}$$
(1)

where subscript "B" refers to unrenormalized quantities, \measuredangle is the gauge parameter, $\mathcal{E} = \frac{4-n}{2}$ and n is the "dimension of space--time". The functions a_r and b_r are independent of \measuredangle /12/. Note that the renormalization constants of the propagators are, as usual, denoted by \mathbb{Z}^{-1} . The renormalized Green function is

$$\Gamma_{R}\left(\frac{\kappa^{2}}{\mu^{2}}, h, g, \star\right) = \lim_{\varepsilon \to 0} \mathbb{Z}_{\Gamma}\left(h, g, \star, \varepsilon\right) \Gamma_{B}\left(\kappa^{2}, h_{B}, g_{B}, \star_{B}, \varepsilon\right). \tag{2}$$

One can obtain the Ovsiannikov equation of the type (I.14) by differentiating (2) with respect to \mathcal{M}^2 , $\left(\mathcal{M}^2 \frac{\partial}{\partial h^2} + \beta_h (h, g) \frac{\partial}{\partial h} + \beta_g (h, g) \frac{\partial}{\partial g} + \delta(h, g, x) d \frac{\partial}{\partial x} - V_r(h, g, x) \right) \int_{\mathcal{R}} \left(\frac{k^2}{h^2}, h, g, x \right) = 0,$ where $\beta_h(h, g) \equiv \lim_{E \to 0} \frac{\partial h}{\partial \ln \mu^2} = \left(h \frac{\partial}{\partial h} + g \frac{\partial}{\partial g} - 1 \right) \partial_1 (g, h),$ $\beta_g(h, g) \equiv \lim_{E \to 0} \frac{\partial g}{\partial \ln \mu^2} = \left(h \frac{\partial}{\partial h} + g \frac{\partial}{\partial g} - 1 \right) \delta_1 (g, h),$ $\delta(h, g, x) \equiv \frac{\partial \ln x}{\partial \ln \mu^2} = \left(h \frac{\partial}{\partial h} + g \frac{\partial}{\partial g} \right) d_1 (h, g, x),$ $V_r(h, g, x) \equiv \frac{\partial \ln Z_r}{\partial \ln \mu^2} = -\left(h \frac{\partial}{\partial h} + g \frac{\partial}{\partial g} \right) C_1 (h, g, x),$ (3)

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where all differentiations are carried out with h_B , g_B and d_B (as well as κ^2 and ϵ) fixed. The functions a, b, c and d can be determined uniquely from (2) order by order in h and g. However we shall evaluate the renormalization group functions (3) using R-operation of $^{/3,5/}$, because it is completely equivalent to 't Hooft's prescription. The functions (3) do not depend on the ratios of external momenta. Therefore, only a certain part of each diagram which is independent of external momenta can contribute to the renormalizations (1). Given any diagram G, we denote this part by $\kappa R^{'}G$,

$$RG = (1 - K)R'G$$

where \mathcal{R} is a symbol of R-operation in the sense of $^{/3,9/}$ and the operator K keeps only the pole terms in the Laurent series in \mathcal{E} . In other words, the operation \mathbb{R}' performs subtraction of all the subgraphs of \mathcal{G} but does not subtract the diagram \mathcal{G} as a whole. The $\mathcal{K}\mathcal{R}'\mathcal{G}$ is the polynomial in the external momenta, so that in the case of logarithmically divergent diagram it does not depend on them at all. This is valid also in the case of linear divergence if we introduce the trivial momentum factor prescribed by the corresponding Feynman rule into the definition of $\mathcal{K}\mathcal{R}'$, for example, $\mathcal{R}\mathcal{G}_{\mathcal{A}} = (\mathcal{I} - \mathcal{K})\mathcal{I}_{\mathcal{A}}\mathcal{K}'\mathcal{G}$. We can represent $\mathcal{R}\mathcal{G}$ in the following form:

$$RG(\rho) = G(\rho) - \Sigma' \frac{A}{\varepsilon^{m}} G'(\rho) - \Sigma' \frac{B}{\varepsilon^{\ell}} = R'G(\rho) - \Sigma' \frac{B}{\varepsilon^{\ell}} , \qquad (4)$$

where G'(p) is the diagram G with some of its subgraphs contracted into a point. Let R'/ be the result of acting of R' upon each diagram of the Green function $\int (R' acts upon tree diagrams and one-$ -loop diagrams as the unity operator). Comparing (4) with (2) yieldsthe required formula to calculate the renormalizations:

$Z_{\Gamma} = 1 - K R' \Gamma .$

This relationship allows us to choose the external momenta in the most convenient manner, for instance, setting some of them equal to zero.

IV. Two-loop calculations in 't Hooft's scheme.

In this section the results of two-loop calculations of the renormalizations (1) and renormalization group functions (3) are presented.

A. Scalar electrodynamics

Consider the interaction of photon A_{μ} and of a scalar isodoublet φ_1 , φ_2 by the Lagrangian

$$\mathcal{L}_{int} = e A_{p} \left(\varphi_{1} \partial_{\mu} \varphi_{2} - \varphi_{2} \partial_{\mu} \varphi_{3} \right) + \frac{e^{2}}{2} A_{\mu} A_{p} \left(\varphi_{1}^{2} + \varphi_{2}^{2} \right) - \frac{h}{4!} \left(\varphi_{1}^{2} + \varphi_{2}^{2} \right)^{2}.$$

The calculations have been performed in a general gauge with the photon propagator chosen in the form

$$\sim -\frac{i}{\kappa^2} \left(\frac{q}{q_{\mu\nu}} + \left| d - 1 \right| \frac{K_{\mu} K_{\nu}}{K^2} \right),$$

where d' is the gauge parameter. We use the following notation for the Green functions and their renormalization constants

Hence for the coupling constants we find

$$e_B^2 = (\mu^2)^{\varepsilon} e^2 Z_{\Gamma_s}^2 Z_{\Delta}^{-2} Z_{\Sigma}^{-1} , \quad h_B = (\mu^2)^{\varepsilon} h Z_{\Box} Z_{\Delta}^{-2}$$

Taking into account the Ward identities

$$Z_{\Gamma_{ij}} = Z_{\Gamma_{ij}} = Z_{\Delta}$$

we obtain

$$e_B^2 = (\mathcal{A}^2)^{\varepsilon} e^2 \mathcal{Z}_{\mathcal{A}}^{-1}$$

Thus we need only the renormalizations of ∂ , Δ and \square . To simplify the calculations the momentum arguments of the Green function \square are chosen in the following way



so that we obtain the propagator-type integral

$$\int \frac{d\rho \, dq \, f(\rho, q, \kappa)}{\rho^2 \, q^2 \, (\rho-q)^2 \, (\kappa-\rho)^2 \, (\kappa-q)^2} ,$$

where $f(\rho, q, \kappa) = a\kappa^2 + b\rho^2 + cq^2 + d(\kappa-\rho)^2 + e(\kappa-q)^2 + f(\rho-q)^2$
with $a, b...$ being the numerical constants. The integral associated
with the first term in $f'(\rho, q, \kappa)$ is convergent and has to be omit-
ted while the others can easily /10/ be evaluated. The results of
two-loop calculations are given below

$$\begin{aligned} e_{B}^{2} &= \left(\mu^{2}\right)^{\varepsilon} \left(e^{2} + \frac{e^{4}}{3\varepsilon(4\pi)^{2}} + \frac{e^{6}}{(4\pi)^{4}} \left(\frac{1}{9\varepsilon^{2}} + \frac{2}{\varepsilon}\right)\right) ,\\ \beta_{e}\left(e^{2}\right) &= \frac{1}{3} \frac{e^{4}}{(4\pi)^{2}} + 4 \frac{e^{6}}{(4\pi)^{4}} ,\\ d_{B} &= d\left(1 - \frac{e^{2}}{3\varepsilon(4\pi)^{2}} - \frac{2e^{4}}{\varepsilon(4\pi)^{4}}\right) ,\\ \delta'(d, e^{2}) &= -\frac{1}{3} \frac{e^{2}}{(4\pi)^{2}} - 4 \frac{e^{4}}{(4\pi)^{4}} .\end{aligned}$$

These functions get contributions only from diagrams of the photon propagator. The cancellation of the corrections to its longitudinal part was directly confirmed. Note that the second term in $oldsymbol{eta}_{e}$ has the same sign as the first, like in the case of spinor electrodynamics. The remaining results are listed below $Z_{A}^{-1} = 1 - \frac{e^{2}(3-d)}{\varepsilon^{1/4}\varepsilon^{1/2}} + \frac{e^{4}}{\varepsilon^{2/4}\varepsilon^{1/4}} \left(\frac{d^{2}}{2} - 3d + 4\right) + \frac{1}{\varepsilon^{4}(4\varepsilon)^{4}} \left(\frac{5}{3}e^{4} + \frac{h^{2}}{18}\right),$ $Z_{D} = 1 + \frac{1}{\varepsilon(4\pi)^{2}} \left(\frac{5}{3}h - 2de^{2} + 18\frac{e^{4}}{h} \right) + \frac{1}{s^{2}/(\varepsilon)^{4}} \left(\frac{25}{9}h^{2} - he^{2} \left(\frac{10}{3}d + 5 \right) \right) +$ $+e^{4}(2d^{2}+30)-12\frac{e^{6}}{h}(3d-5)\Big|-\frac{1}{E(4\pi)^{4}}\left(\frac{16}{9}h^{2}-\frac{14}{3}he^{2}-23e^{4}+104\frac{e^{6}}{h}\right),$ $\beta_{h}(e^{2},h) = \frac{1}{(4\pi)^{2}} \left(\frac{5}{3}h^{2} - 6he^{2} + 18e^{4} \right) + \frac{1}{(4\pi)^{4}} \left(-\frac{10}{3}h^{3} + \frac{28}{3}he^{2} + \frac{158}{3}he^{4} - 208e^{6} \right).$ Just as it was expected /10,12,13/ both the Gell-Mann-Low functions have appeared to be independent of the gauge parameter. As have been mentioned there is no dependence of the ratios of the exter nal momenta in the above equations as well. We are now in a position to write down the Ovsiannikov equations (in two-loop approximation) for the Green functions and invariant charges of scalar electrodynamics in 't Hooft's approach. Using the normalization conditions one can represent the solutions to these equations as the perturbation series in "the effective coupling constants"/5/. However the most convenient form of renormalization group equations seems to be the Lie form, so it is attractive to proceed in the following way. From the two-loop Ovsiannikov equations and one-loop normalization functions (calculated for the particular momentum dependence of the Green function under consideration) one obtains two-loop Lie equations using the conversion formulas of Ref.1. The Lie equations of the form (I.9), (I.15) are valid for

an arbitrary renormalization scheme, because the function Ψ_r does not vary from one scheme to another. For instance, one can solve these equations in the frame of λ -scheme, which is attractive by the triviality of the normalization conditions. However to simplify the calculations one can choose to work with the equations in the form (I.9a), (I.15a).

It should be noted that $\beta_e(e^2)$ in two-loop approximation coincides with the corresponding function $f_e(e^2)$ of the Lie equation. It is a consequence of h -independence of β_e . Hence the function $f_e(e^2)$ has no zeros outside the origin, so there occurs the well--known ghost-pole trouble in two-loop scalar electrodynamics.

B. Pseudoscalar Yukawa interaction.

The interaction Lagrangian of termion and pseudoscalar boson fields is

$$\mathcal{L}_{int} = g \overline{\Psi} \delta_{s} \varphi \Psi - \frac{h}{4!} \varphi^{4} .$$

We use the following notation. Z_{g}^{-1} and Z_{f}^{-1} are the renormalizations of boson and fermion propagators respectively, $Z_{f'}$ and Z_{D} are the renormalizations of three and four-point vertices, so that $g_{B}^{2} = [f_{f'}]^{E} g^{2} Z_{f'}^{2} Z_{g}^{-1} Z_{f'}^{-2}$ and $h_{B} = [f_{f'}]^{E} h Z_{D} Z_{g}^{-2}$ are charge renormalizations. The two-loop calculations result in the following

$$\begin{split} \overline{Z}_{B}^{-1} &= 1 + \frac{2g^{2}}{\varepsilon^{(1+\epsilon)^{2}}} + \frac{\overline{\gamma}g^{4}}{\varepsilon^{2}(1+\epsilon)^{4}} + \frac{1}{\varepsilon^{(1+\epsilon)^{4}}} \left(\frac{h^{2}}{24} - \frac{5}{2}g^{4} \right) , \\ \overline{Z}_{F}^{-1} &= 1 + \frac{g^{2}}{2\varepsilon(1+\epsilon)^{2}} + \frac{11g^{4}}{8\varepsilon^{2}(1+\epsilon)^{4}} - \frac{13g^{4}}{16\varepsilon(1+\epsilon)^{4}} , \\ \overline{Z}_{\Gamma} &= 1 + \frac{g^{2}}{\varepsilon^{\frac{2}{(1+\epsilon)^{2}}}} + \frac{3g^{4}}{\varepsilon^{\frac{2}{(1+\epsilon)^{4}}}} - \frac{1}{2\varepsilon(1+\epsilon)^{4}} \left(\frac{3g^{4}}{3g^{4}} + \frac{g^{2}h}{2} \right) , \end{split}$$

$$\begin{aligned} \Xi_{\Box} &= 1 + \frac{3h}{2\varepsilon(4\pi)^2} - \frac{24g^4}{\varepsilon(4\pi)^2h} + \frac{1}{\varepsilon^2(4\pi)^4} \left(\frac{g}{4}h^2 + 3g^2h - 36g^4 - 72g^6\right) \\ &- 72\frac{g^6}{h} + \frac{1}{\varepsilon(4\pi)^4} \left(-\frac{3}{2}h^2 - 3g^2h + 12g^4 + 96\frac{g^6}{h}\right). \end{aligned}$$

From these, using (3), one can easily obtain the anomalous dimensions. Multiplication of the corresponding renormalization constants allows us to find

$$\begin{split} \mathcal{B}_{g}\left(g^{2},h\right) &= \frac{5g^{4}}{(4\pi)^{2}} + \frac{1}{(4\pi)^{4}} \left(\frac{h^{2}g^{2}}{12} - 2g^{4}h - \frac{57}{4}g^{6}\right), \\ \mathcal{B}_{h}\left(g^{2},h\right) &= \frac{1}{(4\pi)^{2}} \left(\frac{3}{2}h^{2} + 4g^{2}h - 24g^{4}\right) + \frac{1}{(4\pi)^{4}} \left(-\frac{47}{6}h^{3} - 6g^{2}h^{2} + 14g^{4}h + 192g^{6}\right). \\ \end{split}$$

Now we can write the Ovsiannikov equation for the Green functions and then, according to the above prescriptions, transform it into the two-loop Lie equation of (I.15a) -type.

First of all we have to investigate the system

$$\frac{\partial \xi_{g}\left(X,g^{2},h\right)}{\partial \ell_{m} x} = \beta_{g}\left(\xi_{g}\left(X,g^{2},h\right),\xi_{h}\left(X,g^{2},h\right)\right),$$

$$\frac{\partial \xi_{h}\left(X,g^{2},h\right)}{\partial \ell_{m} x} = \beta_{h}\left(\xi_{g}\left(X,g^{2},h\right),\xi_{h}\left(X,g^{2},h\right)\right).$$

With the use of the explicit form of β_g and β_h it can be shown that there is only one ultraviolet stable rixed point on the whole phase plane (ξ_g, ξ_h) , namely $\frac{d}{(\xi_H)^2}\xi_g^{\infty} \approx \frac{d}{\xi}$, $\frac{d}{(\xi_H)^2}\xi_h^{\infty} \approx d$. The values ξ_g^{∞} and ξ_h^{∞} are limits of $\xi_g(x, g^2, h)$ and $\xi_h(x, g^2, h)$, respectively as X tends to infinity (see rig.1). Hence a certain Green function $f'(x, g^2, h)$, as it follows from (I.15a), in the asymptotic region behaves as $\chi = \overline{\Psi_r}(\xi_g^{\infty}, \xi_h^{\infty})$. To find the asymptotic behaviour of the boson and fermion propagators we are to evaluate the one-loop contribution to the corresponding $p(g^2, h)$ and to use (I.16). The calculations result rinally in



Fig. 1. The phase plane of the Yukawa model in two-loop approximation. The arrows indicate the direction of the momentum increase. $\mathcal{D}_{\mathcal{B}}(\kappa^{4}) \sim (\kappa^{2})^{-0.6} , \quad \mathcal{D}_{\mathcal{F}}(\kappa^{2}) \sim (\kappa^{2})^{-0.2}$ The two-loop approximation of the Yukawa theory have already been considered in paper /14/ where the Feynman cut-off method was used and the asymptotic form of a one-parameter set of Green functions was calculated. The momentum dependence of each member of the family was characterized by a parameter \mathcal{A} . The number of fixed points was found to change with changing \mathcal{A} . One can see from the conversion formulas that in the non-perturbative approach the number of zeros of the Gall-Mann-Low function cannot change, but in any given order it can, as was directly observed by the authors of paper /14/. The conversion formulas may be used to compare the results of the calculations in /14/ with those of the present paper. We only notice that the expression (5) must not depend on the renormalisation scheme used so that the conversion formulas cannot explain the discrepancy between (5) and the analogous equation in /14/.

It has been mentioned, that the equations $\mu^2 \frac{\partial \mu^2}{\partial h^2} = \beta_0 \left(g^2, h\right)$ and $\mu^2 \frac{\partial h}{\partial h^2} = \beta_h \left(g^2, h\right)$ describe the change of g^2 and h as μ^2 varies, all observable quantities remaining constant. It is interesting that in the Yukawa theory (in contrast with scalar electrodynamics, for instance) there are two solutions of these equations which pass through the origin of the phase plane of the charges g^2 and h. Hence suggesting that the quartic meson interaction is generated by the Yukawa fermion-meson coupling, we can consider h to be dependent on g^2 . This dependence is actually given by the solution, passing through the origin. The first two terms in the expansion of h in powers of g^2 can be obtained from (5) and (6):

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$$h(g^{2}) = \lambda g^{2} + \beta g^{4} + O(g^{6}),$$

$$\alpha = \frac{1 \pm 145}{3}, \quad \beta = \frac{\frac{35}{26} \alpha^{3} + 2\lambda^{2} - \frac{443}{6} \alpha - 96}{3(\alpha - 2)}$$

C. The Wess and Zumino model

The Lagrangian of this supersymmetric model is $^{/15/}$ $\mathcal{L}_{int} = g \overline{\Psi} \mathcal{J}_5 \mathcal{B} \Psi - g \overline{\Psi} \mathcal{A} \Psi - \frac{g^2}{2} (\mathcal{A}^2 + \mathcal{B}^2)^2$, where Ψ is a Majorana spinor, \mathcal{A} and \mathcal{B} are scalar and pseudoscalar boson fields respectively. The notation of renormalizations is the same as of the previous example. The calculation yields

$$Z_{\Gamma} = 1 ,$$

$$Z_{B}^{-1} = Z_{F}^{-1} = Z_{D} = 1 + \frac{4g^{2}}{\epsilon(4\pi)^{2}} + \frac{32g^{4}}{\epsilon^{2}(4\pi)^{4}} - \frac{16g^{4}}{\epsilon(4\pi)^{4}} ,$$

$$\beta(g^{2}) = 12 \frac{g^{4}}{(4\pi)^{2}} - 96 \frac{g^{6}}{(4\pi)^{4}} .$$

First of all, we see that the ward identities /16/ have proved to be valid at the two-loop level. Besides that there is an ultraviolet stable fixed point in this theory due to the existence of zero in $\beta(q^2)$ at the point $\frac{1}{(4\pi)^2} g_{\infty}^2 = \frac{1}{g}$. The above expression for $\beta(q^2)$ is applicable to an arbitrary scheme of renormalizations. Therefore the was and Zumino model analyzed in the two-loop approximation exhibits the finite asymptotic behaviour. The invariant charge is a product of the propagators. Hence one can find, using the ward identity $\mathcal{D}_{\beta}(\kappa^2) = \mathcal{D}_{F}(\kappa^2)$, that the asymptotic behaviour of the Green functions is also finite. Indeed, $\overline{g}^2 \sim \mathcal{D}_{F}^2 \mathcal{D}_{B}^{=}$ $= \mathcal{D}_{F}^3 = \mathcal{D}_{B}^3$, so that $\mathcal{D} \rightarrow \text{const}$ as $\overline{g}^2 \rightarrow g_{\infty}^2$, and similarly for the function \square . In other words the corresponding Green function obeys the equality $\Psi(g_{\infty}^2=0)$. In both the Yukawa-type models considered above, by analogy with the scalar electrodynamics, we put some external momenta equal to zero to simplify the calculations of the diagrams. The only requirement on the choice of the momentum dependence of the given diagram is to prevent the appearance of the spurious infra-red divergences. It is easily achieved, for instance, in the following way

It should be noted that it is only 't Hooft's scheme that enables us to make such a simplification without changing the results.

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