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QCD VACUUM EFFECT ON HADRON ELECTROMAGNETIC MASS DIFFERENCES

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In quark models /1-3/ the mass differences between members of the same isotopic-spin multiplets are usually caused by three factors: 1) the electromagnetic interaction of quarks in a hadron ΔM_{EM}, 2) the dependence of the quark kinetic energy ΔM_{kin} on their masses and 3) the effect of u-, d -quark mass difference on the strong interaction potential ΔM_s.

This is the case of electromagnetic interaction of particles in the presence of strong interactions. A consistent calculation of these contributions is possible within the relativistic bag model /4/. Thus, the authors of ref. /5/ have calculated ΔM_{EM} and ΔM_{kin} in the case of hadron ground states. Further in ref. /3/ the dependence of the one-gluon exchange potential ΔM_g on quark masses was taken into account. However, the comparison of the theoretical results /6/ with experiment /7/ (Table I of our work) shows that the MIT bag model used in /5,6/ describes unsatisfactorily the electromagnetic mass differences of hadrons.

Recently /8,9/, we have proposed the version of the bag model based on the idea that the interaction of valence quarks with fluctuating vacuum fields in the bag plays the dominating role. As we have shown, this nonperturbative interaction between quarks, strongly depending on their mass, defines the spectroscopy of the ground states of hadrons.

The aim of this paper is to estimate the effect of the QCD vacuum on isotopic mass differences.

In the model /8,9/ the total energy difference between two members of a multiplet is given by the sum

$$\begin{aligned} \Delta M_{tot} &= \Delta M_{EM} + \Delta M_{kin} + \Delta M_s , \\ \Delta M_s &= \Delta M_g + \Delta M_{vac} + \Delta M_{inst} , \end{aligned} \qquad (1)$$

where ΔM_q is due to the QCD hyperfine interaction of quarks inside a bag and ΔM_{vac} and ΔM_{inst} are due to the interaction of quarks with vacuum fields.

Detailed calculations of the contributions ΔM_{EM}, ΔM_{kin} and ΔM_q were carried out in /5,6/, where the standard MIT values of the strange quark mass m_s and gauge coupling constant α_s were used

$$m_s = 280\,\mathrm{Mev}\ ,\ \alpha_s = 2{,}2\,. \tag{2}$$

In our calculations we take the values

$$m_s = 220\,\mathrm{Mev}\ ,\ \alpha_s = 0{,}7 \tag{3}$$

obtained in describing the hadron spectrum within the model /8,9/.

Let us consider the contributions ΔM_{vac} and ΔM_{inst}. The first is caused by the interaction of quarks with low-frequency vacuum field, which gives the confinement of quarks. The Hamiltonian of this interaction is expressed by /8/:

$$H_{vac} = \frac{\omega_q}{2}(\bar{Q}\gamma_0 q + \bar{q}\gamma_0 Q)\,, \tag{4}$$

where Q, q are the vacuum and valence quark fields, respectively, ω_q is the one-particle energy of a quark in the bag of radius R. Expression (4) follows directly from the QCD (bag) Lagrangian by singling a vacuum component out of the quark field /8,9/:

$$\psi(x) = q(x) + Q(x)\,. \tag{5}$$

A correction to the hadron mass is calculated with using the perturbation theory with the interaction Hamiltonian (4). In this case we take the wave functions of the bag model as a solution of the zeroth order approximation. The resulting formula is /8/:

$$M_{vac} = -\frac{\pi}{12}\sum_{flav} N_i \frac{\langle 0|\bar{Q}_i Q_i|0\rangle R^4(\omega_i + m_i)^2\omega_i}{\varkappa_i^2[2\omega_i(\omega_i R - 1) + m_i]}\,, \tag{6}$$

where N_i is the number of quarks of the type i in the hadron, $\omega_i = (m_i^2 + \varkappa_i^2/R^2)^{1/2}$ is the one-particle energy, m_i is the current quark mass, $\langle 0|\bar{Q}_i Q_i|0\rangle$ is the condensate of the i-th quark, and $\varkappa_i$ is determined from the solution of the equation:

$$tg\,\varkappa_i = \varkappa_i/[1 - m_i R - (\varkappa_i^2 + (m_i R)^2)^{1/2}]\,. \tag{7}$$

By expanding (6) in a series with respect to the small quark mass of u-, d- quarks, we obtain

$$M_{vac} = M_{vac}^0 + 0{,}035\sum_{flav} N_i m_i \langle 0|\bar{Q}_i Q_i|0\rangle R^3 + \ldots \tag{8}$$

Expressions (6) and (8) take into account only such contributions to the energy of a hadron that are caused by the one-particle interaction of the large-scale vacuum fluctuations. Many particle interactions of such a type are much weaker than the one-particle ones /9/; therefore their influence on the hadron masses is insignificant.

Many-particle interactions, as was shown in /8,9/, have in principle a small-distance character and may be approximated by the t'Hooft interaction /10/ induced by the high-frequency part of vacuum fluctuations-instantons.

Within the instanton liquid model of the QCD vacuum /11,12/, the Hamiltonian of this interaction is expressed by /8,13/

$$H_{inst} = -\sum_{\substack{i\neq j\\ flav}} \eta_{ij}\Big[\bar{q}_{iR}q_{iL}\bar{q}_{jR}q_{jL} + (R \leftrightarrow L) + \frac{3}{32}\big(\bar{q}_{iR}\lambda^a q_{iL}\bar{q}_{jR}\lambda^a q_{jL} - \frac{3}{4}\bar{q}_{iR}\sigma_{\mu\nu}\lambda^a q_{iL}\bar{q}_{jR}\sigma^{\mu\nu}\lambda^a q_{jL}\big)\Big]\,, \tag{9}$$

where

$$q_{R,L} = \frac{1}{2}(1 \mp \gamma_5)q\ ,\ m^* = \frac{2\pi}{3}\langle 0|\bar{u}u|0\rangle\rho_c^2\,,$$

$$m_i^* = m_i + m^*,\ \eta_{ij} = \frac{4}{3}\pi^2\rho_c^2 m^{*2}/(m_i^* m_j^*)\,,$$

ρ_c is the characteristic size of an instanton in the QCD vacuum. In the first order with respect to small finite masses of u, d quarks we obtain from (9) the energy difference resulting from instantons

$$\Delta M_{inst} = -\sum_{flav} \frac{m_i}{m^*} E_{inst} , \tag{10}$$

where E_{inst} is the correction for massless u, d quarks calculated in /8/ with the MIT bag model wave functions

$$E_{inst} = \langle h | H_{inst} | h \rangle . \tag{11}$$

The values of E_{inst} for the terms of hadron multiplets are the following:

$$E^{\pi}_{inst} = -\lambda_0 / R_0^3 , \quad E^{K}_{inst} = -\lambda_s / R_0^3 , \quad E^{N}_{inst} = -3\lambda_0 / 4R_0^3 ,$$
$$E^{\Sigma}_{inst} \doteq E^{\Xi}_{inst} = -3\lambda_s / 4R_0^3 , \tag{12}$$

where $\lambda_s \simeq 0{,}65 \lambda_0 ,$

$$\lambda_0 = \frac{\pi \varkappa_0^2 \rho_c^2}{4 j_0^2(\varkappa_0)(\varkappa_0 - 1)^2} \int_0^1 dx \, x^2 \left[j_0^2(\varkappa_0 x) + j_1^2(\varkappa_0 x) \right]^2 . \tag{13}$$

For the members of the meson vector nonet and baryon decouplet corrections (11) are equal to zero /8,14/. This selection rule is related with the fact that the instanton mechanism of interaction within a hadron takes place only if two quarks are in the state with the zero total spin (formation of a diquark) /10/.

In using (11), it is supposed that the instanton interaction is taken into account only in the first order of perturbation theory. That is why the value of R_0 in (12) should be different from the radius R in the calculation of $\Delta M_{EM}, \Delta M_{kin}, \Delta M_q, \Delta M_{vac}$. This difference ΔR arises from (9). (R_0 is the equilibrium bag radius without instanton interaction and its values are listed in Table 3 of /8/).

Let us estimate $\Delta R = R_0 - R$ as it was done in /15/, where the change of the bag size caused by the pion field was considered,

$$\Delta R = 3 E_{inst}(R_0) / [M_0''(R_0) R_0] . \tag{14}$$

In this expression M_0 is the energy of a hadron without the contribution of (12). The results of calculations of various contributions to electromagnetic mass differences are presented in the Table.

In the calculations we used Table 3 from /8/ and the results of the works /5,6/. The Table contains also the relevant results obtained within the nonrelativistic models in the works /2,3/ and also in the bag model calculations /6/. The parameters of the QCD vacuum are chosen the following /8/:

$$\langle 0 | \bar{u} u | 0 \rangle = \langle 0 | \bar{d} d | 0 \rangle = -(250 \, Mev)^3 ;$$
$$\rho_c = 2 \, Gev^{-1} . \tag{15}$$

The numbers in Table 1 correspond to the difference of masses of u and d quarks to be equal to

$$m_d - m_u = 3{,}69 \, Mev ,$$

that was obtained from the most precise experimental measurement of the difference $M_p - M_n$. By comparing the calculated values with the experimental ones and the MIT bag model results of /6/, we see that the interaction through the QCD vacuum gives an essential contribution to the isotopic mass differences of hadrons belonging to a baryon octet and of pseudoscalar mesons. Moreover, the sum of the contributions of the QCD bag perturbative theory (ΔM_{EM} , ΔM_{kin} , ΔM_q) and of the interaction through nonperturbative fluctuations of vacuum describes well the mass differences of the terms of the hadron isotopic spin multiplets.

The only expception is the differences $\pi^{\pm} - \pi^{0}$ and $K^{*+} - K^{*0}$. However, if in the first case the deviation from the experiment may be explained by mixing with the η , η' states /9/, in the second case this discrepancy looks strange. In this connection we think that new, more precise experiments on determining electromagnetic mass differences of resonances are necessary.

We should like to emphasize the work /16/ where the isotopic mass differences are estimated from the expression included in the sum of the terms usual for the MIT approach /5/ and the term caused by instantons. The agreement of the results was excellent. However, as was marked in /9/, we consider that the dependence of the instanton contribution on quark masses is wrong in the work /16/.

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Table I. The electromagnetic mass differences of Hadrons (MeV)
ΔM_{NR} are from /3/, ΔM_{MIT} from /6/

Particles	R (GeV^{-1})	ΔM_{kin}	ΔM_{EM}	ΔM_{g}	ΔM_{vac}	ΔM_{inst}	ΔM_{tot}	ΔM_{exp}	ΔM_{MIT}	ΔM_{NR}
1	2	3	4	5	6	7	8	9	I0	II
$p-n$	4.42	-1.77	0.57	0.09	-0.18	0.00	-1.29	-1.29343 ± 0.00004	-I.29	-I.3
$\Sigma^+-\Sigma^0$	4.25	-1.77	0.37	-0.02	-0.15	-1.84	-3.41	-3.10 ± 0.14	-I.83	-3.5
$\Sigma^0-\Sigma^-$	4.25	-1.77	-1.51	-0.02	-0.15	-1.84	-5.29	-4.860 ± 0.076	-3.45	-4.5
$\Xi^0-\Xi^-$	4.44	-1.77	-1.67	-0.13	-0.18	-1.43	-5.18	-6.34 ± 0.74	-4.02	-6.3
$\Delta^0-\Delta^{++}$	6.04	3.54	-2.22	-0.16	0.89	0.00	2.05	2.70 ± 0.30	I.I5	3.0
$\Delta^0-\Delta^+$	6.04	1.77	-0.38	-0.08	0.45	0.00	1.76	-	I.38	2.3
$\Delta^0-\Delta^-$	6.04	-1.77	-0.96	0.08	-0.45	0.00	-3.10	-	-2.86	-3.9
$\Sigma^{*0}-\Sigma^{*+}$	5.68	1.77	-0.37	-0.07	0.37	0.00	1.70	4 and 4	I.44	2.0
$\Sigma^{*0}-\Sigma^{*-}$	5.68	-1.77	-1.06	0.07	-0.37	0.00	-3.13	-2.0 ± 2.4	-2.94	-3.7
$\Xi^{*0}-\Xi^{*-}$	5.51	-1.77	-1.13	0.06	-0.34	0.00	-3.18	-2.92 ± 0.91	-3.0I	-3.8
$\pi^{\pm}-\pi^0$	2.00	0.00	1.94	0.00	0.00	0.00	1.94	4.6043 ± 0.0037	I.6I	-
K^+-K^0	2.00	-1.77	1.87	-0.21	-0.02	-2.78	-2.91	-3.92 ± 0.14	-I.62	-6.0
$\rho^{\pm}-\rho^0$	6.16	0.00	0.72	0.00	0.00	0.00	0.72	-0.3 ± 2.2	0.94	-0.6
$K^{*+}-K^{*0}$	5.68	-1.77	0.60	0.00	-0.37	0.00	-1.47	-6.7 ± 1.3	-I.II	-2.7

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Дорохов А.Е., Кочелев Н.И. E2-86-790

Влияние вакуума КХД на электромагнитные разности масс адронов

В составной модели, учитывающей взаимодействие с вакуумными конденсатами КХД, вычислены электромагнитные расщепления масс адронов. Полная разность масс определяется суммой вкладов: 1) электромагнитным взаимодействием кварков, 2) зависимостью кинетической энергии кварков от разности их масс и 3) влиянием разности масс кварков на потенциал сильного взаимодействия. Показано, что учет взаимодействия кварков с вакуумом КХД дает удовлетворительное описание экспериментальных данных. Оценена разность масс u и d кварков: $m_d - m_u = 3{,}7$ МэВ.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Dorokhov A.E., Kochelev N.I. E2-86-790

QCD Vacuum Effect on Hadron Electromagnetic Mass Differences

Mass splitting within hadron isotopic spin multiplets is calculated in the model that takes into account quark interaction with QCD vacuum fields. The mass difference between u and d quarks is estimated to be: $m_d - m_u = 3.7$ MeV.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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