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EXACT SOLUTION OF THE MULTIFLAVOR GROSS – NEVEU MODEL

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The two-dimensional model of the principal chiral field $\Omega \in SU(N_c)$ with the lagrangian

$$L = \frac{1}{2} \lambda^{-1} \operatorname{tr} [(\partial_\mu \Omega^{-1}) \partial_\mu \Omega] \tag{1}$$

is interesting, particularly, because of its similarity to four-dimensional gauge theories (asymptotic freedom, dimensional transmutation, dynamical mass generation, instanton structure, etc.). Polyakov and Wiegmann/1/ have shown the equivalence of model (1) to the multiflavor ($N_f \to \infty$) chiral-invariant Gross-Neveu model

$$L = \bar{\psi}_j^a \gamma_\mu i \partial_\mu \psi_j^a - \lambda (\bar{\psi}_j^a \gamma_\mu T_A^{ab} \psi_j^b)^2, \tag{2}$$

where ψ_j^a are Dirac fermions with color ($a = 1 \dots N_c$) and flavor ($j = 1 \dots N_f$) indices, and T_A $(A=1 \dots N_c^2-1)$ are the color $SU(N_c)$ generators, $\operatorname{tr}(T_A T_B) = \frac{1}{2}\delta_{AB}$, $T_A^{ab} T_A^{cd} = \frac{1}{2}(\delta^{ad}\delta^{cb} - N_c^{-1}\delta^{ab}\delta^{cd})$. Although this model can directly be solved using the Bethe-ansatz technique, Polyakov and Wiegmann chose to solve another model where fermions have one flavor but transform under the rank-N_f symmetric representation of the color group (for $N_c=2$, representation of isospin $S = \frac{1}{2} N_f$). Equivalence of this higher-spin model to the multiflavor one (2) is much more subtle /2/. In fact, Wiegmann /3/ alleged its equivalence to a nonrelativistic substitute /4/ for model (2) in which even the number of degrees of freedom is half as small. Therefore, we are going to study the multiflavor relativistic model directly without referring to its nonrelativistic or higher-spin versions.

Now we give a more accurate formulation of the problem. First of all, model (2) with a single coupling constant λ cannot be renormalized in a consistent fashion. Perturbative quantum corrections lead to new counterterms, absent in the initial lagrangian. The most general renormalizable interaction lagrangian, invariant under the $SU(N_c) \otimes SU(N_f)$ and chiral transformations, involves four independent charges,

$$L_{int} = [f - g/N_c - \tilde{g}/N_f - h/(N_c N_f)](\bar{\psi}^a_j \gamma_\mu \psi^a_j)^2 - 2(g + h/N_f)(\bar{\psi}^a_j \gamma_\mu T_A^{ab} \psi^b_j)^2$$
$$- 2(\tilde{g} + h/N_c)(\bar{\psi}^a_j \gamma_\mu \tilde{T}^J_{jk} \psi^a_k)^2 - 4h(\bar{\psi}^a_j \gamma_\mu T_A^{ab} \tilde{T}^J_{jk} \psi^b_k)^2$$
$$= 4\, \psi^{*a}_{+j} \psi^{*b}_{-k} (f \psi^b_{-k} \psi^a_{+j} - g \psi^a_{-k} \psi^b_{+j} - \tilde{g} \psi^b_{-j} \psi^a_{+k} - h \psi^a_{-j} \psi^b_{+k}), \tag{3}$$

where $\gamma_0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\gamma_1 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$, $\psi = \begin{pmatrix} \psi_- \\ \psi_+ \end{pmatrix}$, $\bar{\psi} = \psi^\dagger \gamma_0 = (\psi_+^*, \psi_-^*)$ and $\tilde{T}^J$ are flavor $SU(N_f)$ generators. As we shall see below, there are three closed integrable subsectors with only two nonzero charges: f and any one of $g, \tilde{g}, h$.

The argument of ref. /1/ can be applied to the (f , g) subsector with external sources for flavor-singlet currents. As $N_f \to \infty$, this subsector is equivalent to model (1) with $\lambda = 2g$ plus two decoupled massless flavorless fields: the colorless one is due to the abelian $(\bar{\psi}^a_j \gamma_\mu \psi^a_j)^2$ interaction, and the $U(N_c)$ one to the fermion determinant. However, for $N_f \to \infty$, the latter is decoupled from the source terms too; so, integrating it off yields simply a constant. Thus, in addition to model (1), only colorless massless excitations are left.

More precisely, in this way one can get only L-singlet states of model (1) which is invariant under the global transformations $\Omega \to L \Omega R$. The reason is the following. For $N_f \to \infty$, in the auxiliary nonabelian field coupled to the fermion current, only the pure gauge $\Omega^{-1} \partial_\mu \Omega$ remains essential (the other part decouples into the massless sector). But this expression is just an L singlet, and its R transformation corresponds to the fact that the fermions belong to the fundamental representation of R.

The Bethe-ansatz diagonalization of the hamiltonian corresponding to eq.(3) proceeds in a standard fashion /5-9/. We seek eigenvectors of the form

$$|V\rangle = \int dx_1 \ldots dx_N \exp\left(i \sum_{n=1}^{N} k_n x_n\right) \sum_{Q \in S_N} \theta(x_{Q1} < \ldots < x_{QN}) V_Q \begin{pmatrix} a_1 \ldots a_N \\ j_1 \ldots j_N \end{pmatrix} \left[\prod_{n=1}^{N} \psi^{*a_n}_{\alpha_n j_n}(x_n) \right] |0\rangle \tag{4}$$

with a definite number $N_\pm$ $(N = N_+ + N_-)$ of pseudoparticles of chirality $\alpha_n = \pm 1$. In eq.(4), $\theta(\ldots) = 1$ if the condition is satisfied, and 0 otherwise. The hamiltonian acts on the wave function as a differential operator,

$$H = -i \sum_{n=1}^{N} \alpha_n \partial/\partial x_n + 4 \sum_{n=2}^{N} \sum_{m=1}^{n-1} \theta(\alpha_m \neq \alpha_n) \delta(x_m - x_n)(-f + g \mathcal{P}_{mn} + \tilde{g} \tilde{\mathcal{P}}_{mn} + h \mathcal{P}_{mn} \tilde{\mathcal{P}}_{mn}), \tag{5}$$

where $\mathcal{P}_{mn}$ and $\tilde{\mathcal{P}}_{mn}$ exchange, respectively, the color (a_m, a_n) and flavor (j_m, j_n) indices of V_Q.

Vector (4) is an eigenvector of the hamiltonian (5) with the eigenvalue E and momentum P,

$$E = \sum_{n=1}^{N} \alpha_n k_n , \qquad P = \sum_{n=1}^{N} k_n , \tag{6}$$

if the kinetic term acting on the step function cancels the interaction term. This occurs if the following relation holds for any neighboring permutation $Q^{(n)}$ that differs from Q by exchanging Qn with $Q(n-1)$:

$$V_{Q^{(n)}} = S_{Qn,\, Q(n-1)} V_Q , \tag{7}$$

where the S-matrix for pseudoparticles of different chiralities, when $\Delta_{mn} \equiv \frac{1}{2}(\alpha_m - \alpha_n) \neq 0$, is

$$S_{mn} = 2i \Delta_{mn} (i \Delta_{mn} - f + g \mathcal{P}_{mn} + \tilde{g} \tilde{\mathcal{P}}_{mn} + h \mathcal{P}_{mn} \tilde{\mathcal{P}}_{mn})^{-1} - 1. \tag{8}$$

In deriving eqs. (7), (8), we adopted the convention $\delta(x_m - x_n) \times \theta(x_m < x_n) = \frac{1}{2} \delta(x_m - x_n)$.

The set of conditions (7) imposed on V_Q is consistent only when S-matrix is factorizable /10/

$$S_{mn} S_{nm} = 1, \quad S_{lm} S_{ln} S_{mn} = S_{mn} S_{ln} S_{lm}, \quad \{k,l\} \cap \{m,n\} = \emptyset \Rightarrow [S_{kl}, S_{mn}]_- = 0. \tag{9}$$

Equations (9) restrict the arbitrary as yet S_{mn} for $\Delta_{mn} = 0$. In our case (8), the S-matrix is generally a linear combination of the four operator terms with coefficients depending on $\Delta = 0, \pm 1$ and the coupling constants. The analysis of eqs. (9) shows that all the four charges cannot be present simultaneously, and only three nontrivial cases are allowed by factorizability:

(i) $f, g \neq 0;\ \tilde{g}=h=0;\quad S^{\Delta}=e^{i\eta\Delta}\,\frac{2\zeta\Delta+i\mathcal{P}}{2\zeta\Delta+i};$

$$\zeta=\zeta(f,g)\equiv\tfrac{1}{4}(1+f^2-g^2)/g\ ;\quad \eta=\eta(f,g)\equiv 2\,\mathrm{atan}(g-f).$$

(ii) $f, \tilde{g}\neq 0;\ g=h=0;\quad S^{\Delta}=e^{i\eta\Delta}\,\frac{2\zeta\Delta+i\tilde{\mathcal{P}}}{2\zeta\Delta+i};\ \zeta,\eta=\zeta,\eta(f,\tilde{g}).$

(iii) $f, h\neq 0;\ g=\tilde{g}=0;\quad S^{\Delta}=e^{i\eta\Delta}\,\frac{2\zeta\Delta+i\mathcal{P}\tilde{\mathcal{P}}}{2\zeta\Delta+i};\ \zeta,\eta=\zeta,\eta(f,h).$

The only arbitrariness left is the change $S^0\to -S^0$, and in cases (i) and (ii), also $S^0\to\pm\mathcal{P}\tilde{\mathcal{P}}$. We are going to investigate the (f , g) subsector, case (i), which includes eq.(2), and hence, relates to eq.(1). The S-matrix (i) is diagonal in flavor indices which come into play only in filling the Dirac sea.

The periodic boundary conditions imposed on the wave function lead to the following eigenvalue equations for $V_{identity}$ and $\{k_n\}$:

$$Z_n V=e^{ik_nL}V,\qquad Z_n=S_{n,n+1}\cdots S_{nN}\,S_{n1}\cdots S_{n,n-1}\ . \tag{10}$$

For $N_c>2$, eqs.(10) lead to a Bethe-ansatz hierarchy /3,9,11/. Here, we concentrate our attention on $N_c=2$ /5-8, 10/. Eigenvectors of color isospin $\frac{1}{2}N-M$ are parametrized by sets of $\lambda_m (m=1\ldots M)$ satisfying the Bethe-ansatz equations

$$\left(\frac{\lambda_m-\zeta+\frac{1}{2}i}{\lambda_m-\zeta-\frac{1}{2}i}\right)^{N_+}\left(\frac{\lambda_m+\zeta+\frac{1}{2}i}{\lambda_m+\zeta-\frac{1}{2}i}\right)^{N_-}=-\prod_{l=1}^{M}\frac{\lambda_m-\lambda_l+i}{\lambda_m-\lambda_l-i}\ . \tag{11}$$

Eigenvalues are given by

$$e^{ik_nL}=\exp[\,i\eta\alpha_n N_{(-\alpha_n)}]\prod_{m=1}^{M}\frac{\lambda_m-\zeta\alpha_n+\frac{1}{2}i}{\lambda_m-\zeta\alpha_n-\frac{1}{2}i}\ . \tag{12}$$

Equations (11) have been analyzed in ref./8/. Antiferromagnetic solutions for $N_\pm\to\infty$ involve a sea of $R\approx\frac{1}{2}N$ real roots with H holes at x_h, and the following configurations of complex-conjugate pairs: W wide pairs, $|\mathrm{Im}\,\lambda|>1$; Q quartets, each consisting of two pairs with $\mathrm{Re}\,\lambda_1=\mathrm{Re}\,\lambda_2+O(e^{-cN})$, $|\mathrm{Im}\,\lambda_1|+|\mathrm{Im}\,\lambda_2|=1+O(e^{-cN})$; T two-strings, $|\mathrm{Im}\,\lambda|=\frac{1}{2}+O(e^{-cN})$. Complex pairs cannot be present without holes. The total number of roots

$$M=R+2W+4Q+2T=\tfrac{1}{2}N-\tfrac{1}{2}H+2W+2Q+T\leq\tfrac{1}{2}N \tag{13}$$

must be an integer; this connects oddness of N and H. Logarithm of eq.(12) determines

$$k_n=k_{\alpha_n}+2\pi L^{-1}l_n\ , \tag{14}$$

$$k_\pm L=\pm\Lambda N_\mp\pm\tfrac{3}{2}\pi N_\pm\ \pm 2\sum_{h=1}^{H}\mathrm{atan}\,\exp[\pi(-\zeta\pm x_h)], \tag{15}$$

$$\Lambda=i^{-1}\ \ln\frac{\Gamma(1+i\zeta)\,\Gamma(\frac{1}{2}-i\zeta)}{\Gamma(1-i\zeta)\,\Gamma(\frac{1}{2}+i\zeta)}+\eta-\tfrac{1}{2}\pi, \tag{16}$$

where l_n are integers.

Since the energy (6) is not restrained from below, we need an ultraviolet cut-off:

$$2\pi L^{-1}\alpha_n l_n\geq -K\ . \tag{17}$$

Arbitrariness of the cut-off procedure consists in the choice of branches in eq.(15). We have considered a general case when a multiple of $2\pi(N_\pm, M, H, W, Q, T, 1)$ is added to eq.(15). The coefficients can be fixed from physical requirements that the vacuum should have chirality and momentum zero, be a color and flavor singlet without holes, and excitations should have finite energy and momentum. This leads to a choice equivalent to eq. (15).

After introducing the cut-off (17), we can construct the ground state. Anticommutativity of the fermion operators in eq.(4) requires that for equal chiralities $(\alpha_m=\alpha_n)$ and flavors $(j_m=j_n)$ the wave vectors must be different $k_m\neq k_n$; hence, then $l_m\neq l_n$ in eq.(14). For both chiralities, each of the lowest vacancies allowed by eq.(17) can be occupied only by N_f particles of different flavors, which form together a flavor singlet. If $N_\pm=\mu_\pm N_f-\nu_\pm$ with an integer $\mu_\pm$ and $0\leq\nu_\pm<N_f$, then at the densest packing, $\mu_\pm-1$ layers are filled entirely, and on the last level, $\nu_\pm$ places remain empty. The energy and momentum (6) will then be

$$E=L^{-1}(\tfrac{3}{2}\pi+\pi/N_f)(N_+^2+N_-^2)+(2\Lambda/L)N_+N_--(K+\pi/L)N$$
$$+\pi/(LN_f)[\nu_+(N_f-\nu_+)+\nu_-(N_f-\nu_-)]+(2/L)\sum_{h=1}^{H}\{N_+\mathrm{atan}\exp[\pi(-\zeta+x_h)]+N_-\mathrm{atan}\exp[\pi(-\zeta-x_h)]\}, \tag{18}$$

$$P = L^{-1}\left(\tfrac{3}{2}\pi+\pi/N_f\right)\left(N_+^2-N_-^2\right)-\left(K+\pi/L\right)\left(N_+-N_-\right)$$
$$+\pi/(LN_f)\left[\nu_+(N_f-\nu_+)-\nu_-(N_f-\nu_-)\right]+(2/L)\sum_{h=1}^{H}\left\{N_+\,\mathrm{atan}\exp[\pi(-\zeta+x_h)]-N_-\mathrm{atan}\exp[\pi(-\zeta-x_h)]\right\}. \quad (19)$$

Minimizing the energy (18) with respect to $H, \nu_\pm, N_\pm$ determines the vacuum:

$$H_* = \nu_\pm^* = 0, \quad N_\pm^* = \frac{1}{2}\frac{KL+\pi}{\frac{3}{2}\pi+\pi/N_f+\Lambda}, \quad E_* = -\frac{1}{2}\frac{(KL+\pi)^2/L}{\frac{3}{2}\pi+\pi/N_f+\Lambda}, \quad P_* = 0. \quad (20)$$

Subtracting the vacuum energy, for excitations with $N_\pm = N_\pm^* + n_\pm$ we now let $K\to\infty$, leaving $n_\pm$ finite. The charges (i) become functions of the cut-off parameter so as to eliminate divergencies from physical quantities. Finiteness of the minimum hole energy, at $x_h = O(1/K)$, requires that

$$\zeta = \pi^{-1}\ln(K/\mu) + o(1), \quad (21)$$

where μ is a scale parameter. In the momentum, eq.(19), besides terms known to be finite, a K term is left

$$P_K = \Lambda(n_- - n_+)\; K/\left(\tfrac{3}{2}\pi+\pi/N_f+\Lambda\right). \quad (22)$$

To avoid divergency when $n_+ \neq n_-$ (these states are physically attainable because the energy is finite for any $n_\pm$), we have to set $\Lambda = O(1/K)$ or $o(1/K)$ so as to make P_∞ vanish. Through eq. (16), this entails a correlation /6,7/ between the two charges to all orders in perturbation theory

$$f = \tfrac{1}{2}g - \tfrac{3}{4}g^3 + \dots, \quad g>0. \quad (23)$$

Now, as $K\to\infty$, energy and momentum of excitations take the form

$$E-E_* = L^{-1}\left(\tfrac{3}{2}\pi+\pi/N_f\right)(n_+^2+n_-^2)+\pi/(LN_f)\left[\nu_+(N_f-\nu_+)+\nu_-(N_f-\nu_-)\right]+m\sum_{h=1}^{H}\cosh(\pi x_h), \quad (24)$$

$$P = L^{-1}\left(\tfrac{3}{2}\pi+\pi/N_f\right)(n_+^2-n_-^2)+\pi/(LN_f)\left[\nu_+(N_f-\nu_+)-\nu_-(N_f-\nu_-)\right]+m\sum_{h=1}^{H}\sinh(\pi x_h),$$

where, $m = 2\mu/\left(\tfrac{3}{2}\pi+\pi/N_f\right)$ is a renormalized mass. If $n_\pm \propto L$, the first terms in eqs.(24) correspond to relativistic massless particles, and the $\nu_\pm$ terms are bounded and vanish as $L\to\infty$.

Changes in ℓ_n also lead to massless, colorless, flavorless excitations.

Thus, massive spectrum of subsector (i) of model (3) includes relativistic fermions with rapidities πx_h. They have color spin $\frac{1}{2}$ because each hole diminishes M by $\frac{1}{2}$. The spins combine in different ways depending on the presence of complex pairs. The transformation under SU(N_f) can be arbitrary according to $n_\pm$. In the simplest case when we add one pseudoparticle for each hole, we obtain the fundamental representation. However, at even H, the numbers $n_\pm$ may be multiples of N_f, which leads to flavor singlets. These flavorless bosonic states represent the L-singlet sector of the SU(2) principal chiral field (1) in agreement with the treatment of the model by Faddeev and Reshetikhin /12/.

The S-matrix for holes is diagonal in flavor indices and can be calculated /6-8/, irrespective of N_f. It has the form as for the $N_f=1$ chiral-invariant Gross-Neveu model /6-9,13/ or for the spin - $\frac{1}{2}$ XXX magnet /14/. The free massless particles decouple. Such an S-matrix for holes does not corroborate the assumption /15/ that in the $O(4)$ σ model, fundamental massive particles are in the vector representation of $O(4)$. Thus, the fermionization program can be performed in a rigorous fashion and leads to the results that differ from those obtained by solving the Bethe-ansatz equations for higher spin $S\to\infty$ /1,3,12/.

Now, we would like to compare the exact solution with perturbation theory. Equations (21) and (23) allow us to find the renormalization-group β functions:

$$\beta_g \equiv \partial g/\partial \ln K^2 = -(2/\pi)g^2 + o(g^3), \quad (25)$$

and β_f is expressed through β_g via eq.(23). The $o(g^3)$ term of eq.(25) does not influence the $K\to\infty$ limit.

Our computations by the method of ref./16/ give the following one-loop β functions for model (3):

$$\pi\beta_f = -g^2-\tilde{g}^2-h^2, \quad \pi\beta_g = -N_c g^2 + 2\tilde{g}h,$$
$$\pi\beta_{\tilde{g}} = -N_f\tilde{g}^2+2gh, \quad \pi\beta_h = -N_c N_f h^2 - 2N_c gh - 2N_f\tilde{g}h. \quad (26)$$

In subsector (i) the special solution to eqs.(26) with proportional charges is $f=g/N_c$. This agrees with the SU(N_c) exact result of refs./6,7/ for $N_f=1$ and with eq.(23) at $N_c=2$. Since the β functions (25) and (26) in subsector (i) do not depend on N_f, it is worth comparing eq.(25) with the two-loop result at $N_f = 1$. We have computed the β functions for a generalized model with the interaction lagrangian

$$L_{int} = f(\bar{\psi}^{\alpha}\gamma_{\mu}\psi^{\alpha})^2 + g_1(\bar{\psi}^{\alpha}\psi^{\alpha})^2 - g_2(\bar{\psi}^{\alpha}\gamma_{*}\psi^{\alpha})^2, \quad (27)$$

where $\gamma_* = \gamma_0\gamma_1$. At $g_1 = g_2 = g$, eq.(27) is reduced through the Fierz rearrangement to eq.(3) with one flavor ($\tilde{g} = h = 0$); and at $f = g_2 = 0$, to the Gross-Neveu model /17/. To avoid difficulties with γ_*, we used the analytic renormalization /18/. The two-loop result has the form

$$\beta_f = -\pi^{-1} g_1 g_2 + \pi^{-2}[(N_c - 1) f (g_1 - g_2)^2 + g_1 g_2 (g_1 + g_2)],$$
$$\beta_1 = \pi^{-1}[-N_c g_1^2 + (2f + g_1)(g_1 - g_2)] + \pi^{-2}\{N_c g_1 (g_1^2 + g_2^2) + [2N_c f^2 - 2f(g_1 + g_2) - g_1^2](g_1 - g_2)\}, \quad (28)$$

β_2 is obtained from β_1 by exchanging g_1 and g_2. This corrects and extends to two loops our previous result /19/. The chiral-invariant one-charge solution to eq.(28) is $g_1 = g_2 = N_c f$ up to two loops, and

$$\beta_1 = -N_c g_1^2/\pi + 2N_c g_1^3/\pi^2. \quad (29)$$

Besides, there are only two chiral-noninvariant solutions at $N_c = 2$: $g_1 = -g_2 = \pm 2f$, $\beta_1 = \pm 2g_1^2/\pi + 4g_1^3/\pi^2$. At $f = g_2 = 0$, eq.(28) agrees with ref./20/.

As has been foreseen by Destri /2/, eq.(29) with $N_c = 2$ differs from eq.(25). However, the renormalization schemes of the Bethe ansatz and perturbation theory are different as well, and the charge should be recalculated from one scheme to another. All schemes must agree in the tree approximation; therefore, $g_1 = g + o(g)$, but generally, no constraint on $o(g)$ can be set beforehand. Of course, if g_1 was a power series in g, the two-loop β functions would be the same /21/, which is just the case for usual perturbative renormalization schemes in single-charge theories. The difference between eqs. (25) and (29) indicates that the Bethe-ansatz scheme is nonanalytically related to conventional perturbation theory $g_1 = g - (2/\pi)g^2 \ln g + o(g^2 \ln g)$, like in the Kondo model /7,22/. However,

such a recalculation is permitted by the conformity principle.

In conclusion, we point out possible generalizations of the present work. For the SU(N_c) principal chiral field with $N_c > 2$ /3/ all our reasonings up to eq.(10) remain valid. One may try to solve the Bethe-ansatz hierarchy without invoking the "string" hypothesis. Obviously, like for $N_c = 2$, the results will be analogous to the $N_f = 1$ case: the mass spectrum, the S-matrix /9/, and the correlation between the charges /6,7/. The scheme can be extended to the anisotropic (XXZ, XYZ) principal chiral field /23/ as well. Another situation takes place when we try to introduce the Wess-Zumino term. After fermionization, one obtains a theory with different numbers of left and right fermions /24/. In such a theory there is in fact an anomaly, which does not allow one to renormalize the model consistently. In filling the Dirac sea, any choice of branches leads to infinities in momentum of some excited states. There are difficulties in perturbation theory too.

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Л.В.Авдеев, М.В.Чижов E2-86-663

Точное решение многокомпонентной модели Гросса-Невё

Фермионизация по Полякову - Вигману главного кирального поля с группой SU(2), т.е. $0(4)$-σ-модели, проводится строгим образом до конца. Вместо того, чтобы решать фермионную модель высшего спина, мы изучаем многокомпонентную кирально-инвариантную модель Гросса - Невё, эквивалентность которой исходной бозонной теории была показана строго, если число компонент стремится к бесконечности. Выполняется диагонализация релятивистской модели с помощью анзатца Бете, проводится заполнение моря Дирака, построены физический вакуум и возбуждения с конечной энергией и импульсом. Обнаружено, что две константы взаимодействия в модели должны быть связаны. Для массивных физических возбуждений S-матрица оказывается такой же, как и в однокомпонентном случае. Это не подтверждает предположения о том, что фундаментальные частицы σ-модели находятся в векторном представлении $0(4)$. Ренормгрупповая β-функция согласуется с однозарядным решением по теории возмущений.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Exact Solution of the Multiflavor Gross-Neveu Model

The Polyakov - Wiegmann fermionization of the SU(2) principal chiral field in two dimensions, i.e., the 0(4) sigma model, is fulfilled in a rigorous fashion to the end. Instead of solving a higher-spin fermionic theory, we study the multiflavor chiral-invariant Gross - Neveu model, strictly shown to be equivalent to the initial bosonic theory if the number of flavors tends to infinity. The Bethe-ansatz diagonalization of the relativistic model is performed, the filling of the Dirac sea is accomplished, the physical vacuum and excitations with finite energy and momentum are constructed. A correlation between the two couplings in the model is found necessary. The S-matrix for the massive physical excitations proves to be the same as in the one-flavor case. This does not corroborate the assumption that the fundamental particles of the sigma model are in the vector representation of 0(4). The renormalization-group beta function agrees with a one-charge solution in perturbation theory.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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