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SCATTERING OF COMPOSITE PARTICLES AND QUASIPOTENTIAL APPROACH IN QUANTUM FIELD THEORY

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SCATTERING OF COMPOSITE PARTICLES AND QUASIPOTENTIAL APPROACH IN QUANTUM FIELD THEORY

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Рассеяние составных частиц и квазипотенциальный подход в квантовой теории поля

Рассматривается рассеяние составных частиц в рамках квазипотенциального подхода в переменных "светового фронта".

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Scattering of Composite Particles and Quasipotential Approach in Quantum Field Theory

Scattering of composite particles is considered on the basis of the "light-front" form of the quasipotential approach in quantum field theory.

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In connection with the development of composite models of elementary particles a problem of the description of their interactions becomes of special interest.

In the present paper a method for investigation of such problems is proposed. Considerations are based on the quasipotential approach in quantum field theory /1/. This method has been developed and generalized as applied to the problems of constructing the form-factors of composite particles /2-8/.

Below we present a description of the scattering of two composite particles in terms of the "light front" form of quasipotential approach suggested in ref. /7/. It will be shown that some simple assumptions on the hadron interactions in the scattering process allow the results of the dimensional analysis together with quark-structure assumption of hadrons /9/ to be reproduced.

Let us proceed now to the description of these results.

Consider the eight-point Green function

$$G(x_1, x_2, x_3, x_4; y_1, y_2, y_3, y_4) =$$

$$= \langle 0| T(\phi_1(x_1)\phi_2(x_2)\phi_3(x_3)\phi_4(x_4)\phi_1^+(y_1)\phi_2^+(y_2)\phi_3^+(y_3)\phi_4^+(y_4))|0\rangle =$$

$$= [(2\pi)^{-4}]^8 \int \prod_{i=1}^{4} d^4p_i\, d^4q_i \exp[-i\sum_{i=1}^{4}(p_i x_i - q_i y_i)]\, G(p_1, p_2, p_3, p_4, q_1, q_2, q_3, q_4) =$$

$$= [(2\pi)^{-4}]^8 \int d^4P^{(12)} d^4p^{(12)} d^4P^{(34)} d^4p^{(34)} d^4Q^{(12)} d^4q^{(12)} d^4Q^{(34)} d^4q^{(34)} \times$$

$$\times \exp[-i(P^{(12)}X^{(12)} + P^{(34)}X^{(34)} + p^{(12)}x^{(12)} + p^{(34)}x^{(34)} -$$

$$- Q^{(12)}Y^{(12)} - Q^{(34)}Y^{(34)} - q^{(12)}y^{(12)} - q^{(34)}y^{(34)})] \times$$

$$\times G(P^{(12)}, p^{(12)}; P^{(34)}, p^{(34)}; Q^{(12)}, q^{(12)}; Q^{(34)}, q^{(34)}) \ . \qquad (1)$$

In (1) the momenta $P^{(12)}$, $p^{(12)}$, $P^{(34)}$, $p^{(34)}$, $Q^{(12)}$, $q^{(12)}$, $Q^{(34)}$, $q^{(34)}$ are introduced according to the following formulas

$$P^{(12)} = p_1 + p_2, \quad p^{(12)} = \frac{p_1 - p_2}{2}; \quad Q^{(12)} = q_1 + q_2, \quad q^{(12)} = \frac{q_1 - q_2}{2};$$

$$P^{(34)} = p_3 + p_4, \quad p^{(34)} = \frac{p_3 - p_4}{2}; \quad Q^{(34)} = q_3 + q_4, \quad q^{(34)} = \frac{q_3 - q_4}{2}.$$

Passing to the quasipotential description in the spirit of ref. /7/ we introduce the "light front" variables and define the quantity

$$\tilde{\tilde{G}}(P^{(12)}, p_+^{(12)}, \vec{p}_\perp^{(12)}; P^{(34)}, p_+^{(34)}, \vec{p}_\perp^{(34)}; Q^{(12)}, q_+^{(12)}, \vec{q}_\perp^{(12)}; Q^{(34)}, q_+^{(34)}, \vec{q}_\perp^{(34)}) =$$

$$= \int_{-\infty}^{\infty} dp_-^{(12)} dp_-^{(34)} dq_-^{(12)} dq_-^{(34)} G(P^{(12)}, p^{(12)}; P^{(34)}, p^{(34)}; Q^{(12)}, q^{(12)}; Q^{(34)}, q^{(34)}), \qquad (2)$$

where $p_\pm = p_0 \pm p_3$, $\vec{p}_\perp = (p_1, p_2)$.

Introduce now the quantity M by the formula

$$G(P^{(12)}, p^{(12)}; P^{(34)}, p^{(34)}; Q^{(12)}, q^{(12)}; Q^{(34)}, q^{(34)}) =$$

$$= \int d^4p^{(12)'} d^4p^{(34)'} d^4q^{(12)'} d^4q^{(34)'} G_{12}(P^{(12)}; p^{(12)} p^{(12)'}) G_{34}(P^{(34)}; p^{(34)} p^{(34)'}) \times$$

$$\times M(P^{(12)}, p^{(12)'}; P^{(34)}, p^{(34)'}; Q^{(12)}, q^{(12)'}; Q^{(34)}, q^{(34)'}) G_{12}(Q^{(12)}; q^{(12)'}, q^{(12)}) \times$$

$$\times G_{34}(Q^{(34)}; q^{(34)'}, q^{(34)}) \equiv (G_{12} G_{34}) M (G_{12} G_{34}) \ .$$

The quantity $\tilde{\tilde{G}}$ can be presented in the form:[x/]

$$\tilde{\tilde{G}}(P^{(12)}, P^{(34)}, Q^{(12)}, Q^{(34)}) = \qquad (3)$$

$$= G_{12}(P^{(12)}) G_{34}(P^{(34)}) * M(P^{(12)}, P^{(34)}, Q^{(12)}, Q^{(34)}) * G_{12}(Q^{(12)}) G_{34}(Q^{(34)}) \ .$$

The symbol * in the formula (3) has to be understood in the following sense

$$\tilde{\tilde{A}} * \tilde{\tilde{B}} = \int_{-P_+^{(12)}}^{P_+^{(12)}} dp_+^{(12)} \int_{-P_+^{(34)}}^{P_+^{(34)}} dp_+^{(34)} \int d\vec{p}_\perp^{(12)} d\vec{p}_\perp^{(34)} \times$$

$$\times \tilde{\tilde{A}}(\ldots; p_+^{(12)}, \vec{p}_\perp^{(12)}; p_+^{(34)}, \vec{p}_\perp^{(34)}) \tilde{\tilde{B}}(p_+^{(12)}, \vec{p}_\perp^{(12)}; p_+^{(34)}, \vec{p}_\perp^{(34)}) ,$$

and dots correspond to the set of other arguments on which the operators $\tilde{A}$ and $\tilde{B}$ can depend.

Knowing the pole singularities of the two-particle Green functions $\tilde{G}_{12}$, $\tilde{G}_{34}$ one can show that in the vicinity of these poles the function $\tilde{G}$ is written as follows:

$$\tilde{\tilde{G}}(P^{(12)}, P^{(34)}, Q^{(12)}, Q^{(34)}) \cong \qquad (4)$$

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x/ In what follows we shall omit the arguments which are related to the relative moments and this will not cause any misunderstanding.

$$\approx [i(2\pi)^4]^4 \frac{\Psi(P^{(12)})\,\Psi(P^{(34)})\,\Psi^+(Q^{(12)})\,\Psi^+(Q^{(34)})}{(P^{(12)^2}-M_{12}^2)\,(P^{(34)^2}-M_{34}^2)\,(Q^{(12)^2}-M_{12}'^2)(Q^{(34)^2}-M_{34}'^2)} \times$$

$$\times \Psi^+(P^{(12)})\,\Psi^+(P^{(34)}) * \overset{\approx}{M}_{1234}(P^{(12)},P^{(34)},Q^{(12)},Q^{(34)}) * \Psi(Q^{(12)})\,\Psi(Q^{(34)}). \qquad (4)$$

Here

$$\overset{\approx}{M}_{1234} = \lim_{\substack{P^{(12)^2}\to M_{12}^2,\ P^{(34)^2}\to M_{34}^2 \\ Q^{(12)^2}\to M_{12}'^2,\ Q^{(34)^2}\to M_{34}'^2}} G_{12}^{-1}(P^{(12)})\,G_{34}^{-1}(P^{(34)}) *$$

$$* G_{12}G_{34}MG_{12}G_{34}(P^{(12)},P^{(34)},Q^{(12)},Q^{(34)})\,G_{12}^{-1}(Q^{(12)})\,G_{34}^{-1}(Q^{(34)}) .$$

M_{12}, M_{34}, M_{12}', M_{34}' are the masses of the corresponding states.

From formulas (2) and (4) we get the following expression for the scattering amplitude

$$T(P^{(12)},P^{(34)},Q^{(12)},Q^{(34)}) = \Psi^+(P^{(12)})\,\Psi^+(P^{(34)}) * \overset{\approx}{M}_{1234}(P^{(12)},P^{(34)},Q^{(12)},Q^{(34)}) * \Psi(Q^{(12)})\,\Psi(Q^{(34)}). \qquad (5)$$

The formula (5) gives a general expression for the scattering amplitude in the case of scattering of composite particles. The detailed form of the scattering amplitude depends on the interaction mechanism in the intermediate state and on a specific form of the wave functions of the scattered objects. Below we consider two possible mechanisms: 1) scattering via the exchange of some intermediate particle (Fig.1), 2) constituent interchange mechanism (Fig. 2).

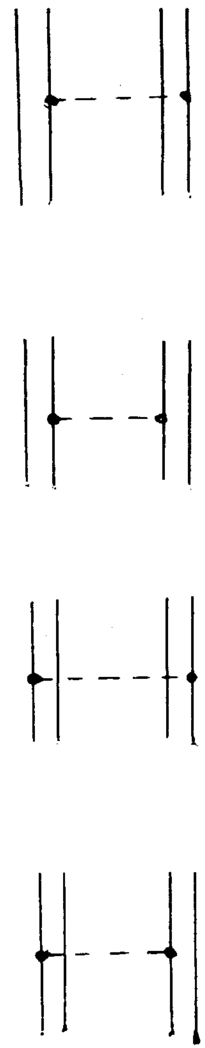

Fig. 1

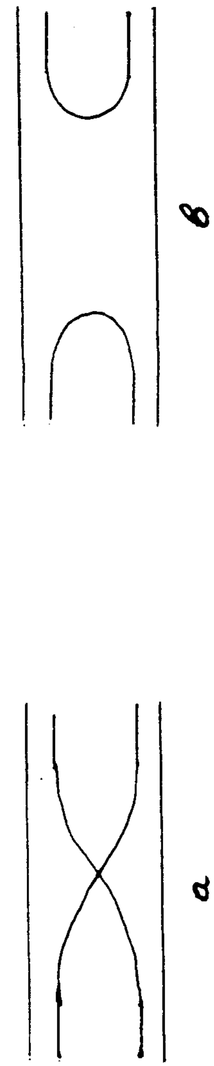

Fig. 2

For the sake of simplicity we consider the case of scalar particles. It can be shown that in the first case in the reference frame, where

$$P_+^{(12)} = P_+^{(34)}, \quad Q_+^{(12)} = Q_+^{(34)},$$

$$t = (P^{(12)} - Q^{(12)})^2 = -\vec{\Delta}_\perp^{(t)2},$$

one has the following result for the scattering amplitude

$$T = \frac{g^2}{4(\vec{\Delta}^{(t)2} + \mu^2)} [F_{12}^{(1)}(\vec{\Delta}_\perp^{(t)}) F_{34}^{(1)}(\vec{\Delta}_\perp^{(t)}) +$$

$$+ F_{12}^{(1)}(\vec{\Delta}_\perp^{(t)}) F_{34}^{(2)}(\vec{\Delta}_\perp^{(t)}) + F_{12}^{(2)}(\vec{\Delta}_\perp^{(t)}) F_{34}^{(1)}(\vec{\Delta}_\perp^{(t)}) +$$

$$+ F_{12}^{(2)}(\vec{\Delta}_\perp^{(t)}) F_{34}^{(2)}(\vec{\Delta}_\perp^{(t)})],$$

where $F_{12}^{(i)}$, $F_{34}^{(i)}$ are scalar form factors of the scattered particles (cf. with /10/), which are expressed in terms of the wave functions with zero transversal components of the total momentum in the following way:

$$F_{12}^{(1)}(\vec{\Delta}_\perp^{(t)}) =$$

$$= \frac{(2\pi)^3}{2} \int_0^1 \frac{dx}{x^2(1-x)} \int d\vec{p}_\perp \, \Phi^+_{\vec{P}_\perp^{(12)}=0}(x, \vec{p}_\perp + (1-x)\vec{\Delta}_\perp^{(t)}) \, \Phi_{\vec{P}_\perp^{(12)}=0}(x, \vec{p}_\perp),$$

$$F_{34}^{(1)}(\vec{\Delta}_\perp^{(t)}) =$$

$$= \frac{(2\pi)^3}{2} \int_0^1 \frac{dx}{x^2(1-x)} \int d\vec{p}_\perp \, \Phi^+_{\vec{P}_\perp^{(34)}=0}(x, \vec{p}_\perp - (1-x)\vec{\Delta}_\perp^{(t)}) \, \Phi_{\vec{P}_\perp^{(34)}=0}(x, \vec{p}_\perp),$$

$$F_{12}^{(2)}(\vec{\Delta}_\perp^{(t)}) =$$

$$= \frac{(2\pi)^3}{2} \int_0^1 \frac{dx}{x(1-x)^2} \int d\vec{p}_\perp \, \Phi^+_{\vec{P}_\perp^{(12)}=0}(x, \vec{p}_\perp - x\vec{\Delta}_\perp^{(t)}) \, \Phi_{\vec{P}_\perp^{(12)}=0}(x, \vec{p}_\perp),$$

$$F_{34}^{(2)}(\vec{\Delta}_\perp^{(t)}) =$$

$$= \frac{(2\pi)^3}{2} \int_0^1 \frac{dx}{x(1-x)^2} \, d\vec{p}_\perp \, \Phi^+_{\vec{P}^{(34)}=0}(x, \vec{p}_\perp + x\vec{\Delta}_\perp^{(t)}) \, \Phi_{\vec{P}^{(34)}=0}(x, \vec{p}_\perp),$$

where for instance

$$\Phi_{P^{(12)}}(x, \vec{p}_\perp) = P_+^{(12)} x(1-x) \, \Psi_{P^{(12)}}(p_+^{(12)}, \vec{p}_\perp^{(12)}).$$

Considering the constituent interchange mechanism (Fig.2a) one gets the following expression for the scattering amplitude

$$T = \frac{-1}{2(2\pi)^3} \int_0^1 \frac{dx}{x^2(1-x)^2} \int d\vec{p}_\perp \, \Phi^{+(12)}_{\vec{P}_\perp=0}(x, \vec{p}_\perp - x\vec{\Delta}_\perp^{(u)} + (1-x)\vec{\Delta}_\perp^{(t)}) \times$$

$$\times \Phi^{+(34)}_{\vec{P}_\perp=0}(x, \vec{p}_\perp)[M_{12}^2 + M_{34}^2 - S(x, \vec{p}_\perp + x\vec{\Delta}_\perp^{(t)} - (1-x)\vec{\Delta}_\perp^{(u)}) - S(x, \vec{p}_\perp)] \times$$

$$\times \Phi^{(12)}_{\vec{P}_\perp=0}(x, \vec{p}_\perp - x\vec{\Delta}_\perp^{(u)}) \, \Phi^{(34)}_{\vec{P}_\perp=0}(x, \vec{p}_\perp + (1-x)\vec{\Delta}_\perp^{(t)});$$

$$\vec{\Delta}_\perp^{(u)2} = -u. \tag{6}$$

Here the notation has been introduced

$$S(x, \vec{p}_\perp) = \frac{m_1^2 + \vec{p}_\perp^2}{1-x} + \frac{m_2^2 + \vec{p}_\perp^2}{x},$$

and the following properties of wave functions

$$\Phi(x,\vec{p}_\perp) = \Phi(x,-\vec{p}_\perp)$$
$$\Phi(x,\vec{p}_\perp) = \Phi(1-x,\vec{p}_\perp)$$

have been used.

Let us choose now the wave functions of the composite particles in the form

$$\Phi_N(x,\vec{p}_\perp) = \frac{\phi_N(x)}{[S(x,\vec{p}_\perp)]^N}, \quad N = A,B,C,D. \tag{7}$$

A , B and C, D denote the hadrons before and after the scattering, respectively.

Inserting the wave functions (7) into formula (6) for the scattering amplitude one gets in the asymptotic region

$$T \underset{\substack{s\to\infty \\ |t|\to\infty}}{\sim} \frac{1}{s^{A+C+D-1}} \left(\frac{1+z}{2}\right)^{-C} \left(\frac{1-z}{2}\right)^{-D} f(z), \tag{8}$$

where

$$f(z) = \int_0^1 \frac{dx\,\phi_A^+(x)\,\tilde{\phi}_B^+(x)\,\phi_C(x)\phi_D(x)}{[(1-x)^2\frac{1-z}{2} + x^2\frac{1+z}{2}]^A}\left[(1-x)^2\frac{1+z}{2} + x^2\frac{1-z}{2}\right]\times$$

$$\times \frac{[(1-x)]^{A+B+C+D-3}}{x^{2C}(1-x)^{2D}},$$

$$\tilde{\phi}^+(x) = -\frac{(2\pi)^3}{2}\int d\vec{p}_\perp \frac{\Phi_B^+(x,\vec{p}_\perp)}{[x(1-x)]^B},$$

$z = \cos\theta$, , θ is the scattering angle in the c.m.s.

$$-t \simeq \frac{s}{2}(1-z), \quad -u \simeq \frac{s}{2}(1+z).$$

Formula (8) is in close connection with the results of the dimensional analysis of the scattering processes /9/ assuming the quark-structure of hadrons and with those of parton model in the infinite momentum frame /11/.

The contribution to the scattering amplitude from the annihilation diagrams of the type given in Fig. 2b in the lowest order of perturbation theory is identically equal to zero due to the projective properties of the wave functions. Instead of calculating the higher orders one can use the ($s \to u$) crossing symmetry property of the physical scattering amplitude. Thus, imposing this requirement one obtains from formulas (6) and (8) the contribution we are seeking for:

$$T \underset{\substack{s\to\infty \\ |t|\to\infty}}{\sim} \frac{1}{s^{A+C+D-1}}(1-z)^{-D}\tilde{f}(z). \tag{9}$$

Note, that by straightforward calculation of the diagrams of Fig. 2b, assuming the following property for the operator

$$\tilde{M}(P^{(12)},x^{(12)},\vec{p}_\perp^{(12)}:P^{(34)},x^{(34)},\vec{p}_\perp^{(34)}:Q^{(12)},y^{(12)},\vec{q}_\perp^{(12)}:Q^{(34)},y^{(34)},\vec{q}_\perp^{(34)}) =$$

$$= \tilde{M}(P^{(12)},-x^{(12)},\vec{p}_\perp^{(12)}:P^{(34)},-x^{(34)},\vec{p}_\perp^{(34)}:Q^{(12)},-y^{(12)},\vec{q}_\perp^{(12)}:Q^{(34)},-y^{(34)},\vec{q}_\perp^{(34)}),$$

when

$$-1 < x^{(12)} < 0, \quad -1 < x^{(34)} < 0, \quad -1 < y^{(12)} < 0, \quad -1 < y^{(34)} < 0$$

we obtain the result of the type (9).

In this case the function $\tilde{f}(z)$ in (9) is of the form:

$$\tilde{f}(z) = \frac{1}{2^{C+D-A-1}}\int_0^1 \frac{dx\,\phi_A^+(x)\,\tilde{\phi}_B^+(x)\,\phi_C(x)\,\phi_D(x)}{[4x^2+(1-x)^2(1-z)]^{C-1}}\times$$

$$\times \frac{[x(1-x)]^{A+B+C+D-3}}{x^{2A}(1-x)^{2D}}.$$

It is interesting to investigate the dependence of the power of s in formulas (8), (9) on the number and on spins of the constituents, to study in more detail the form of the wave functions of the scattered objects (see in this connection /12/) and to compare the corresponding results with experimental data.

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