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D.I.Kazakov, A.V.Kotikov

CORRECTION TO DIS CROSS SECTION RATIO $R = \sigma_L / \sigma_T$ IN QCD

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1. Introduction

In recent years the nonleading effects in DIS have been intensively studied in order to check the QCD predictions [1-3]. The present paper is devoted to the calculation of α_s correction to the ratio of cross sections for the longitudinal and transverse polarized photon scattering on a hadron

$$R(x, Q^2) = \sigma_L / \sigma_T ,$$

where $Q^2 = -q^2 > 0$ and $x = 2pq/Q^2$, q and p being the photon and hadron momentum respectively.

As far as in the parton model $R = 0$, the observed nonzero value of R is due to quark interactions and is described in QCD. However, in the leading order the QCD predictions deviate from experimental values for $x > 0.4$. The effect of the next α_s correction has been studied in refs. [1-2]. There were some variations in final results.

We here present the result of analytical calculation of α_s correction to the longitudinal structure function of DIS and discuss its influence on $R = \sigma_L/\sigma_T$. For the analysis of experimental data we apply the scheme-invariant perturbation theory. As will be shown below, the account of α_s correction improves the agreement with experiment.

2. The Formalism

1. The hadronic part of DIS cross-section can be represented in the form

$$F_{\mu\nu} = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) F_1(x, Q^2) - \left(p_\mu - \frac{(pq)q_\mu}{q^2}\right)\left(p_\nu - \frac{(pq)q_\nu}{q^2}\right)\frac{2x}{q^2} F_2(x, Q^2), \quad (1)$$

where $F_i(x, Q^2)$ ($i = 1,2$) are the structure functions. The value of R we are interested in is expressed through F_i by

$$R(x, Q^2) = \frac{F_2(x, Q^2) - 2x F_1(x, Q^2)}{2x F_1(x, Q^2)} \equiv \frac{F_L(x, Q^2)}{2x F_1(x, Q^2)}, \quad (2)$$

where $F_L(x, Q^2)$ is called the longitudinal structure function .

By the optical theorem the tensor $F_{\mu\nu}$ is connected with the amplitude of elastic forward scattering

$$T_{\mu\nu}(p,q) = i\int d^4z\, e^{iqz}\langle p|\, T(J_\mu(z) J_\nu^+(0))|p\rangle = \tag{3}$$

$$= e_{\mu\nu} T_L(x, Q^2) + d_{\mu\nu} T_2(x, Q^2),$$

where

$$e_{\mu\nu} = g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}, \quad d_{\mu\nu} = -\left[g_{\mu\nu} + \frac{(p_\mu q_\nu + p_\nu q_\mu)2x}{q^2} + p_\mu p_\nu \frac{4x^2}{q^2}\right].$$

If we expand the invariant amplitudes T_L and T_2 in inverse powers of x

$$T_k = \sum_{n=0}^{\infty}\left(\frac{1}{x}\right)^n T_{k,n}, \qquad k = 2, L. \tag{4}$$

the moments $T_{k,n}$ coincide (for even n) with those of the structure functions F_k :

$$T_{k,n} = M_{k,n} = \int_0^1 dx\, x^{n-2} F_k(x, Q^2), \quad \begin{matrix} k=2,L \\ n=2m \end{matrix}. \tag{5}$$

On the other hand, the product of currents in eq. (3) can be expressed through the set of local operators $O_{\mu_1\ldots\mu_n}$ with the coefficient functions $C_n(z)$:

$$T(J(z)J^+(0)) = \sum_n C_n(z^2)\, O_{\mu_1\ldots\mu_n} \cdot z^{\mu_1}\ldots z^{\mu_n}.$$

Hence the nonsinglet (NS) parts of the moments can be expressed through the product of the coefficient functions $C_{k,n}^{NS}(Q^2/\mu^2)$ and the matrix element

$$\langle p|\, O_{\mu_1\ldots\mu_n}|p\rangle = p_{\mu_1}\ldots p_{\mu_n} A_n^{NS}(\mu^2) \tag{6}$$

$$T_{k,n}^{NS}(Q^2) = C_{k,n}^{NS}(Q^2/\mu^2)\, A_n^{NS}(\mu^2).$$

Hereafter we consider only the nonsinglet operators, so we omit NS in the formulas.

2. The coefficient function obeys the following renormalization group equation

$$\left(\mu\frac{\partial}{\partial\mu} + \beta_g\frac{\partial}{\partial g} - \gamma_n\right) C_{k,n}(Q^2/\mu^2, g) = 0, \tag{7}$$

where γ_n is an anomalous dimension of $O_{\mu_1\ldots\mu_n}$. Upon solving this equation, we get

$$T_{k,n} = A_n C_{k,n}(1, \bar\alpha(Q^2)) \exp\left(-\int_{\alpha}^{\bar\alpha} d\alpha' \frac{\gamma_n(\alpha')}{2\beta(\alpha')}\right), \quad k=2,L, \tag{8}$$

where $\alpha \equiv g^2/16\pi^2$. Eq. (8) is valid for any other of PT. Let us expand all the quantities in powers of α. We get

$$\begin{aligned} C_{2,n}(1,\alpha) &= 1 + B_{2,n}\alpha + \cdots, \\ C_{L,n}(1,\alpha) &= b_{L,n}\alpha(1 + B_{L,n}\alpha + \cdots), \\ \beta(\alpha) &= -\beta_0\alpha^2 - \beta_1\alpha^3 - \ldots, \\ \gamma_n(\alpha) &= \gamma_n^0\alpha + \gamma_n^1\alpha^2 + \ldots. \end{aligned} \tag{9}$$

With account of eq. (9) we can rewrite eq. (8) as follows

$$\begin{aligned} T_{2,n}(Q^2) &= A_n[\bar\alpha(Q^2)]^{\gamma_n^0/2\beta_0}\left[1 + \bar\alpha\left(B_{2,n} + \frac{\gamma_n^1}{2\beta_0} - \frac{\gamma_n^0\beta_1}{2\beta_0^2}\right)\right] \\ T_{L,n}(Q^2) &= A_n b_{L,n}[\bar\alpha(Q^2)]^{\frac{\gamma_n^0}{2\beta_0}+1}\left[1 + \bar\alpha\left(B_{L,n} + \frac{\gamma_n^1}{2\beta_0} - \frac{\gamma_n^0\beta_1}{2\beta_0^2}\right)\right], \end{aligned} \tag{10}$$

where all the coefficients except $B_{L,n}$ are known.

3. As is well known, the results of PT calculations depend on the renormalization scheme applied. To eliminate the scheme ambiguities, in refs. /9-11/ there has been proposed and developed the so-called scheme invariant perturbation theory (SIPT). In this approach any physical quantity is accompanied by its own coupling constant. For example, in our case the coupling constant is associated with each type and the number of the moment of structure functions. In SIPT the moments of structure functions eq. (10) are expressed in the form /12/

$$T_{2,n}(Q^2) = A_n \left[a_{2,n}(Q^2) \right]^{\gamma_n^0 / 2\beta_0} ,$$

$$T_{L,n}(Q^2) = A_n \left[a_{L,n}(Q^2) \right]^{\frac{\gamma_n^0}{2\beta_0} + 1} \qquad (11)$$

where $a_{2,n}$ and $a_{L,n}$ are determined by the solutions of transcendental equations, respectively:

$$\frac{1}{a_{2,n}} + \frac{\beta_1}{\beta_0} \ln a_{2n} = \beta_0 \ln \frac{Q^2}{\Lambda^2_{\overline{MS}}} - \left(\frac{2\beta_0}{\gamma_n^0} B_{2,n}(\overline{MS}) + \frac{\gamma_n^1}{\gamma_n^0} - \frac{\beta_1}{\beta_0} \right),$$

$$\frac{1}{a_{L,n}} + \frac{\beta_1}{\beta_0} \ln a_{L,n} = \beta_0 \ln \frac{Q^2}{\Lambda^2_{\overline{MS}}} - \left(\frac{2\beta_0}{\gamma_n^0 + 2\beta_0} B_{L,n}(\overline{MS}) + \frac{\gamma_n^1}{\gamma_n^0 + 2\beta_0} - \frac{\beta_1 \gamma_n^0}{\beta_0(\gamma_n^0 + 2\beta_0)} \right)$$

As we see, the use of SIPT improves the agreement with experimental data.

3. The results

1. The diagrams contributing to the longitudinal structure function are represented in Fig.1.

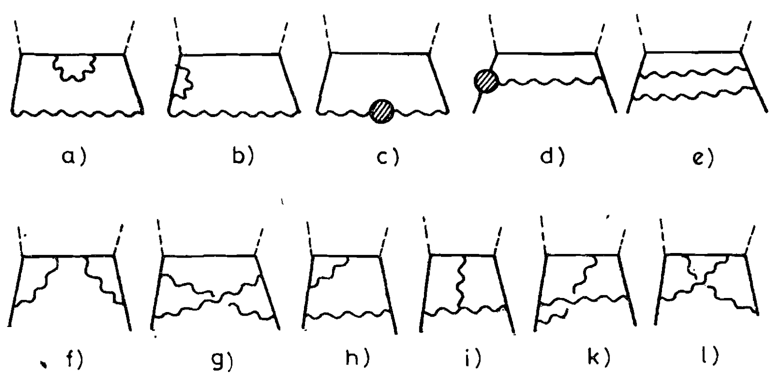

Fig.1

Here the solid, wary and dotted lines denote the quark, gluon and photon, respectively.

The digrams have been calculated by the method by "gluing" /4/ and the method of uniqueness /5-7/. The details of calculations can be found in /8/.

The method of gluing transforms the diagram to the propagator type. For this purpose it is multiplied by the propagator of a special form containing the traceless product of momenta $q^{\mu_1 \cdots \mu_n}$ and integrated over q. The resulting diagram has an additional loop whereas a coefficient function of a simplest pole in $\varepsilon = 4-D$ defines the coefficient function of the n-th moment.

The method of uniqueness is the method to calculate Feynman integrals. They are evaluated in $D = 4-2\varepsilon$ dimensions, and we are usually interested in the coefficients of negative and some positive powers of ε.

The total contribution of the diagrams Fig.1 can be represented as follows:

$$\begin{aligned} B_{L,n}(\overline{MS}) = {} & (2C_F - C_A)\Big[8P(n) - 4K_3(n) - 4S_3(n) + \\ & + 12\zeta(3) - \frac{6}{5}\Big\{ \frac{1-\delta_n^2}{n-2}\left(4K_2(n) - 3\right) + \delta_n^2\left(6\zeta(3) - 7\right)\Big\} - \\ & - 8K_2(n)\left(1 + \frac{1}{n} - \frac{1}{n+1} - \frac{3}{5}\frac{1}{n+3}\right) - S_1(n)\cdot\frac{23}{3} - \\ & - \frac{215}{18} + \frac{11}{3}\frac{1}{n} + \frac{11}{3}\frac{1}{n+1} - \frac{18}{5}\frac{1}{n+3}\Big] + \\ & + 2C_F\Big[S_1^2(n) - S_2(n) + S_1(n)\left(\frac{19}{6} - \frac{1}{n} - \frac{1}{n+1}\right) + \\ & + \frac{377}{36} - \frac{7}{6}\frac{1}{n} - \frac{19}{6}\frac{1}{n+1} + \frac{1}{n^2} - \frac{1}{(n+1)^2}\Big] - \frac{4}{3}T_F\Big[S_1(n) + \\ & + \frac{19}{6} - \frac{1}{n} - \frac{1}{n+1}\Big], \qquad n = 2m \end{aligned} \qquad (12)$$

where

$$S_i(n) = \sum_{k=1}^{n} \frac{1}{k^i}, \quad K_i(n) = \sum_{k=1}^{n} \frac{(-)^{k+1}}{k^i}, \quad P(n) = \sum_{k=1}^{n} \frac{K_2(k)}{k},$$

and

$$C_F = \frac{N^2 - 1}{2N}, \quad C_A = N, \quad T_F = f/2$$

for the SU(N) gauge group and f quark flavours.

The contribution of crossed diagrams doubles the even moments and cancels the odd ones.

Eq. (12) has a more simple analytical form than that of ref. /3/ and coincides with it numerically. The contributions of individual diagrams are given in the Appendix.

2. To reconstruct the structure function out of the moments, we use the technique of Yndurain /13/. The idea is to take N known moments (we take $N=6$) and numerically reconstruct the structure function according to the following formulas

$$x_{N,k} = \frac{k+1}{N+2}, \quad F(x_{N,k}) = \tilde{F}(x_{N,k}) + \delta(x_{N,k}), \tag{13}$$

$$\tilde{F}(x_{N,k}) = \frac{(N+1)!}{k!}\sum_{l=0}^{N-k}\frac{(-)^{l}}{l!\,(N-k-l)!}\,M_{k+l+2},$$

$$\delta(x_{N,k}) = \frac{1}{2}\,\frac{(N+1)!\,(N-k+1)}{k!\,(N+2)^2(N+3)}\sum_{l=0}^{N-k}\frac{(-)^{l}(k+l)(k+l-1)}{l!\,(N-k-l)!}\,M_{k+l}.$$

In what follows we use the moments $M_{k,n}$ for even as well as for odd values of n. In the leading order for the analytical expression for $M_{k,n}$ there have been used the values of $T_{k,n}$ for even n. In our case this is not so simple because of $K_i(n)$ having different analytical results for even and odd values of n. This difficulty is removed by the redefinition of $K_i(n)$:

$$K_i(n) \to \tilde{K}_i(n) = \begin{cases} K_i(n) & n=2m \\ \frac{1}{2}\left[K_i(n+1)+K_i(n-1)\right], & n=2m+1 \end{cases}$$

Further on it is more useful to rewrite eqs. (10), (11) in the form

$$T_{2,n} = T_{2,n}(Q_0^2)\left[\frac{\bar{\alpha}(Q^2)}{\bar{\alpha}(Q_0^2)}\right]^{\frac{\gamma_n^0}{2\beta_0}}\left[1+\left(\bar{\alpha}(Q^2)-\bar{\alpha}(Q_0^2)\right)\left(B_{2,n}+\frac{\gamma_n^1}{2\beta_0}-\frac{\gamma_n^0\beta_1}{2\beta_0^2}\right)\right], \tag{10a}$$

$$T_{L,n} = T_{2,n}(Q^2)\,b_{L,n}\,\bar{\alpha}(Q^2)\left[1+(B_{L,n}-B_{2,n})\bar{\alpha}(Q^2)\right].$$

$$T_{2,n}(Q^2) = \left[\frac{a_{2,n}(Q^2)}{a_{2,n}(Q_0^2)}\right]^{\frac{\gamma_n^0}{2\beta_0}} T_{2,n}(Q_0^2),$$

$$T_{L,n}(Q^2) = a_{L,n}(Q^2)\,b_{L,n}\left[\frac{a_{L,n}(Q^2)}{a_{L,n}(Q_0^2)}\right]^{\frac{\gamma_n^0}{2\beta_0}} T_{2,n}(Q_0^2) \tag{11a}$$

with $a_{L,n}$ and $a_{2,n}$ defined earlier.

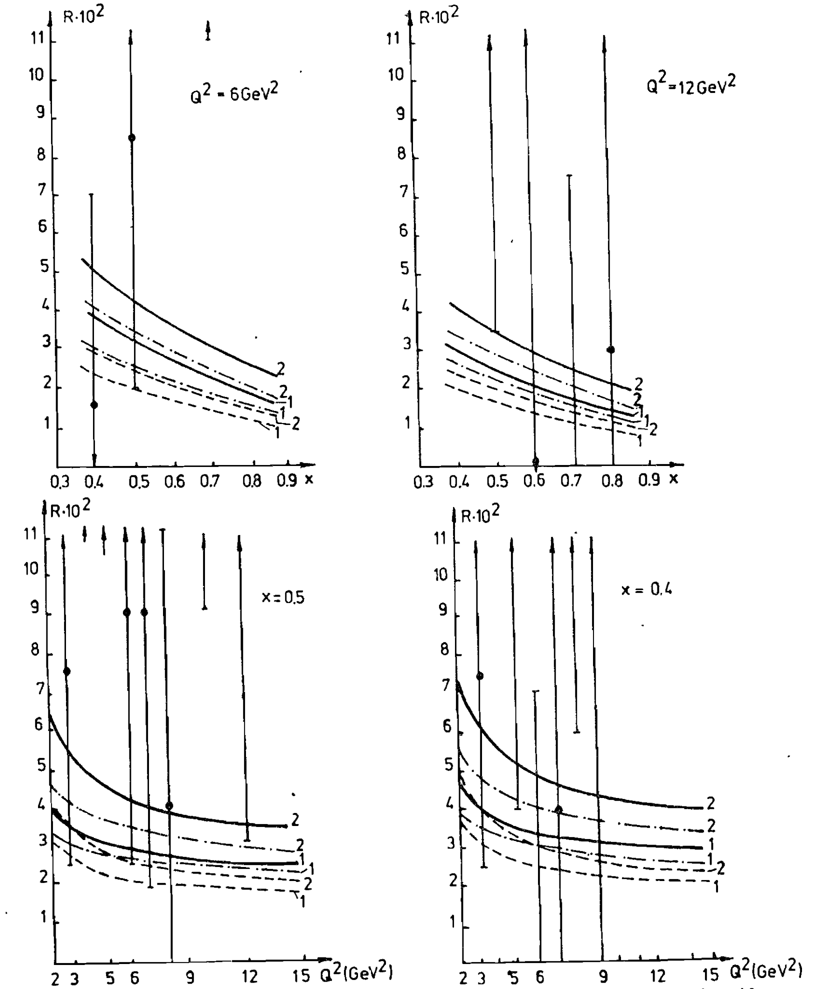

Fig. 2. Here the dashed, dotted and solid lines correspond to the leading order, two-loop MS and two-loop SIPT contributions respectively. The indices 1 and 2 reflect two different values of Λ.

As an input, like in ref. [1] we use the parametrization of F_2 at $Q_0^2 = 5\ \text{Gev}^2$

$$F_2(x, Q_0^2) = 2x^{1/2}(1-x)^3 .$$

Below the x and Q^2 dependence of $R(x,Q^2)$ is shown in the range

$$2\ \text{Gev}^2 \le Q^2 \le 15\ \text{Gev}^2 , \quad 0{,}3 \le x \le 0{,}9$$

We have used the following values of $\Lambda_{\overline{MS}}$ (and the corresponding Λ_{LO})

$\Lambda^{(1)}_{\overline{MS}} = 200$ Mev $(\Lambda^{(1)}_{LO} = 150$ Mev$)$, $\Lambda^{(2)}_{\overline{MS}} = 400$ Mev $(\Lambda^{(2)}_{LO} = 300$ Mev$)$.

4. Discussion

The account of a next-to-leading correction and the use of SIPT improve the agreement with experiment, especially for small x . This effect is more visible for large Λ . All the two-loop curves lie above those of the leading order contrary to the statement of ref. [3] . They cross the experimental points within the error but lie below the mean values. Large experimental uncertainties make it difficult the comparison with theoretical predictions so far. A contribution from higher twist terms probably is also important in a given energy range [3,14].

Appendix. Individual diagram contribution to $B_{1,n}(\overline{MS})$ for $n = 2m$

a) $$-C_F\left[S_1(n) + 3/2\right],$$

b) $$2C_F\left[-5/2 + \frac{1}{n+1}\right],$$

c) $$\frac{5C_A}{3}\left[S_1(n) + \frac{107}{30} - \frac{1}{n} - \frac{1}{n+1}\right] - \frac{4T_F}{3}\left[S_1(n) + \frac{19}{6} - \frac{1}{n} - \frac{1}{n+1}\right],$$

d) $$(2C_F - C_A)\left[2S_1(n)\left(\frac{2}{n}-1\right) - \frac{15}{2} + \frac{6}{n} + \frac{3}{n+1} - \frac{2}{n^2}\right] + C_A\left[3S_1^2(n) - S_2(n) + 2S_1(n)\left(2-\frac{1}{n+1}\right) + \frac{3}{2} + \frac{1}{n} - \frac{1}{n+1}\right],$$

e) $$-2C_F\left[2S_1(n)\left(\frac{1}{n} - \frac{1}{n+1}\right) + \frac{3}{2}\frac{1}{n} - \frac{3}{2}\frac{1}{n+1} - \frac{1}{n^2} + \frac{1}{(n+1)^2}\right],$$

f) $$-2C_F\left[S_1(n)\right],$$

g) $$(C_A - 2C_F)\left[\frac{4S_1(n)}{n} + \frac{3}{n} + \frac{2}{n+1} - \frac{2}{n^2}\right],$$

h) $$2C_F\left[4\left(\frac{2}{n+3} - \frac{1}{n+2}\right)\left\{S_3(n) + S_2(n)S_1(n) - 2T(n) + 6\zeta(3)\right\} + 4S_2(n)\left(\frac{1}{n+2} - \frac{3}{n+3}\right) + 2S_1(n)\left(\frac{1}{n+1} - \frac{2}{n+2} - \frac{5}{n+3}\right) + \frac{1}{2} - \frac{2}{n+1} + \frac{2}{n+3}\right],$$

i) $$C_A\left[4\left(\frac{1}{n+2} - \frac{2}{n+3}\right)\left\{S_3(n) + S_2(n)S_1(n) - 2T(n) + 6\zeta(3)\right\} + 4S_2(n)\left(-\frac{1}{n+2} + \frac{3}{n+3}\right) - S_1(n)\left(1 + \frac{2}{n+1} + \frac{4}{n+2} - \frac{10}{n+3}\right) + 1 - \frac{2}{n+3}\right],$$

j) $$(2C_F - C_A)\left[4\left(\frac{1}{n+2} - \frac{2}{n+3}\right)\left\{S_3(n) + S_2(n)S_1(n) - 2T(n) + 6\zeta(3)\right\} + 3S_1^2(n) - S_2(n)\left(1 + \frac{4}{n+2} - \frac{12}{n+3}\right) + 4K_2(n) + S_1(n)\left(5 - \frac{4}{n+1} - \frac{4}{n+2} + \frac{10}{n+3}\right) + 1 - \frac{2}{n+3}\right],$$

k) $$(2C_F - C_A)\left[8P(n) - 4K_3(n) - 4S_3(n) + 12\zeta(3) - \frac{5}{6}\left\{\frac{1-\delta_n^2}{n-2}\cdot(4K_2(n)-3) + \delta_n^2(6\zeta(3)-7)\right\} - 8K_2(n)\left(\frac{3}{2} + \frac{1}{n} - \frac{1}{n+1} - \frac{3}{5}\frac{1}{n+3}\right) - 6S_1(n) + 3 - \frac{18}{5}\frac{1}{n+3}\right],$$

where

$$T(n) = \sum_{k=1}^{n} \frac{S(k)}{k^2} .$$

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Казаков Д.И., Котиков А.В. E2-86-715

Поправка к отношению сечений глубоконеупругого рассеяния $R = \sigma_L/\sigma_T$ в КХД

Вычислена α_s поправка к продольной структурной функции ГНР. Результат применен для нахождения величины $R = \frac{\sigma_L}{\sigma_T}$ с использованием схемно-инвариантной теории возмущений. Показано, что учет нелидирующего вклада улучшает согласие с экспериментом.

Работа выполнена в Лаборатории Теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1986

Kazakov D.I., Kotikov A.V. E2-86-715

Correction to DIS Cross Section Ratio $R = \sigma_L/\sigma_T$ in QCD

The α_s correction to the longitudinal structure function of DIS is calculated within QCD. It is applied to the determination of $R = \sigma_L/\sigma_T$ in the framework of scheme invariant perturbation theory. The account of non-leading correction improves the agreement with experimental data.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1986