> СООБЩЕНИЯ
> ОБЪЕАИНЕННОГО ИНСТИТУТА ЯАЕРНЫХ ИССАЕАОВАНИЙ

АУБНА

C324. 1
V-82

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## APPLICATION

OF 'T HOOFT'S RENORMALIZATION SCHEME TO TWO-LOOP CALCULATIONS

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APPLICATION<br>OF 'T HOOFT'S RENORMALIZATION SCHEME TO TWO-LOOP CALCULATIONS

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Применение перенормировочной схемы' $т$ Хоофіта для
двухпетлевых вычислений
Продемонстрированы несомненные преимушества схемы 'т Хоофта для асимптотических расчетов в ренормализационной группе. Выполнены двухпетлевые вычисления в трех ренормируемых моделях: в скалярной электродинамике, псевдоскалярной юкавской теории и суперсимметричной модели Весса и Зумино.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

## Сообщение Объединенного института ядерных исследований Дубна 1975

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E2 - 8649
Application of 't Hooft's Renormalization Scheme to Two-Loop Calculations
The manifest advantages of 't Hooft's scheme for the asymptotic calculations are demonstrated. The two-loop computations are carried out in three particular models: scalar electrodynamics, pseudoscalar Yukawa theory and supersymmetric model by Wess and Zumino.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

## I. Introduction

In the previous paper $/ 1 /$ the connection between the different renormalization approaches was studied and the conversion iormulas were derived. These iormulas allow us to reconstruct renormalization group functions or any renormalization scheme 1 rom those of 't Hoor't's scheme with the use of only lower-order extra information. A special role of 't Hooft's scheme is based on its remarkable features, which are discussed in Sec.II. The necessary relations of this scheme are presented in Sec.III, where also the most convenient way to perform the R -operation is described Then, in Sec. IV the 't Hooit scheme is used ior two-loop calculations in three renormalizable theories: scalar electrodynamics, pseudoscalar theory with Yukawa-type coupling and supersymmetric Wess-Zumino model.

Note that this paper is a sequel to Rer. 1 and should be read in conjunction with it, especially when the properties of renormalization group equations are concerned. Hereaiter the predix I will refer to equations of Ref. 1.

I wish to express my gratitude to D.V.Shirkov ior interest in this work and helplul discussions.

## II._-_t Hooit's gcheme ofrenormalization

The dimensional regularization method / 2/ has generally been recognized mainly because of its remarkable property to maintain the initial aymmetry of the Lagrangian in the regularized expressions. The proof of this property, though not for the most general case, can be found in $/ 2,3 /$. The different kinds of subtraction procedure can be used to obtain the finite results. Probably the most natural and convenient scheme was proposed by 't Hoof $\mathrm{t} / 4 /$. It is manifestly gauge invariant and leads to a new form of renormalization group equations $/ 5,6,7 /$. Some interesting properties of these equations will be considered later.

The proof of Bogoliubov-Yarasiuk's theorem /8/for 't Hooft's scheme is given in 13,9/, where also the following important fact (which is the necessary condition of renormalizability) is established: all R-operation counterterme have only the polynomial dependence on the external momenta. Consequently, the renormalization constants do not depend on the ration of external momenta at all $/ 6 /$, and we can simplify the calculations by setting some of the external momenta to be zero/10/. It is ahown in paper/11/ that the counterterms are also polynomials in masses if there is no normal ordering in the theory. It results in the mass-independence of all renormalization constants, all the masses being renormalized multiplicatively. However in the present paper only the asymptotic torm of renormalization group equations is atudied so that all masses can be put equal to zero from the very beginning. III._Renormalization_Group_equation_in_it_Hooft'sagcheme.

It is necessary now to write down the basic iormulas of
't Hoon't's approach to clarify the deinitions which will ve used in the following section. Consider the two-craree theory with a gauge rield. Ihe renormalizations look as follows:

$$
\begin{align*}
& h_{B}=\left(\mu^{2}\right)^{\varepsilon}\left(h+\sum_{v=1}^{\infty} \frac{a_{v}(h, g)}{\varepsilon^{\nu}}\right) \\
& g_{B}=\left(\mu^{2}\right)^{\varepsilon}\left(g+\sum_{v=1}^{\infty} \frac{b_{v}(h, g)}{\varepsilon^{v}}\right) \\
& \alpha_{B}=\alpha\left(1+\sum_{v=1}^{\infty} \frac{d_{v}(h, g, \alpha)}{\varepsilon^{v}}\right) \\
& z=1+\sum_{v=1}^{\infty} \frac{c_{v}(h, g, \alpha)}{\varepsilon^{v}} \tag{1}
\end{align*}
$$

where subscript "B" relers to unreriomalized quantit:es, $\alpha$ is the gauge parameter, $\varepsilon=\frac{4-n}{2}$ and $n$ is the "dimension of space--time". The functions $a_{r}$ and $b_{r}$ are independent of $\alpha / 12 /$. Note that the renormalization constants of the propacators are, as usual, denoted by $Z^{-1}$. the renormalized Green function is

$$
\begin{equation*}
\Gamma_{R}\left(\frac{k^{2}}{\mu^{2}}, h, g, \alpha\right)=\lim _{\varepsilon \rightarrow 0} z_{\Gamma}(h, g, \alpha, \varepsilon) \Gamma_{B}\left(\kappa^{2}, h_{B}, g_{B}, \alpha_{B}, \varepsilon\right) . \tag{2}
\end{equation*}
$$

One can obtain the Ovsiannikov equation oi the type (I.14) by dirferentiatinc (2) with respect to $\mu^{2}$,
$\left(\mu_{\text {where }}^{2} \frac{\partial}{\partial \mu^{2}}+\beta_{h}(h, g) \frac{\partial}{\partial h}+\beta_{g}(h, g) \frac{\partial}{\partial g}+\delta(h, g, \alpha) \alpha \frac{\partial}{\partial \alpha}-\gamma_{r}(h, g, \alpha) / \Gamma_{R}\left(\frac{k^{2}}{\mu^{2}}, h, g, \alpha\right)=0\right.$, $\beta_{h}(h, g) \equiv \lim _{\varepsilon \rightarrow 0} \frac{\partial h}{\partial \ln \mu^{2}}=\left(h \frac{\partial}{\partial h}+g \frac{\partial}{\partial g}-1 / a_{1}(g, h)\right.$,
$\beta g(h, g) \equiv \lim _{\varepsilon \rightarrow 0} \frac{\partial g}{\partial \ln \mu^{2}}=\left(h \frac{\partial}{\partial h}+g \frac{\partial}{\partial g}-1\right) b_{1}(g, h)$,
$\delta(h, g, \alpha) \equiv \frac{\partial \ln \alpha}{\partial \ln \mu^{2}}=\left(h \frac{\partial}{\partial h}+g \frac{\partial}{\partial g}\right) d_{1}(h, g, \alpha)$,
$\gamma_{\Gamma}(h, g, \alpha) \equiv \frac{\partial \ln Z_{\Gamma}}{\partial \ln \mu^{2}}=-\left(h \frac{\partial}{\partial h}+g \frac{\partial}{\partial g}\right) c_{1}(h, g, \alpha)$,
where all diriferentiations are carried out with $h_{B}, g_{B}$ and $\alpha_{B}$ (as well as $K^{2}$ and $\varepsilon$ ) Iixed. The iunctions $a, b, c$ and $d$ can be determined uniquely $\operatorname{ram}(2)$ order by order in $h$ and $g$. However We shall evaluate the renormalization group functions (3) using R-operation $01 / 3, y /$, because it is completely equivalent to 't Hoort's prescription. The functions (3) do not depend on the ratios or external momenta. Therefore, only a certain part of each diagram which is independent of external momenta can contribute to the renormalizations (1). Given any diagram $G$, we denote this part by $K R^{\prime} G$,

$$
R G=(1-K) R^{\prime} G,
$$

where $R$ is a symbol of R-operation in the sense of $13,9 /$ and the operator $K$ keeps only the pole terms in the Laurent series in $\mathcal{E}$. In other words, the operation $R^{\prime}$ performs subtraction of all the subgraphs oi $\mathcal{G}$ vut does not subtract the diagram $\mathcal{G}$ as a whole. The $K R^{\prime} G$ is the polynomial in the external momenta, ao that in the case of logarithmically divergent diagram it does not depend on them at all. This is valid also in the case of linear divergence i1 we introduce the trivial momentum factor preacribed by the corresponding Feynman rule into the delinition of $K R^{\prime}$, for example, $R G G_{\mu}=(1-k) \rho_{\mu} R^{\prime} G$. We can ropresent $R G$ in the following rorm:
$R G(p)=G(p)-\sum \frac{A}{\varepsilon^{m}} G^{\prime}(\rho)-\sum \frac{B}{\varepsilon^{l}}=R^{\prime} G(\rho)-\sum \frac{B}{\varepsilon^{l}}$,
where $G^{\prime}(\rho)$ is the diagram $G$ with some or its subgraphs contracted into a point. Let $R^{\prime} \Gamma$ be the result of acting of $R^{\prime}$ upon each diagram of the Green tunction $\Gamma\left(R^{\prime}\right.$ acts upon tree diagrame and one--loop diegrame as the unity operator). Comparing (4) with (2) yields the required iormula to calculate the renormalizations:

$$
Z_{\Gamma}=1-K R^{\prime} \Gamma
$$

This relationship allows us to choose the external momenta in the most convenient manner, for instance, setting some of them equal to zero.

## IV._Two-loop_calculations_in 'tiHooft's_scheme.

In this section the resulta of two-loop calculations of the renormalizations (1) and renormalization group iunctions (3) are presented.

## A._Scalar_electrocyynamica

Consider the interaction of photon $A_{\mu}$ and of a scalar isodoublet $\varphi_{1}, \varphi_{2}$ by the Lagrangian
$\mathcal{L}_{\text {int }}=e A_{\mu}\left(\varphi_{1} \partial_{\mu} \varphi_{2}-\varphi_{2} \partial_{\mu} \varphi_{1}\right)+\frac{e^{2}}{2} A_{\mu} A_{\mu}\left(\varphi_{1}^{2}+\varphi_{2}^{2}\right)-\frac{h}{4_{!}}\left(\varphi_{1}^{2}+\varphi_{2}^{2}\right)^{2}$.
The calculations have been performed in a general gauge with the photon propagator chosen in the form

$$
-\frac{i}{k^{2}}\left(g_{\mu v}+(d-1) \frac{k_{\mu} k_{v}}{k^{2}}\right)
$$

where $d$ is the gauge parameter. We use the following notation for the Green functions and their renormalization constants

| м四 | 2 | $D_{R}=z_{D}^{-1} D_{B}$ |
| :---: | :---: | :---: |
| $\varphi_{1}$ | $\triangle$ | $\Delta_{R}=z_{\Delta}^{-1} \Delta_{B}$ |
|  | $\Gamma$ | $\Gamma_{3 R}=z_{\Gamma_{3}} \Gamma_{38}$ |
|  | $\Gamma_{4}$ | $\Gamma_{4 R}=z_{\Gamma_{4}} \Gamma_{4 B}$ |
|  | $\square$ | $\square_{R}=Z_{\square} \square_{8}$ |

Hence Ior the coupling constants we find

$$
e_{B}^{2}=\left(\mu^{2}\right)^{\varepsilon} e^{2} z_{r_{3}}^{2} z_{\Delta}^{-2} z_{D}^{-1}, \quad h_{B}=\left(\mu^{2}\right)^{\varepsilon} h z_{\square} z_{\Delta}^{-2}
$$

Taking into account the Ward identities

$$
z_{\Gamma_{4}}=z_{\Gamma_{3}}=z_{\Delta}
$$

we obtain

$$
e_{B}^{2}=\left(\mu^{2}\right)^{\varepsilon} e^{2} z_{D}^{-1}
$$

Thus we need only the renormalizations of $D, \Delta$ and $\square$. To simplii'y the calculations the momentum arguments of the Green function $\square$ are chosen in the following way

so that we obtain the propagator-type integral

$$
\int \frac{d p d q f(p, q, k)}{p^{2} q^{2}(p-q)^{2}(k-p)^{2}(k-q)^{2}}
$$

where $\quad f(p, q, k)=a k^{2}+b p^{2}+c q^{2}+d(k-p)^{2}+e(k-q)^{2}+f(p-q)^{2}$ with $a, B \ldots$ being the numerical constants. The integral associated with the lirat term in $F(p, q, k)$ is convergent and has to be omitted while the others can easily $/ 10 /$ be evaluated. The resulta of two-loop calculations are given below
$e_{B}^{2}=\left(\mu^{2}\right)^{\varepsilon}\left(e^{2}+\frac{e^{4}}{3 \varepsilon(4 \pi)^{2}}+\frac{e^{6}}{(4 \pi)^{4}}\left(\frac{1}{9 \varepsilon^{2}}+\frac{2}{\varepsilon}\right)\right)$,
$\beta_{e}\left(e^{2}\right)=\frac{1}{3} \frac{e^{4}}{(4 \pi)^{2}}+4 \frac{e^{6}}{(4 \pi)^{4}}$,
$d_{B}=d\left(1-\frac{e^{2}}{3 \varepsilon(4 \pi)^{2}}-\frac{2 e^{4}}{\varepsilon(4 \pi)^{4}}\right)$,
$\delta\left(d, e^{2}\right)=-\frac{1}{3} \frac{e^{2}}{(4 \pi)^{2}}-4 \frac{e^{4}}{(4 \pi)^{4}}$.

These functions get contributions only from diagrams of the photon propagator. The cancellation or the corrections to its longitudinal part was directly conrirmed. Note that the second term in $\beta_{e}$ has the same sign as the i'irst, like in the case 0. spinor electrodiynamics. The remaining results are listed below $z_{\Delta}^{-1}=1-\frac{e^{2}(3-d)}{\varepsilon(4 \Sigma)^{2}}+\frac{e^{4}}{\varepsilon^{2}(4 \Sigma)^{4}}\left(\frac{d^{2}}{2}-3 d+4\right)+\frac{1}{\varepsilon(4 \Sigma)^{4}}\left(\frac{5}{3} e^{4}+\frac{h^{2}}{18}\right)$, $Z_{\square}=1+\frac{1}{\varepsilon(4 x)^{2}}\left(\frac{5}{3} h-2 d e^{2}+18 \frac{e^{4}}{h}\right)+\frac{1}{\varepsilon^{2}(4 x)^{4}}\left(\frac{25}{9} h^{2}-h e^{2}\left(\frac{10}{3} d+5\right)+\right.$ $\left.+e^{4}\left(2 d^{2}+30\right)-12 \frac{e^{6}}{h}(3 d-5)\right)-\frac{1}{\varepsilon(4 x)^{4}}\left(\frac{16}{9} h^{2}-\frac{14}{3} h e^{2}-23 e^{4}+104 \frac{e^{6}}{h}\right)$, $\beta_{h}\left(e^{2}, h\right)=\frac{1}{(4 \pi)^{2}}\left(\frac{5}{3} h^{2}-6 h e^{2}+18 e^{4}\right)+\frac{1}{(4 \pi)^{4}}\left(-\frac{10}{3} h^{3}+\frac{28}{3} h^{2} e^{2}+\frac{158}{3} h e^{4}-208 e^{6}\right)$. Just as it was expected $10,12,13 /$ both the Gell-hiann-Low functions have appeared to be independent of the sauge paraneter. As have been mentioned there is no dependence oi the ratios oi the exter nal momenta in the above equations as well. We are now in a position to write down the Ovsiannikov equations (in two-loop approximation) for the Green iunctions and invariant charges or scalar electrodynamics in 't Hooft's approach. Using the normalization conditions one can represent the solutions to these equations as the perturbation series in "the eflective coupling constants"/5/. However the most convenient rorm of renormalization group equations seems to be the Lie iorm, so it is attractive to proceed in the iollowing way. From the two-loop Ovsiannikov equations and one-loop normalization iunctions (calculated for the particular momentum dependence of the Green lunction under consideration) one obtains two-loop Lie equations using the conversion formulas of Ref.1. The Lie equations of the form (I.9), (I.15) are valid for
an arbitrary renormalization scheme, because the runction $\Psi_{\Gamma}$ does not vary irom one scheme to another. For instance, one can solve these ecuations in the irame or $\quad \lambda$-scheme, which is attractive iy the triviality of the normalization conditions. However to gimplily the calculations one can choose to work with the equations in the -orm (I.9a), (I.15a).

It should be noted that $\beta_{e}\left(e^{2}\right)$ in two-loop approximation coincides with the corresponding tunction $f e\left(e^{2}\right)$ of the Lie equation. Itis aconsequence of $h$-independence of $\beta_{e}$. Hence the Iunction $f_{e}\left(e^{2}\right)$ has no zeros outside the origin, so there occurs the well--known 反host-pole trouble in two-loop scalar electrodynamics.

## B._Pseudoscalar_Yukawanteraction.

The interaction Lagrangian of iermion and pseudoscalar boson fields is

$$
\mathscr{L}_{\text {int }}=g \bar{\psi} \gamma_{5} \varphi \psi-\frac{h}{4!} \varphi^{4}
$$

We use the rollowing notation. $Z_{B}^{-1}$ and $Z_{F}^{-1}$ are the renormalizations of boson and fermion propagators respectively, $Z_{\Gamma}$ and the renormalizations of three and four-point vertices, so that $g_{B}^{2}=\left(H^{2}\right)^{\varepsilon} g^{2} z_{\Gamma}^{2} Z_{B}^{-1} Z_{f}^{-2}$ and $h_{B}=\left(h^{2}\right)^{\varepsilon} h Z_{D} Z_{B}^{-2}$ are charge renormalizations. The two-loop calculations result in the following
$Z_{B}^{-1}=1+\frac{2 g^{2}}{\varepsilon\left((4 \pi)^{2}\right.}+\frac{7 g^{4}}{\varepsilon^{2}(\sqrt{2})^{4}}+\frac{1}{\varepsilon(4 \pi)^{4}}\left(\frac{h^{2}}{24}-\frac{5}{2} g^{4}\right)$,
$z_{F}^{-1}=1+\frac{g^{2}}{2 \varepsilon(4 \pi)^{2}}+\frac{11 g^{4}}{8 \varepsilon^{2}(4 \pi)^{4}}-\frac{13 g^{4}}{16 \varepsilon(4 \pi)^{4}}$,
$z_{\Gamma}=1+\frac{g^{2}}{\varepsilon\left(\frac{4}{y}\right)^{2}}+\frac{3 g^{4}}{\varepsilon^{2}(4 \pi)^{4}}-\frac{1}{2 \varepsilon(4 \pi)^{4}}\left(3 g^{4}+g^{2} h\right)$,
$Z_{a}=1+\frac{3 h}{2 \varepsilon(4 \pi)^{2}}-\frac{24 g^{4}}{\varepsilon(4 \pi)^{2} h}+\frac{1}{\varepsilon^{2}(4 \pi)^{4}}\left(\frac{g}{4} h^{2}+3 g^{2} h-36 g^{6}-\right.$
$\left.-72 \frac{g^{6}}{h}\right)+\frac{1}{\varepsilon(4 \pi)^{4}}\left(-\frac{3}{2} h^{2}-3 g^{2} h+12 g^{4}+96 \frac{g^{6}}{h}\right)$.
From these, using (3), one can easily obtain the anomalous dimensions. Wultiplication of the corresponding renormalization constants allows us to find

$$
\begin{equation*}
\beta g\left(g^{2}, h\right)=\frac{5 g^{4}}{(4 \pi)^{2}}+\frac{1}{(4 \pi)^{4}}\left(\frac{h^{2} g^{2}}{12}-2 g^{4} h-\frac{57}{4} g^{6}\right) \tag{5}
\end{equation*}
$$

$\beta h\left(g^{2}, h\right)=\frac{1}{(4 \pi)^{2}}\left(\frac{3}{2} h^{2}+4 g^{2} h-24 g^{4}\right)+\frac{1}{(4 \pi)^{4}}\left(-\frac{17}{6} h^{3}-6 g^{2} h^{2}+14 g^{4} h+\frac{192 g^{6}}{(6)}\right)$.
Now we can write the Ovsiannikov equation for the Green Iunctions and then, according to the above prescriptions, transform it into the two-loop Lie equation of (I. 15a) -type.
First of all we have to investigate the system

$$
\begin{aligned}
& \frac{\partial \xi_{g}\left(x, g^{2} h\right)}{\partial \ln x}=\beta_{g}\left(\xi_{g}\left(x, g^{2}, h\right), \xi_{h}\left(x, g^{2}, h\right)\right), \\
& \frac{\partial \xi_{h}\left(x, g^{2}, h\right)}{\partial \ln x}=\beta_{h}\left(\xi_{g}\left(x, g^{2}, h\right), \xi_{h}\left(x, g^{2}, h\right)\right) .
\end{aligned}
$$

With the use of the explicit form of $\beta_{g}$ and $\beta_{h}$ it can be shown that there is only one ultraviolet stable rixed point on the whole phase plane $\left(\xi_{g}, \xi_{h}\right)$, namely $\frac{1}{\left(4_{\pi}\right)^{2}} \xi_{g}^{\infty} \approx \frac{1}{4}, \frac{1}{\left(\xi_{n}\right)^{2}} \xi_{h}^{\infty} \approx 1$. The values $\xi_{g}^{\infty}$ and $\xi_{h}^{\infty}$ are limits of $\xi_{g}\left(x, g^{2}, h\right)$ and $\xi_{h}\left(x, g^{2}, h\right)$, respectively as $X$ tends to infinity (see rig. 1 ). Hence a certain Green iunction $\Gamma\left(x, g^{2}, h\right)$, as it follows frow ( $I \cdot 15 a$ ), in the asymptotic region behaves as $\left.X^{-} \overline{\psi_{r}}\left(\xi_{g}^{\infty},\right\}_{h}^{\infty}\right)$. To find the asymptotic denaviour of the boson and fermion propagators we are to evaluate the one-loop contribution to the corresponding $P\left(g^{2}, h\right)$ and to use (I.l6). The calculations result ininally in


Pig. 1. The phase plane of the Yukawa model in two-loop approximation. The arrows indicate the direction of the momentum increage.

$$
D_{B}\left(k^{2}\right) \sim\left(k^{2}\right)^{-0.6}, D_{F}\left(k^{2}\right) \sim\left(k^{2}\right)^{-0.2}
$$

The two-loop approximation of the Yukawa theory have already been considered in paper /14/ where the Feymman cut-orf method was used and the asymptotic form of a one-parameter set or Green functions was calculated. The momentum dependence of each member of the family was characterized by a parameter $a$. The number ol fixed points was found to change with changing $a$. One can see from the converaion formulas that in the non-perturbative approach the number of zeros of the Gall-Mann-Low function cannot change, but in any given order it can, as was directly observed by the authors oí paper /14/. The conversion iormulas may be used to compare the results of the calculations in /14/ with those of the present paper. We only notice that the expression (5) must not depend on the renormalisation scheme used so that the conversion formulas cannot explain the discrepancy between (5) and the analogous equation in /14/.

It has been mentioned, that the equations $\mu^{2} \frac{\partial g^{2}}{\partial h^{2}}=\beta_{g}\left(g^{2}, h\right)$ and $\mu^{2} \frac{\partial h}{\partial \mu^{2}}=\beta_{h}\left(g^{2}\right)$ haecribe the change of $g^{2}$ and $h$ as $\mu^{2}$ varies, all observable quantities remaining constant. It is interesting that In the Yukawa theory (in contrast with scalar electrodynamics, for instance ) there are two solutions of these equations which pass through the origin of the phage plane of the charges $g^{2}$ and $h$. Hence suggesting that the quartic meson interaction is generated by the Yukawa fermion-meson coupling, we can consider $h$ to be dependent on $g^{2}$. This dependence is actually given by the solution, passing through the origin. The first two terms in the expansion of $h$ in powers of $g^{2}$ can be obtained from (5) and (6):

$$
\begin{aligned}
& h\left(g^{2}\right)=\alpha g^{2}+\beta g^{4}+o\left(g^{6}\right) \\
& \alpha=\frac{1 \pm \sqrt{145}}{3}, \beta=\frac{\frac{35}{24} \alpha^{3}+2 \alpha^{2}-\frac{113}{8} \alpha-96}{3(\alpha-2)} .
\end{aligned}
$$

## C. The Wess_and_Zumino model

The Lagraneian oi this supersymmetric model is /15/

$$
\mathscr{L}_{\text {int }}=g \bar{\psi} \gamma_{5} B \psi-g \bar{\psi} A \psi-\frac{g^{2}}{2}\left(A^{2}+B^{2}\right)^{2},
$$

where $\psi$ is a liajorana spinor, $A$ and $B$ are scalar and pseudoscalar boson lifelds respectively. The notation of renormalizations is the sane as or the previous example. The calculation yields

$$
\begin{aligned}
& Z_{\Gamma}=1, \\
& Z_{B}^{-1}=Z_{f}^{-1}=Z_{0}=1+\frac{4 g^{2}}{\varepsilon(4 \pi)^{2}}+\frac{32 g^{4}}{\varepsilon^{2}\left((\sqrt{5})^{4}\right.}-\frac{16 g^{4}}{\varepsilon(4 \pi)^{4}}, \\
& \beta\left(g^{2}\right)=12 \frac{g^{4}}{\left(/(\sqrt{2})^{2}\right.}-96 \frac{g^{6}}{(4 \pi)^{4}} .
\end{aligned}
$$

First oi all, we see that the ward identities $/ 16 /$ have proved to be valid at the two-loop level. Besides that there is an ultraviolet stable fixed point in this theory due to the existence of zero in $\beta\left(g^{2}\right)$ at the point $\frac{1}{(4 \pi)^{2}} g_{\infty}^{2}=\frac{1}{8}$. The above expression for $\beta\left(g^{2}\right)$ is applicable to an arbitrary scheme or renormalizations. Thereiore the wess and zumino model analyzed in the two-loop approximation exhibits the finite asymptotjc behaviour. Tne invariant charge is a product oi the propagators. Hence one can find, using the vard identity $D_{B}\left(k^{2}\right)=D_{F}\left(k^{2}\right)$, that the agymptotic behaviour oi the Green iunctions is also tinite. Indeed, $\bar{g}^{2} \sim D_{f}^{2} D_{B}=$ $=D_{F}^{3}=D_{B}^{3}$, so that $D \rightarrow$ const as $g^{2} \longrightarrow g_{\infty}^{2}$, and similarly for the Iunction $\square$. In other words the corresponding Green function obeys the equality $\psi\left(g_{\infty}^{2}\right)=0$. In both the Yukawa-type models considered above, by analogy with the scalar electrocynamics, we
put some external momenta equal to zero to simplify the calculations oi the diagrama. The only requirement on the choice $\mathrm{o}_{i}$ the momentum dependence of the given diagrem is to prevent the appearance of the spurious infra-red divergences. It is easily achieved, I'or instance, in the following way



It should be noted that it is only 't Hoort's scheme that enables us to make such a simplification without changing the results.

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