объединенный
институт
ядерных
исследований
дубна

E2-86-816

D.I.Kazakov

FINITE N=1 SUSY GAUGE FIELD THEORIES

Submitted to "Intern. Journal of Modern Physics A"

1986

1. INTRODUCTION

In a recent note [1] we propose the method to construct finite N=1 SUSY gauge field theories within dimensional regularization. The present paper contains a more detailed description and development of the method. Here also some formulas of ref. [1] are specified and some useful consequences are obtained.

The advocated approach is a natural continuation and, from a practical point of view, a simplification of the approach to construct finite field theories proposed in our earlier papers [2,3] (see also [4]). We use here the advantages of dimensional regularization.

In refs. [2-4], an arbitrary N=1 SUSY gauge field theory formulated in terms of N=1 superfields is considered. The action contains both the gauge- and Yukawa-type interactions with the couplings g and y_i, respectively. To reduce the number of independent divergences, the background field gauge is used. In this gauge the only uncorrelated divergences are those of the gauge and the chiral superfield propagators. If these propagators are finite, the anomalous dimensions vanish and so do the β-functions. That means that the theory is finite. This can be achieved by:

i) a proper choice of the matter content of the theory, obeying the sum rule eq.(4a) (see below);

ii) a proper choice of renormalized Yukawa couplings according to eigenvalue solutions of renormalization group equations

$$y_R^i = f_0^i g_R + f_1^i g_R^2 + f_2^i g_R^3 + \dots , \qquad (1)$$

where the coefficients f_n^i are determined order by order of perturbation theory.

Necessary and sufficient conditions for finiteness in all orders of perturbation theory coincide with those obtained below (see eq.(22)) and are determined already in the one-loop order.

The advantage of the approach proposed in ref.[1] is that, first, it is direct. There is no necessity to perform renormalization, to calculate the anomalous dimensions and to try to nullify them with the help of eq.(1). Here the vanishing of divergences imposes the relations between the bare couplings directly. A second advantage is that eqs.(8) (see below) contrary to eqs.(1), are linear in g. All the nonlinearity is transformed into that of the parameter of dimensional regularization ε. At the same time the relations between renormalized couplings remain nonlinear, but the renormalization is not necessary.

2. THE FORMALISM. ONE-LOOP FINITENESS

Consider a general renormalizable N=1 SUSY gauge theory formulated in superfields[5] :

$$S=\int d^4x\Big[\int d^2\theta\, d^2\bar\theta\; \bar\Phi_a\left(e^{gV}\right)^a_b \Phi^b - \frac{tr}{g^2 C_G}\int d^2\theta\, W^\alpha W_\alpha + \int d^2\theta\,\frac{1}{3!}\, d_{abc}\,\Phi^a\Phi^b\Phi^c \tag{2}$$

$$+ \text{gauge-fixing} + \text{ghost} + h.c.\Big].$$

A chiral superfield $\Phi^a(x,\theta)$ is in a reducible representation R of the gauge group G. The index a is a multi-index, it runs over an irreducible representation A and members

of a given irreducible representation s, i.e., $a=\{A,s\}$. Here $V^a_b = V^i (R_i)^a_b$ and $(R_i)^a_b = (R^A_i)^s_t$. Matrices of an irreducible representation satisfy the following conditions

$$[R_i, R_j] = i f_{ijk} R_k\,, \quad R^a_{ib} R^b_{ic} = C_A \delta^a_c\,, \tag{3}$$
$$R^a_{ib} R^b_{ja} = \delta_{ij}\sum_A T_A\,, \quad f_{ijk} f_{ljk} = C_G \delta_{il}\,.$$

Action (2) is invariant under G if

$$d_{abc}(R_i)^c_d + d_{dac}(R_i)^c_b + d_{bdc}(R_i)^c_a = 0,$$

where d_{abc} is totally symmetric in a, b and c.

The chiral self interaction may contain all possible singlet combinations of irreducible representations. Picking out of d_{abc} a purely tensorial structure corresponding to a concrete realization of interaction, we get a set of Yukawa couplings Y_i

$$d_{abc} = \sum_i Y_i\, d^i_{ABC}\, d^i_{stu}\,.$$

To analyse the divergences, we use the symmetry properties of action (2), which essentially reduces the renormalization arbitrariness. The gauge invariance connects the gauge vertices and enables us to choose the gauge, the so-called background gauge, where the problem is reduced to the analysis of divergences of a vector propagator[6]. The presence of nonrenormalization theorems[7] in SUSY theories indicating that the counterterms always have a form of the integral $\int d^4\theta$ means the absence of divergences in chiral vertices. Thus the problem is reduced to chiral propagators.

The theory (2) is finite in the one-loop approximation if the following constraints are fulfilled:[8-10]

$$\sum_A T_A = 3C_G\,, \tag{4a}$$

$$S_A^E = \delta_A^E C_A \, , \tag{4b}$$

where S_A^E is defined by

$$d_{abc}\bar{d}^{ebc} \equiv 2 S_a^e g^2 = 2\delta_s^2 S_A^E g^2 . \tag{5}$$

Eq.(4a) ensures the finiteness of the gauge field propagator. It specifies the matter content of the theory. Eqs.(4b) define the Yukawa couplings and provide the finiteness of chiral propagators.

It is remarkable that the one-loop finiteness automatically leads to the absence of divergences at the two-loop level[8,9] but the above conditions are not enough to achieve three- and higher-loop finiteness[11].

In fact, provided eq.(4a) is fulfilled, the only divergence that one should take care of is that of the chiral field propagators. This is a consequence of supersymmetry that the following theorem holds:

T h e o r e m [12] : If N=1 SUSY gauge theory is finite in L loops (i.e.,all the Green functions are finite), the gauge propagator is finite in (L + 1) loops.

This statement was checked by a direct calculation in three-loop approximation[11] , i.e.,for L = 2.

The same fact follows from the explicit expression[13] relating the gauge β-function with the anomalous dimensions of chiral fields is some renormalization scheme

$$\beta_g = g^2 \frac{(\sum_R T_R - 3C_G) - \sum_R \gamma_R T_R}{1 - 2g C_G} . \tag{6}$$

(Hereafter we introduce the natural expansion parameters, so g stands for $g^2/16\pi^2$ and Y_i for $Y_i^2/16\pi^2$ in a usual fashion).

Hence the fulfillment of eq.(4a) reduces the problem of divergences in SUSY gauge theory to that of chiral propagators. Below we show that they can be made finite in all orders of perturbation theory by a proper choice of Yukawa couplings .

3. THE METHOD TO CONSTRUCT A FINITE THEORY[1]

Consider in a multicoupling theory (2) the unrenormalized expressions for the dimensionless propagators of chiral fields in momentum representation after integrating over grassmann variables. The dimensionally regularized expressions are

$$D_i(p^2, g^{Bare}, \{y_j^{Bare}\}, \varepsilon) = 1 + \sum_{n=1}^{\infty} \frac{1}{(p^2)^{n\varepsilon}} \cdot \left[\frac{C^i_{nn}(g,y)}{\varepsilon^n} + \frac{C^i_{nn-1}(g,y)}{\varepsilon^{n-1}} + \dots + \frac{C^i_{n1}(g,y)}{\varepsilon} + C^i_{n0}(g,y) \right], \tag{7}$$

where n is the order of perturbation theory, the dimension of space-time is taken to be D = 4-2ε , and the couplings g and $\{y_j\}$ are always the bare couplings. We omit the superscript "bare" below. Here i = 1,2,..., N , where N is the number of independently renormalized chiral fields, and j = 1,2,..., M , where M is the number of Yukawa couplings. The coefficients C^i_{nk} , $k \geq 2$ are not independent. Due to the renormalizability of the theory they are polynomials of order n and are governed by $\{C^i_{n1}\}$.

Our aim is to get the vanishing of all C^i_{nk} , $k > 0$, properly choosing the Yukawa couplings Y_j . This can be achieved by the following choice

$$Y_j = f_0^j(\varepsilon)\, g , \tag{8}$$

where the function $f_0^j(\varepsilon)$ is expanded in a Taylor series in ε

$$f_0^j(\varepsilon) = \sum_{k\geq 0} f_{0k}^j \varepsilon^k \qquad (9)$$

and the coefficients f_{0k}^j are obtained by perturbation theory.

To prove that eqs.(8) actually solve the problem of finiteness, we consider eqs.(7) order by order in perturbation expansion.

1 loop

$$C_{11}^i(g,y) = -\left(\sum_j B_{1j}^i y_j + B_{10}^i g\right). \qquad (10)$$

Substituting now eqs.(8), (9) into eqs.(10) we demand

$$\sum_j B_{1j}^i f_{00}^j + B_{10}^i = 0 \,. \quad i = 1,2,\dots,N \quad j = 1,2,\dots,M. \qquad (11)$$

This is a system of linear equations with respect to the coefficients f_{00}^j. They have a solution if the rank of the matrix B_1 is equal to N ($M \geq N$). One should also have $f_{00}^j \geq 0$, because y_j and g in eqs.(8) are nonnegative.

We note that the matrix B_1 is the matrix of anomalous dimensions in the one-loop approximation

$$\gamma_i^{(1)}(g,y) = \sum_j B_{1j}^i y_j + B_{10}^i g$$

and plays a crucial role in the approach to finiteness developed in refs.[2-4].

Eqs.(11) are nothing but eqs.(5) and exibit the relation between the gauge and Yukawa couplings in a classical Lagrangian when $\varepsilon = 0$.

2 loops

$1/\varepsilon^2$: The coefficient $C_{22}^i(g,y)$ should vanish when $y_j = f_{00}^j g$ because otherwise we will obtain a nonlocal

divergence of the type $\frac{C_{22}^i}{\varepsilon} \ln p^2$, which is forbidden due to the renormalizatibility of the theory. This divergence cannot be eliminated in the usual fashion because all one-loop counterterms are absent. So $C_{22}^i(g,y)$ should have the form

$$C_{22}^i(g,y) = F_{2j}^i(g,y)\, C_{11}^j(g,y)\,. \qquad (12)$$

Below we will obtain an explicit expression for C_{nn}^i which confirms our statement.

$1/\varepsilon$: We have

$$C_{21}^i(g,y)\Big|_{y=f_{00}g} + \frac{\partial C_{22}^i(g,y)}{\partial y_j}\Big|_{y=f_{00}g} f_{01}^j g \,. \qquad (13)$$

As follows from eq.(13), even if $C_{21}^i(g,y)|_{y=f_{00}g} \neq 0$, one can achieve finiteness properly choosing the coefficients f_{01}^j. It is possible if the matrix $\frac{\partial C_{22}^i}{\partial y_j}$ has a rank equal to N. According to eqs.(10),(12)

$$\frac{\partial C_{22}^i}{\partial y_j}\Big|_{y=f_{00}g} = -F_{2\,l}^{\,i} B_{1j}^l \Big|_{y=f_{00}g}, \qquad (14)$$

so the existence of a solution to eqs.(13) requires that rank $F_2 = N$. We will show below that it is really so.

The algorithm will obviously work in all orders of perturbation theory. All the coefficients C_{nk}^i, $2\leq k\leq n$ vanish due to renormalizability of the theory and absence of lower-order counterterms. To achieve the absence of a simple pole C_{n1}^i, we have at our disposal a set of parameters $f_{0\,n-1}^j$. Actually in n-loops the coefficients of the simple pole is

$$C_{n1}^i(g,y)\Big|_{y=f_{00}g} + \dots + \frac{\partial C_{nn}^i}{\partial y_j}\Big|_{y=f_{00}g} f_{0\,n-1}^j g \,. \qquad (15)$$

Hence again we have a system of linear equations and to get the vanishing of a simple pole, the matrix $\frac{\partial C_{nn}^i}{\partial y_j}$ should have rank equal to N . But analogously to eq.(12) we have

$$C^{i}_{nn}(g,y) = F_{n}{}^{i}_{j}(g,y)\, C^{j}_{11}(g,y) \tag{16}$$

so

$$\left.\frac{\partial C^{i}_{nn}}{\partial y_j}\right|_{y=f_{oo}g} = -F_{n}{}^{i}_{\ell}\, B_{1}{}^{\ell}_{j}\Big|_{y=f_{oo}g} \tag{17}$$

and the problem is reduced to the proof that rank F_n = N.

To prove this statement, we note that $C^{i}_{nn}(g,y)$ for arbitrary g and y_j is equal to $(-)^n C^{i}_{n(n)}(g,y)$, where $C^{i}_{n(n)}$ is an order n term in the expansion of the renormalization constant Z_i 1):

$$Z_i = 1 + \sum_{\nu=1}^{\infty} \frac{C^{i}_{\nu}(g,y)}{\varepsilon^{\nu}}\ , \quad C^{i}_{\nu}(g,y) = \sum_{k=\nu} C^{i}_{\nu(k)} \{g,y\}^{k}.$$

The coefficients $C^{i}_{\nu}(g,y)$ obey the so-called pole equations[14]. The following equation is valid (remind that $\beta^{(1)}_{g} = 0$ due to eq.(4a))

$$n\, C^{i}_{n(n)} = \beta^{(1)}_{y_j} \frac{\partial}{\partial y_j} C^{i}_{n-1(n-1)} + \gamma^{(1)}_{i} C^{i}_{n-1(n-1)} . \tag{18}$$

Eq.(18) enables us to get an explicit expression for F_n (16). It also follows that

$$C^{i}_{nn}(g,y)\big|_{y=f_{oo}g} = 0$$

and

$$\left.\frac{\partial C^{i}_{nn}}{\partial y_j}\right|_{y=f_{oo}g} = (-)^{n}\frac{1}{n!}\Big[B_1 (f_{oo}A_1)^{n-1}\Big]^{i}_{j} \cdot g^{n}, \tag{19}$$

1) We suggest here the diagonal renormalization of fields. However, what follows is valid also in the case of mixing. What is important is the diagonal form of the one loop approximation due to eq.(5).

where

$$f_{oo} = \mathrm{diag}\,(f^{1}_{oo}, f^{2}_{oo}, f^{3}_{oo}, \dots, f^{M}_{oo}) \tag{20}$$

and the matrix A_1 is the matrix of one-loop β -functions

$$\beta^{(1)}_{y_j}(g,y) = y_j \cdot \Big(\sum_{\ell} A^{j}_{1\ell}\, y_\ell + A^{j}_{10}\, g\Big). \tag{21}$$

The rank of f_{oo} is equal to the number of nonzero coefficients f^{j}_{oo} and rank A_1 = rank B_1 because the rows of A_1 are obtained by linear combitations of that of B_1 ($M \geq N$).

Hence the theory can be made finite with the help of a fine-tuning of Yukawa couplings according to eqs.(8),(9). The necessary and sufficient conditions for finiteness are defined by the one-loop approximation. The finiteness criteria are the following:

$$\text{i) } \sum_{R} T_R = 3C_G ;\quad ii)\ \mathrm{rank}\, B_1 = N ;\quad iii)\ f^{j}_{00} > 0 . \tag{22}$$

The last condition can be weakened and include also zero values of f^{j}_{oo} as it takes place in the approach based on renormalized couplings[2,3] . Establishing the relation between the two approaches enables us, as it will be shown below, to get some useful consequences for the construction of finite realistic models of particle interactions.

4. FINITE RENORMALIZATION

As we have shown, the theory can be made finite without any renormalization. However, a finite renormalization is possible and actually it is present in the approach of ref.[2-4] . Consider the relation of the method discussed here with that of ref.[2-4 2)]. In the latter case the renormalized couplings are considered. They are connected with the bare ones by the usual renormalization equations[1)]

$$Y_i^{Bare} = Y_i^R Z_i(g^R, Y^R, \varepsilon) ,$$
$$g^{Bare} = g^R Z(g^R, Y^R, \varepsilon) . \qquad (23)$$

Imposing the constraints on Yukawa couplings of the form

$$Y_i^R = f_0^i(\varepsilon) g_R + f_1^i(\varepsilon) g_R^2 + f_2^i(\varepsilon) g_R^3 + \dots , \qquad (24)$$

which is a generalization of eqs.(1) for dimension $D = 4 - 2\varepsilon$, the theory becomes finite[3], i.e. the constants Z_i and Z become regular functions of ε

$$Z_i = 1 + \sum_{n \geq 1} g_R^n Z_n^i(\varepsilon) , \quad Z = 1 + \sum_{n \geq 1} g_R^n Z_n(\varepsilon) .$$

Substituting eqs.(23),(24) into eq.(8) we get

$$Y_i^R Z_i = f_0^i g^R Z . \qquad (25)$$

This gives us the relation between $Z_n^i(\varepsilon)$, $Z_n(\varepsilon)$ and $f_n^i(\varepsilon)$. Renormalizabity imposes some constraints on Z_n^i and Z_n and enables us to find explicit relations between $f_n^i(0)$ and $f_0^i(\varepsilon)$ up to lower-order terms. We have

$$f_{10}^i = -(f_{00}A_1)^i_j f^j_{01} ,$$
$$f_{11}^i = -(f_{01}A_1)^i_j f^j_{01} - (f_{00}A_1)^i_j f^j_{02} , \qquad (26)$$
$$\dots$$
$$f_{n0}^i = (-)^n \frac{1}{n!}\left[(f_{00}A_1)^n\right]^i_j f^j_{0n} + \text{lower order terms},$$

2) Quite an analogous approach has been proposed also in ref.[15] Here the theory is reduced to a single-coupling one with the help of the so-called reduction renormalization group equations.[16] The finiteness criterion is also formulated in the one loop approximation.

where we have introduced the notation

$$f_n^i(\varepsilon) = \sum_{k \geq 0} f^i_{nk} \cdot \varepsilon^k . \qquad (27)$$

Eqs.(26) establish the correspondence between the expansions for bare (eq.8) and renormalized (eq.(24)) couplings. Such a correspondence enables us to get an important consequence. Substituting for this purpose eq.(24) into eq.(25) and putting $\varepsilon = 0$ (this is possible due to the regularity of all functions at $\varepsilon = 0$), we get

$$f^i_{00} g_R Z(g_R) = (f^i_{00} g_R + f^i_{10} g_R^2 + f^i_{20} g_R^3 + \dots) Z_i(g_R) . \qquad (28)$$

Hence if for some $i = k$ $f^k_{00} = 0$, then all $f^k_{n0} = 0$. From the point of view of eq.(1) that means that $Y_k = 0$ and plays no role in construction of finite theory. However, as far as in finiteness criteria of ref.[3] f^j_{00} can have any nonnegative values and the only requirement is rank $B_1 = N$, the theory can be made finite with $Y_k = 0$, as well.

Thus, the absence of Y_k on the one hand reduces the number of parameters needed to achieve finiteness and lowers the rank of B_1, but on the other hand the fact that $f^k_{n0} = 0$ for any n means that there exists the following linear connection between the coefficient functions of chiral propagators or between the anomalous dimensions of the fields

$$(B_1^{-1})^k_j \gamma^j \big|_{y = fg} = 0 \quad \text{if } f^k_{00} = 0 . \qquad (29)$$

That means that the number of independently renormalized fields is also reduced.

Thus, the conclusion is that the finiteness criteria (22) include also the case $f^j_{00} = 0$ as in refs.[2,3]. The appearance of a zero value indicates the existence of linear dependence between the chiral propagators. The reduced matrix $\bar{B}_1$ should have rank $\bar{B}_1 = \mathcal{N} - 1$. This particular note solves the problem of the fifth interaction mentioned in refs.[3,4] . Examples of finite models are given in the next section.

5. EXAMPLES OF FINITE THEORIES

We apply the proposed method for constructing some finite SUSY theories. Consider two models discussed in ref.[3] . Eqs.(26) enable to get expansions (9) up to three-loops without any new calculations. Note that as far as the one-loop finiteness leads to the two-loop one all the parameters $f^i_{10} \sim f^i_{01} = 0$. In this case we get from eq.(26)

$$f^i_{20} = \frac{1}{2}\left[(f_{00} A_1)^2\right]^i_j f^j_{02} \tag{30}$$

that enables us to find out the values of f^i_{02} starting from f^i_{20} [3] .

5a. $\mathcal{N} = 4d$ model

This model contains the same set of fields as $\mathcal{N} = 4$ SUSY Yang-Mills theory, namely, one gauge superfield V and three chiral superfields Φ^A in the ajoint representation of a gauge group. The difference from $\mathcal{N} = 4$ theory is that instead of an f-type Yukawa interaction here is a d-type coupling. The superpotential is[11]

$$W_Y = \frac{Y}{3!}\sum_{A=1}^{3} d_{ijk}\,\Phi^i_A \Phi^j_A \Phi^k_A ,$$

where d_{ijk} is a totally symmetric group structure constant. For the SU(n) group (n $\geq$ 3) we have got the following results[3]:

$$f_{00} = \frac{2n^2}{n^2-4}\ , \quad f_{10} = 0\ , \quad f_{20} = -\frac{96\, n^4 (n^2-10)}{(n^2-4)^3}\,\zeta(3)\ .$$

The matrix A_1 in this case is simply $A_1 = 3\,\frac{n^2-4}{2n}$. Substituting these exp. into eq.(30) we find

$$f_{02} = -\frac{64\, n^2 (n^2-10)}{3(n^2-4)^3}\,\zeta(3)$$

or

$$Y = g\,\frac{2n^2}{n^2-4}\left(1 - \frac{32\,(n^2-10)}{3(n^2-4)^2}\,\zeta(3)\,\varepsilon^2 + \cdots\right). \tag{31}$$

This choice of the bare Yukawa coupling provides the finiteness of $\mathcal{N} = 4d$ model up to three loops.

5b. Finite SUSY realistic SU(5) model

Finite SUSY Grand Unified Theories based on the SU(5) gauge group have been considered in a number of papers[17-19] . The two-loop finiteness has been ensured. The superpotential has the form (we use the notation of ref.[18])

$$W_Y = Y_1 \sum_{i=1}^{3} \tilde{\Phi}_{i\alpha} \Psi_{i\beta} \Lambda_i^{\alpha\beta} + \frac{Y_2}{8}\sum_{i=1}^{3} \epsilon^{\alpha\beta\gamma\delta\rho}\, \Phi_i^\alpha \Lambda_i^{\beta\gamma} \Lambda_i^{\delta\rho} + Y_3 \tilde{\Phi}_{4\alpha} \Phi_4^\beta \Sigma^\alpha_\beta + \frac{Y_4}{3} \Sigma^\alpha_\beta \Sigma^\beta_\gamma \Sigma^\gamma_\alpha\ . \tag{32}$$

Here the matter fields Ψ_i and Λ_i (i = 1,2,3) belong to the representations $\bar{5}$ and 10 of SU(5), and higgs fields Φ_i, Φ_4, $\tilde{\Phi}_i$, $\tilde{\Phi}_4$ and Σ belong to the representations 5,5,$\bar{5}$,$\bar{5}$ and 24, respectively. Indices $\alpha, \beta, \ldots = 1,2,\ldots,5$ are the SU(5) ones. The matrix Σ is traceless, and Λ is antisymmetric.

The necessity of fine-tuning of Yukawa couplings have been considered in refs.[3,4] . Note that superpotential (32) contains four Yukawa couplings. At the same time there are at first sight five independently renormalized fields, namely Ψ_i, Φ_i, Φ_4, Λ_i and Σ . The fields $\tilde{\Phi}_i$ and $\tilde{\Phi}_4$ are renormalized like Ψ_i and Φ_4 , respectively. However, the one-loop equations (11) have a solution[18,3,4] ,i.e., the fifth coupling is not needed, it is zero. This fact was verified[3,4] up to three loops. According to the statements of the previous section that means that there is a linear connection between the anomalous dimensions of the fields of type (29). In our case we have

$$\frac{1}{2}\gamma_\psi + \gamma_\phi = \gamma_\Lambda \quad .$$

Hence the propagator of Λ is finite whence it is true for ψ and Φ , and the number of independently renormalized fields is reduced to four.

The matrices B_1, f_{00} and A_1 in this case are[3]:

$$B_1 = \begin{pmatrix} 4 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & \frac{24}{5} & 0 \\ 0 & 0 & 1 & \frac{21}{5} \end{pmatrix}, \quad f_{00} = \begin{pmatrix} 3/5 & & & 0 \\ & 4/5 & & \\ & & 1/2 & \\ 0 & & & 15/14 \end{pmatrix}, \quad A_1 = \begin{pmatrix} 10 & 3 & 0 & 0 \\ 4 & 9 & 0 & 0 \\ 0 & 0 & \frac{53}{5} & \frac{21}{5} \\ 0 & 0 & 3 & \frac{63}{5} \end{pmatrix}.$$

Given the coefficients f'_{20} [3]

$$\frac{f'_{20}}{f'_{00}} = \left\{ \frac{48}{5}, \; -\frac{111}{25}, \; \frac{216}{5}, \; \frac{867}{245} \right\} \zeta(3)$$

we can find f'_{02} with the help of eq.(30).

6. CONCLUSIONS

The proposed method provides us with a wide class of finite $\mathcal{N} = 1$ SUSY field theories. We limited ourselves to the propagators which are diagonal in all indices. This requirement is obligatory in the one-loop approximation due to eq.(5). However, in higher orders it can be weakened. The possibility of nondiagonal transitions is of interest for the description of quark flavour mixing in realistic models of Grand unification. While introducing mixing with the help of unitary matrix[18] the finiteness in higher orders can be achieved through the modification

of the tensor structure of Yukawa-type interaction[4]. In our approach this leads to the following modification of eq.(8) for Yukawa couplings

$$d^{abc} = g\left[d_0^{abc} + \varepsilon d_1^{abc} + \varepsilon^2 d_2^{abc} + \dots \right]. \qquad (33)$$

The finiteness criteria then becomes the convertibility of d_0^{abc} instead of condition ii) of eq.(22).

Note that the cancellation of divergences in our case is not connected with any known symmetry contrary to the known examples of $\mathcal{N} = 4$ or $\mathcal{N} = 2$ extended supersymmetry. A possible symmetry (if any) responsible for this cancellation should be searched in a classical Lagrangian at $\varepsilon = 0$. The appearance of ε-dependence in eqs.(8), (33) may well be a reflection of the supersymmetry breaking by dimensional regularization in higher loops[20].

The author is grateful to A.A. Vladimirov, S.G. Gorishny and O.V. Tarasov for useful discussions.

REFERENCES

1. D.I.Kazakov, Phys.Lett. 179B (1986) 352.
2. A.V.Ermushev, D.I.Kazakov and O.V.Tarasov, Preprint JINR E2-85-794, Dubna 1985.
3. A.V.Ermushev, D.I.Kazakov and O.V.Tarasov, Preprint JINR E2-86-17, Dubna 1986 and Nucl.Phys. B281 (1987) 72.
4. D.R.T.Jones, Preprint IFP-270-UNC 1986 and Nucl.Phys. B279 (1986).
5. S.J.Gates, Jr., M.T.Grisaru, M.Roček, W.Siegel, Superspace: "One Thousand and One Lessons in Supersymmetry" Benjamin/Cummings Publishing Co., Inc., 1983.

6. M.Grisaru, M.Roček, W.Siegel, Nucl.Phys. B159 (1979) 429.

7. M.Grisaru and W.Siegel, Nucl.Phys. B201 (1982) 292.

8. A.J.Parkes, P.C.West, Phys.Lett. 138B (1984) 99;
P.C. West, Phys.Lett. 137B (1984) 371.

9. D.R.T.Jones and L.Mezincescu, Phys.Lett. 138B (1984) 293.

10. S.Hamidi, J.Patera and J.H.Schwarz, Phys.Lett. 141B (1984)349;
S.Rajpoot and J.G.Taylor, Phys.Lett. 147B (1984) 91.

11. A.J.Parkes and P.C.West, Nucl.Phys. B256 (1985) 340;
A.J.Parkes, Phys.Lett. 156B (1985) 73;
D.R.T.Jones and A.J.Parkes, Phys.Lett. 160B (1985) 267.

12. M.T.Grisaru, B.Milewski and D.Zanon, Phys.Lett. 155B (1985) 357.

13. M.A.Shifman and A.I.Vainshtein, Nucl.Phys. B277 (1986)456.

14. G.t'Hooft, Nucl.Phys. B61(1973) 455.

15. O.Pignet and K.Sibold, Phys.Lett. 177B (1986) 373.

16. R.Oehme, K.Sibold and W.Zimmermann, Phys.Lett. 153B (1985) 42.

17. D.R.T.Jones and S.Raby, Phys.Lett. 143B (1984) 137.

18. S.Hamidi and J.H.Schwarz,Phys.Lett. 147B (1984) 301.

19. J.E.Bjorkman, D.R.T.Jones and S.Raby, Nucl.Phys. B259 (1985) 503;
J.Leon et al., Phys.Lett. 156B (1985) 66.

20. L.V.Avdeev and A.A.Vladimirov, Nucl.Phys., B219 (1983) 262.

Received by Publishing Department
on December 17, 1986.

Казаков Д.И. E2-86-816

Конечные N = 1 суперсимметричные калибровочные теории

Дается подробное описание метода построения конечных суперсимметричных калибровочных теорий в формализме N = 1 суперполей в рамках размерной регуляризации. Конечность всех функций Грина основана на наличии суперсимметрии и калибровочной инвариантности и достигается надлежащим выбором состава полей материи, а также юкавских констант связи в виде $Y_i = f_i(\epsilon)g$, где g есть калибровочная константа связи, а функция $f_i(\epsilon)$ регулярна в нуле и вычисляется по теории возмущений. Необходимые и достаточные условия конечности теории определяются уже в однопетлевом приближении. Устанавливается связь с предлагаемым ранее подходом к построению конечных теорий, основанном на особых решениях уравнений ренормгруппы.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1986

Kazakov D.I. E2-86-816

Finite N = 1 Susy Gauge Field Theories

In the present paper we give a detailed description of the method to construct finite N = 1 SUSY gauge field theories in the framework of N = 1 superfields within dimensional regularization. The finiteness of all Green functions is based on supersymmetry and gauge invariance and is achieved by a proper choice of matter content of the theory and Yukawa couplings in the form $Y_i = f_i(\epsilon)g$, where g is the gauge coupling, and the function $f_i(\epsilon)$ is regular at $\epsilon = 0$ and is calculated in perturbation theory. Necessary and sufficient conditions for finiteness are determined already in the one-loop approximation. The correspondence with an earlier proposed approach to construct finite theories based on eigenvalue solutions of renormalization-group equations is established.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1986