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LATTICE VORTICES IN THE TWO-DIMENSIONAL ABELIAN HIGGS MODEL

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1. Introduction

The relevance of instanton configurations /1,2/ in the vacuum of quantized Yang-Mills theories has been newly confirmed by lattice investigations /3-6/, recently. Similarly to what has been exercized originally in the non-linear O(3) σ-model /7,8/, the quantum fluctuations manifested in Monte-Carlo generated equilibrium field configurations have been smoothed away by a relaxation procedure ("cooling"), leaving quasistable background fields with actions and well-defined topological charges in close correspondence to continuum multi-instantons. Moreover, massless staggered lattice fermions find a number of zero-modes carried by these background fields in strict accordance to the Atiyah-Singer index theorem /4/ (modulo fermion doubling) for (anti-) selfdual Yang-Mills fields. A background topological susceptibility has been estimated /4,5,9/ reasonably close to the phenomenologically expected value, in spite of the relatively small lattices simulated so far.

Obviously, it is interesting to investigate on the lattice other models, too, which possess topological excitations. Examples recently studied are the Georgi-Glashow model /10/ and a three-dimensional spin model /11/. In this paper we address the topologically non-trivial excitations of the 2D Abelian Higgs model. The classical field equations of this model are known to have multi-vortex solutions /12-16/. The latter describe translation invariant quantized magnetic flux tube configurations within the 3D phenomenological Ginzburg-Landau theory of superconductivity /17/. To our knowledge, not much is known about the explicit form of general solutions beyond existence theorems and some basic properties of multi-vortices. Therefore, a numerical study like ours may provide some more useful information for those interested in superconductors. However, the scope of our present investigation is merely to demonstrate that "lattice vortices" can be identified at all and possess characteristics known from their continuum counterparts. As far as the Euclidean quantized 1+1 D Abelian Higgs model is concerned, we want to address the question as well, whether a dilute gas picture can be substantiated. A first and preliminary study of this kind was published in Ref./18/.

Additionally, on the technical side, we take the opportunity of the 2D model just to gather more experience with relaxation procedures and with applying various lattice prescriptions for the topological charge. In particular, we shall be able to be more explicit about the role of "dislocations" /19,7/, a notion which comprises shortest-range lattice excitations which still do carry a topological charge (according to one or another lattice prescription). Dislocations are blamed to obscure the topological charge to be ascribed to the generic strongly fluctuating Monte-Carlo configuration. Therefore, they may be the cause for Monte-Carlo overestimates of the topological susceptibility /19/ and for deviations from scaling.

In order to make the paper selfexplanatory, we will reproduce the main facts concerning the continuum theory in Section 2, before we are formulating the model on the lattice in Section 3. Different relaxation procedures are described in Section 4, followed by a presentation of the numerical results in Section 5. The conclusions are to be drawn in Section 6.

2. The Abelian Higgs model and its vortex solutions

We consider an Abelian gauge field A_i coupled to a complex scalar Higgs field ϕ in two (Euclidean) dimensions. The action of the model is defined

$$S \equiv \int d^2x \left\{ \frac{1}{4} F_{ij}^2 + |D_i\phi|^2 + \lambda \left(|\phi|^2 - f^2\right)^2 \right\} \tag{2.1}$$

with $F_{ij} \equiv \partial_i A_j - \partial_j A_i$ and $D_i \equiv \partial_i - igA_i$.

The scalar selfcoupling λ is understood positive. Thus, we are in the Higgs phase describing a neutral scalar and a neutral vector boson with masses

$$m_s = 2f\sqrt{\lambda} \qquad \text{and} \qquad m_V = \sqrt{2}\, fg \ , \tag{2.2}$$

respectively. In the following we will put $f^2 = 1/2$.

All field configurations can be subdivided into equivalence classes labelled by an integer topological charge

$$Q = \frac{g}{4\pi} \int d^2x\ \varepsilon_{ij} F_{ij} = \frac{g}{2\pi} \int d^2x\ F_{12} \ . \tag{2.3}$$

The latter can be rewritten as a sum of winding numbers corresponding to the homotopy group $\pi_1(S_1)$ of mappings from a compact one-dimensional manifold S_1 onto the gauge group U(1). E.g., if

asymptotically

$$\left.\begin{array}{ll} \phi(r,\theta) \xrightarrow[r=|x|\to\infty]{} f\cdot \mathcal{U}(\theta) , & \mathcal{U}(\theta) = e^{i\varphi(\theta)} \in U(1) \\ A_i(r,\theta) \xrightarrow[r\to\infty]{} \frac{1}{ig} \mathcal{U}^{-1}(\theta)\, \partial_i\, \mathcal{U}(\theta) & \end{array}\right\} , \tag{2.4}$$

then we get a contribution to Q from a contour at infinity:

$$W_\infty = \frac{g}{2\pi} \oint_{r\to\infty} dx_i\, A_i = \frac{i}{2\pi} \int_0^{2\pi} d\theta\ \mathcal{U} \frac{d\mathcal{U}^{-1}}{d\theta} \in \mathbb{Z} \ . \tag{2.5}$$

The classical field equations are the following ones

$$\begin{aligned} \partial_i F_{ij} &= 2\, \mathrm{Im}\left(\phi\, \overline{D_j\phi}\right) , \\ D^2\phi &= 2\lambda\phi\left(|\phi|^2 - \tfrac{1}{2}\right) . \end{aligned} \tag{2.6}$$

The character of their solutions strongly depends on the ratio of the couplings λ and g^2

$$b \equiv 2\frac{\lambda}{g^2} \ . \tag{2.7}$$

In the context of the Ginzburg-Landau theory **b = 1** characterizes the transition between type I superconductivity ($b<1$, complete expulsion of magnetic flux) and type II superconductivity ($b>1$, with gradual penetration of magnetic flux tubes repelling each other).

In this paper we concentrate on the case $b = 1$, for which Bogomolny has proven the bound /13/

$$S \geq \pi N \ , \qquad |Q| = N \ , \qquad N = 0,1,2,\ldots \tag{2.8}$$

and for which the only smooth solutions with finite action are known to be either exact N-vortex or N-antivortex configurations, saturating the Bogomolny bound in (2.8) /15/. Thus, for $b = 1$ there is a close analogy to the 4D Yang-Mills theory. The multi-(anti-) vortices are localized around the pointlike zeros of the Higgs field ϕ. They satisfy an exponential bound

$$\left.\begin{array}{r} (f^2 - |\phi|^2) \leq M \exp\left(-(1-\varepsilon)\, m_S |x|\right) \\ F_{12} \leq M \exp\left(-(1-\varepsilon)\, m_V |x|\right) \end{array}\right\} \tag{2.9}$$

we demand finite action, i.e. the following pure gauge behaviour with certain numbers $0<\varepsilon$ and $0<M(\varepsilon)<\infty$ /12,15/.

3. The Abelian Higgs model on the lattice

We start with the Wilson-like lattice action

$$S_L = \sum_n \Big\{ \beta_1 (1-\cos\Theta_p(n)) + \beta_2 \sum_{i=1}^{2} \big[R_{n+\hat{i}}^2 + R_n^2 - 2R_n R_{n+\hat{i}} \cos(\varphi_{n+\hat{i}} - \varphi_n + \Theta_{n,i})\big] + \beta_3 (R_n^2 - f^2)^2 \Big\}, \quad (3.1)$$

where $\Theta_{n,i}$ represents the compact link variables $U_{n,i} = \exp i\Theta_{n,i}$ $(i = 1,2)$, and the Higgs fields reside on the lattice sites $n \equiv x_n$: $\phi_n = R_n \exp i\varphi_n$. $\Theta_p(n)$ abbreviates the plaquette angle, i.e.,

$$\Theta_p(n) = \Theta_{n,1} + \Theta_{n+\hat{1},2} - \Theta_{n+\hat{2},1} - \Theta_{n,2} \quad . \quad (3.2)$$

In the naive continuum limit (with lattice spacing $a \to 0$) one realizes that (3.1) turns into (2.1) with the identification of the coupling constants

$$\beta_1 = \frac{1}{a^2 g^2} \quad , \quad \beta_2 = 1 \quad \text{and} \quad \beta_3 = \lambda a^2 \, . \quad (3.3)$$

Adhering to the case of $b = 1$, we find that the (classical) continuum limit corresponds to simultaneously $\beta_1 \to \infty$ and $\beta_3 \to 0$ with

$$2\,\beta_1 \beta_3 = 1 \quad (3.4)$$

kept fixed.

To our knowledge the continuum limit of the corresponding quantized theory has not yet been investigated. We do not know, therefore, about the existence of any "scaling window" and how β_1 should be related to the lattice spacing a. In our numerical investigations we took $\beta_1 \geqslant 0.5$ and worked on a 16^2 lattice with periodic boundary conditions.

Several topological charge definitions on a lattice have been discussed recently by Panagiotakopoulos /20/. He argued that Lüscher's prescription to construct the charge locally in terms of transition functions relating the gauges defined throughout adjacent cells /21/, is equivalent in the present context to

$$Q_L \equiv \frac{1}{2\pi} \sum_n \, [\, \Theta_p(n)] \; , \quad (3.5)$$

where $[\Theta_p]$ instructs to reduce the plaquette angles to the interval $[-\pi,+\pi]$. Q_L is invariant under continuous deformations of the lattice field as long as no exceptional configurations are involved. Such configurations, for which the topological charge cannot be assigned unambiguously have $[\Theta_p(n)] = \pm\pi$ for one plaquette at least.

The so-called "background charge" operator proposed by Panagiotakopoulos /20/ disregards all local excitations with large plaquette actions

$$1 - \cos\Theta_p > \varepsilon \quad , \qquad 0 < \varepsilon < 2 \quad (3.6)$$

and replaces their contribution by

$$Q_B^{\varepsilon} = \frac{1}{2\pi} \sum_{\wp^\varepsilon} \Big[\sum_{n\in\wp^\varepsilon} \Theta_p(n) \Big] + \frac{1}{2\pi} \sum_{n'} [\, \Theta_p(n')] \, . \quad (3.7)$$

$\wp^\varepsilon$ denotes the maximally connected clusters of "excited" plaquettes falling under inequality (3.6) and those having a common link with them. The remaining sum in (3.7) runs over the "normal" plaquettes. Q_B^{ε} happens to be unambiguously defined as long as

$$\Big[\sum_{n\in\wp^\varepsilon} \Theta_p(n) \Big] \neq \pm\pi \; .$$

Obviously, it is only in the continuum limit $\beta_1 \to \infty$ that "excited" plaquettes receive a vanishing statistical weight. Thus, strictly only in this limit one is allowed to let ε tend to zero.

4. Cooling the gauge and Higgs fields

Having at our disposal generic Monte-Carlo generated equilibrium field configurations, our aim is to smooth the quantum fluctuations by minimizing the action S_L . Approximate solutions of the classical equations of motion should become gradually discernible before one runs into relatively long-living field configurations, being quasi-stable under the cooling iterations.

The first procedure which comes in mind is the Metropolis Monte-Carlo algorithm modified as to accept only those (small) stochastic changes that lower the action. But, whenever we applied this method we found the fields trapped in artificial "stationary" points with action large compared to few multiples of π we expect according to Eq. (2.8).

One does not encounter this problem using the following method, that we call "deterministic" cooling in contradistinction to the previous one. This procedure has been successfully applied in the Yang-Mills case before /3,4/. It is a local process, too, in so far one tries to solve the coupled system of the four equations of motion

$$\frac{\partial S_L}{\partial \chi_n^\alpha} = 0 \qquad \text{with} \quad \chi_n^\alpha = (\theta_{n,1}, \theta_{n,2}, R_n, \varphi_n) \tag{4.1}$$

leaving the other degrees of freedom (χ_m^α , $m \neq n$) untouched. The Eqs. (4.1) have to be supplemented with the second derivative constraints ($\partial^2 S_L / \partial \chi_n^\alpha \partial \chi_n^\beta$ positive definite). It is difficult to solve the equations (4.1) simultaneously. Therefore, we decided to replace successively the old variables $\theta_{n,1}$, $\theta_{n,2}$, R_n, φ_n by the solutions of the corresponding equation out of (4.1), before stepping to the next site in a standard sequence. The dependence on the particular succession of exposing the different degrees of freedom to relaxation is certainly a disadvantage of this cooling procedure. The equation containing $\partial S_L / \partial R_n$ leads to a cubic equation . From its coefficients one can infer that the existence of a zero solution, for instance in the vortex soul, presupposes that

$$\frac{2\beta_2}{\beta_3} - f^2 \geq 0. \tag{4.2}$$

Indeed, this can be assumed to be the case in the continuum limit ($\beta_3 \longrightarrow 0$). The deterministic cooling changes the action drastically and, actually, drives the configuration rapidly into the trivial one.

Therefore, we have applied, instead, the smooth "relaxation" procedure, which accomplishes an evolution of the field configuration in "time" τ according to the relaxation equations

$$\frac{d\chi_n^\alpha}{d\tau} = -\frac{\partial S_L}{\partial \chi_n^\alpha} \tag{4.3}$$

which one can imagine as Langevin equations (widely applied in simulating quantum systems) having the noise switched off. Actually, the fields evolve according to successive replacements

$$\chi_n^\alpha \longrightarrow (\chi_n^\alpha)_{new} = (\chi_n^\alpha)_{old} - \Delta\tau \frac{\partial S_L}{\partial \chi_n^\alpha}\Big|_{old} \tag{4.4}$$

for all degrees of freedom $\chi_n^\alpha = (\theta_{n,1}, \theta_{n,2}, R_n, \varphi_n)$ simultaneously. The time step $\Delta\tau$ can be chosen appropriately to ensure a gentle relaxation.

The practical experience with this method showed, however, that all configurations stabilized intermediately with uniformly almost vanishing Higgs field, such that

$$S_L \gtrsim S_{crit} = \sum_n \beta_3 f^4 \tag{4.5}$$

and the small actual difference is accounted for by the gauge field action alone. It was interesting to notice that the topological charge (3.5) remains constant through the whole critical region. In order to accelerate the adjustment of the Higgs field to the frozen topology of the gauge field, we applied deterministic cooling within the region

$$1.25\, S_{crit} \geq S_L \geq S_{crit} , \tag{4.6}$$

where it changes the configuration still gently. Afterwards, the cooling procedure switches back to relaxation.

The driving "forces" in Eq. (4.3) can be used at the same time to quantify the degree of violation of the equations of motion by the actual lattice configuration. We define the quantity

$$\Delta \equiv \sum_n \left(\left(\frac{\partial S_L}{\partial \theta_{n,1}}\right)^2 + \left(\frac{\partial S_L}{\partial \theta_{n,2}}\right)^2 + \left(\frac{\partial S_L}{\partial R_n}\right)^2 + \left(\frac{\partial S_L}{\partial \varphi_n}\right)^2 \right) \tag{4.7}$$

as a measure of "classicality" of a given configuration.

5. Results

For couplings $\beta_1 = 0.5$ and 1.0 we have generated equilibrium samples of 50 and 100 configurations, respectively, by the Metropolis method. The hot start we began

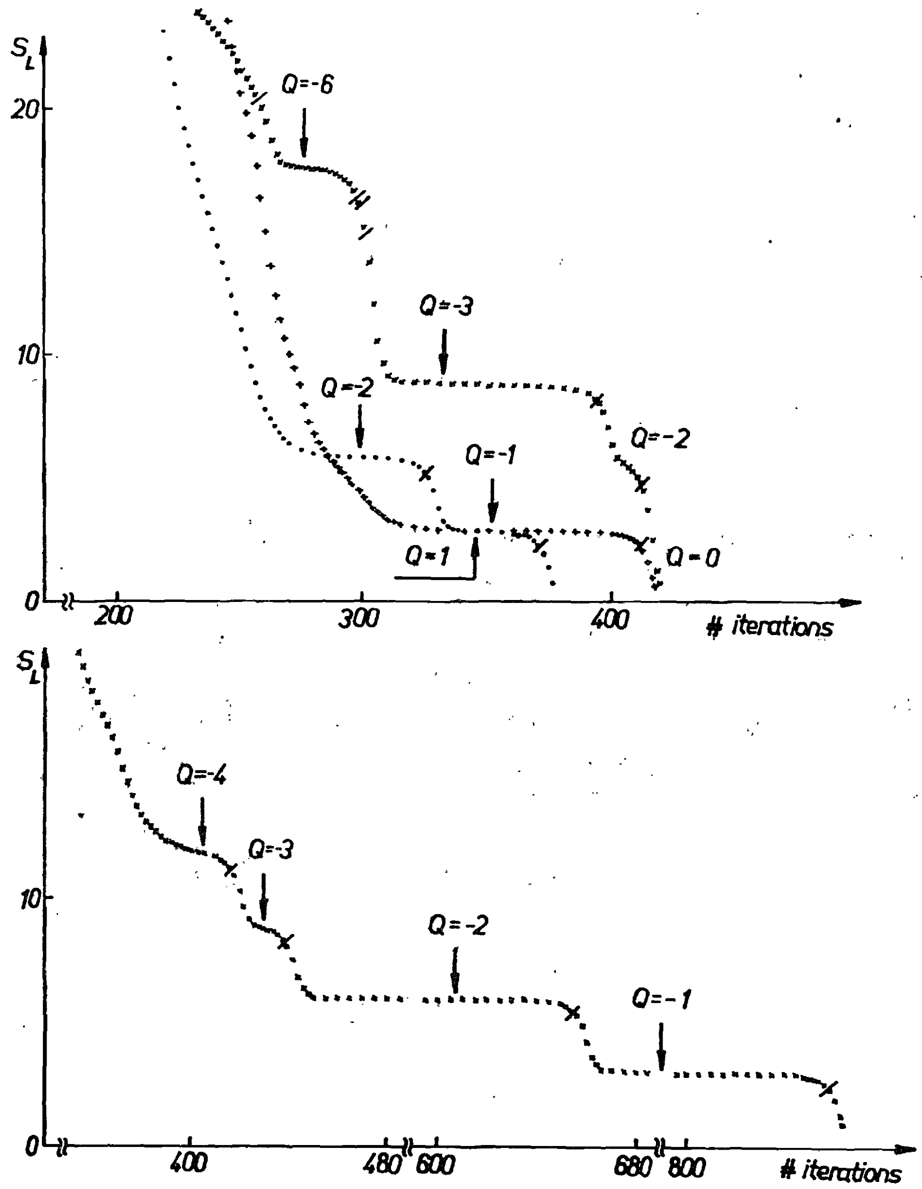

Fig. 1. Typical relaxation histories exhibiting quasi-stable background configurations ($\beta_1 = 0.5$).

from obtained angle variables uniformly distributed over $(-\pi,+\pi)$ and the radial modes chosen according to an ultralocal distribution (depending on the Higgs potential alone). The standard Metropolis Monte-Carlo was performed with maximally 20 hits per degree of freedom. 50 sweeps have been discarded for "equilibration" and the samples above were formed by configurations drawn successively after each 10 sweeps in between. Each configuration from the respective sample has been subject to relaxation according to (4.4) with $\Delta\tau = 0.1$ (except for the intermediate deterministic cooling). After each relaxation step the total action (3.1), the topological charge (3.5) and the "classicality" Δ (4.7) were monitored globally and locally.

Typical relaxation histories are shown in Fig. 1. We see cascading plateaus with quantized action values and with (integer) topological charges related one to another as

$$S_L = |Q_L| \cdot \tilde{\pi}_L(\beta_1) \tag{5.1}$$

The quantum of action, $\tilde{\pi}_L$, has been measured to be

$$\tilde{\pi}_L(0.5) \simeq 2.96 \quad , \qquad \tilde{\pi}_L(1.0) \simeq 3.06 \ .$$

From a very few configurations generated at higher β_1 values we have found by the same cooling procedure

$$\tilde{\pi}_L(2.0) \simeq 3.10 \quad , \qquad \tilde{\pi}_L(10.0) \simeq 3.14 \ .$$

This suffices to see that $\tilde{\pi}_L$ tends to π in the continuum limit $\beta_1 \longrightarrow \infty$. To be more precise, we determined the ratio $S_L / |Q_L|$ always for those plateau configurations (marked by arrows in Fig. 1) which minimally violate the equations of motion (4.1). The corresponding values of Δ for long-living plateaus (several hundred iterations) are of the respective order of magnitude

$$\Delta(\beta_1 = 0.5) \simeq O(10^{-5}) \ , \quad \Delta(\beta_1 = 1.0) \simeq O(10^{-6}) .$$

We also observe that the average life-time (plateau length) increases with β_1.

If we keep always the first (highest) shoulder, we are able

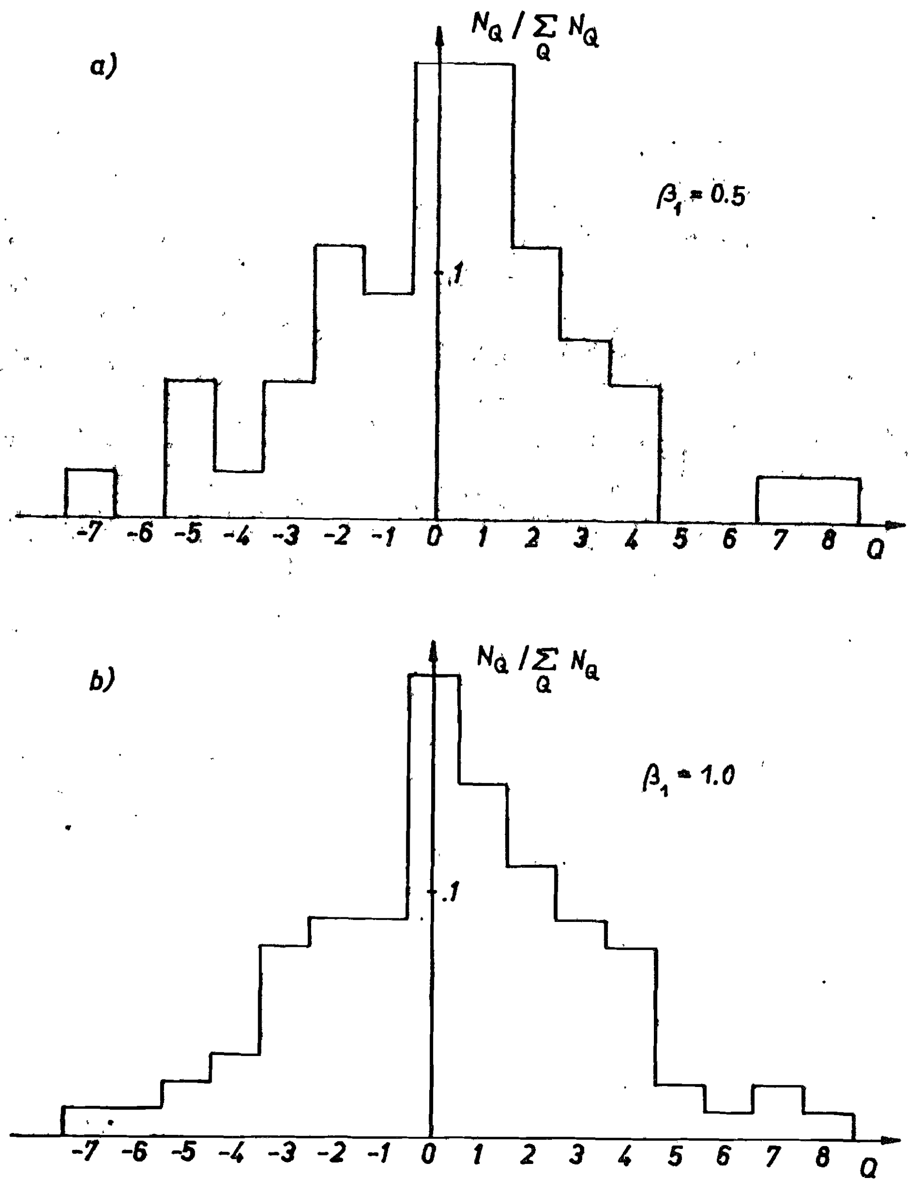

Fig. 2. Topological background charge distributions (a) $\beta_1 = 0.5$, b) $\beta_1 = 1.0$).

to assign a "background topological charge" to any of the Monte Carlo equilibrium configurations (not to be confused with definition (3.7)). In this way we have found the "background charge" distributions of Figs. 2a,b. Of course, relaxation histories, which do not pass through any plateau at all, are the most probable ones. Distributions of this kind can serve in principle to determine a "background topological susceptibility", similar to what has been done in Refs. /4,5,9/.

For the sample of configurations generated at $\beta_1 = 0.5$ we have tested also the reliability of the background charge operator Q_B^{ε} (according to Eq. (3.7)), that had the purpose to circumvent ill-defined topologies for configurations containing excessively excited plaquettes. Already for $\varepsilon = 0.5 \div 0.65$ we realized on the plateaus always

$$Q_B^{\varepsilon} = Q_L , \qquad (5.2)$$

i.e., the quasi-stable plateau configurations possess an unambiguously defined topology.

We show in Fig. 3 the densities of the action (a) and of the topological charge Q_L (b) for a $Q_L = -1$ configuration (generated and cooled at $\beta_1 = 2.0$), which turns out to be a clearly localized, vortex-like object. It is interesting to see how this structure is reflected by the modulus of the Higgs field $|\phi_n| = R_n$. In Fig.4a we have plotted $f^2 - R_n^2$ as a function of x_1 and x_2 . Fig. 4b presents the same quantity logarithmically versus distance from the assumed vortex center. This plot shows a high degree of rotational invariance. The straight line corresponds to an exponential decay (2.9) with a slope

$$m_S = \sqrt{\frac{2f^2}{\beta_1}}\ \frac{1}{a} . \qquad (5.3)$$

An analogous behaviour has been found for configurations generated and cooled at $\beta_1 = 1.0$. We feel justified to call the quasi-stable field configurations "lattice (anti-) vortices". We should mention that the clear pattern of rotational symmetry is visible only in the Higgs field modulus. The phases φ_n generically fluctuate strongly from lattice site to site! We did not try to use gauge freedom to smooth this behaviour to be left with a pure, monotonic rise of the phases along pathes circumfering the vortex soul.

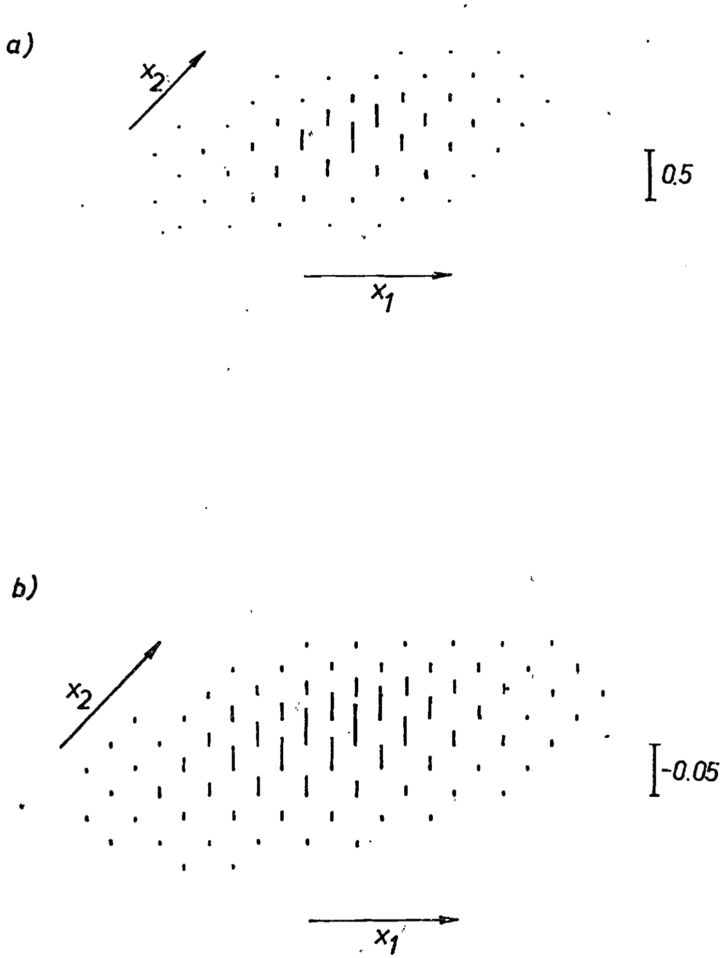

Fig. 3. Spatial distributions of the
a) action density acc. to Eq. (3.1),
b) topological charge density acc. to Eq. (3.5)
for a plateau configuration with $Q_L = -1$ at $\beta_1 = 2.0$.

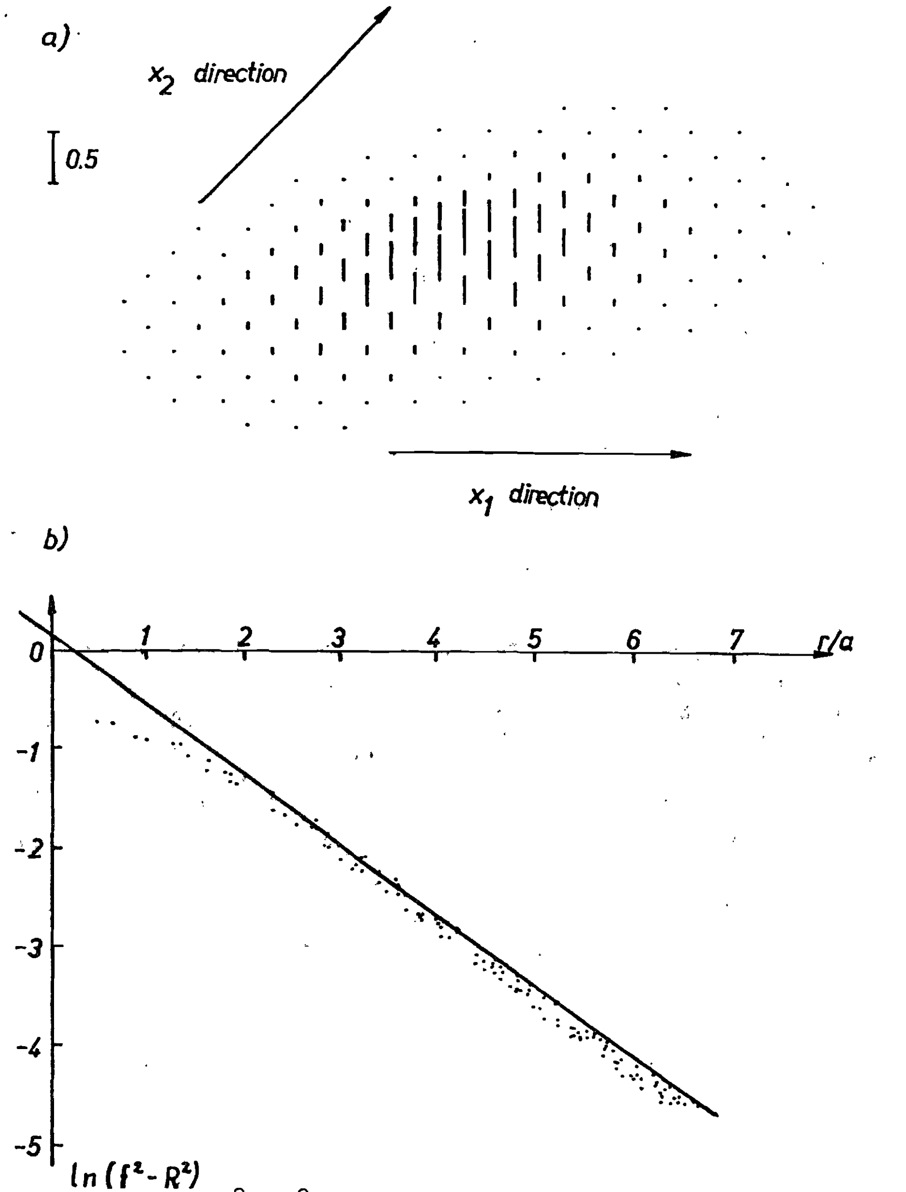

Fig. 4. a) $f^2 - R_n^2$ as a function of x_1 and x_2,
b) $\ln(f^2 - R_n^2)$ as a function of the distance r/a to the estimated center of the lattice vortex, both of them for the event shown in Fig. 3 .

For comparison, we show in Fig. 5 the densities of action and topological charge and the deviation $f^2 - R_n^2$ for a $Q_L = -2$ configuration obtained at $\beta_1 = 2.0$. One sees it clearly consisting of two separated lumps. It is only for the case b = 1 that two vortices do not have an interaction potential (cf. Eq. (2.8)).

A dilute gas picture within the semi-classical approach to the quantized 1+1 D Abelian Higgs model takes for granted the statistical significance of vortex-antivortex configurations, too. In our relaxation studies we have, actually, observed a few cases where a clear plateau was found at values

$$S_L = (|Q_L| + 2K) \tilde{\pi}_L \quad . \tag{5.4}$$

The investigation of their topological charge density showed that K can be read as the number of (well separated) vortex-antivortex pairs. Less well separated lumps of oppositely signed topology revealed themselves by characteristic cooling histories, which lower action with roughly linear slope versus "time" τ, without developing a quasi-stable shoulder.

Finally, we want to comment on the instability of the lattice vortices. It is common that along a plateau at least one of the vortices shrinks, turning finally into a dislocation. This is a maximally localized excitation which, according to the topological charge definition adopted, still carries one unit of charge. In Fig. 1 the step is marked by bars where this unit finally disappears. One generic dislocation is portrayed in Fig. 6. It lives approximately on a single plaquette and violates strongly the equations if motion ($\Delta_{disloc} \simeq 1.4$). The action of this kind of dislocation can be estimated to be

$$S_{disloc} \simeq 2.36 < \tilde{\pi}_L \tag{5.5}$$

at $\beta_1 = 0.5$. In contrast to this, narrow excitations which are still appreciated by the background charge prescription Q_B^{ε} (Eq. (3.7) with ε values in the yet conservative range $\varepsilon = 0.25 \div 0.32$) have a minimal action distinctly higher, $S_L \simeq 2.9$, already close to that of maximally classical lattice vortices.

The lesson to be drawn from this observation is that the

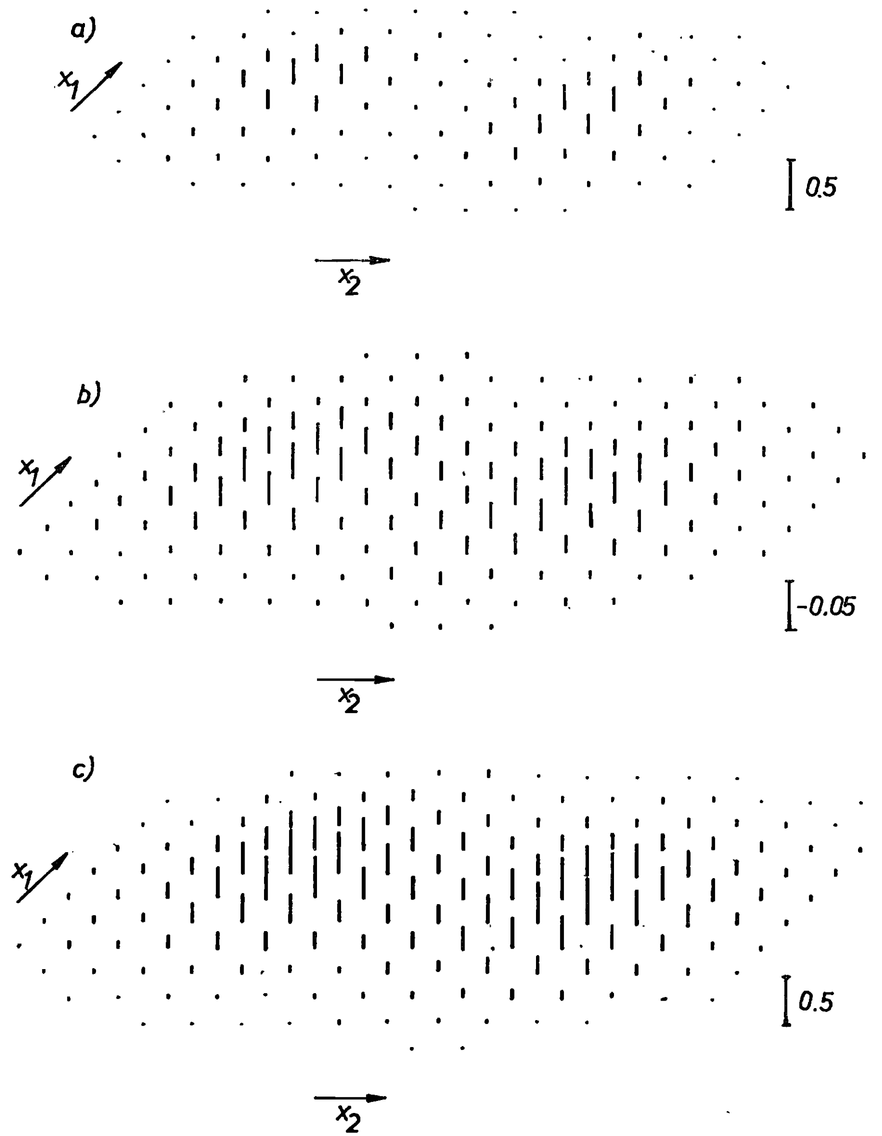

Fig. 5. Two-antivortex event
a) action density, b) topological charge density,
c) $f^2 - R_n^2$.

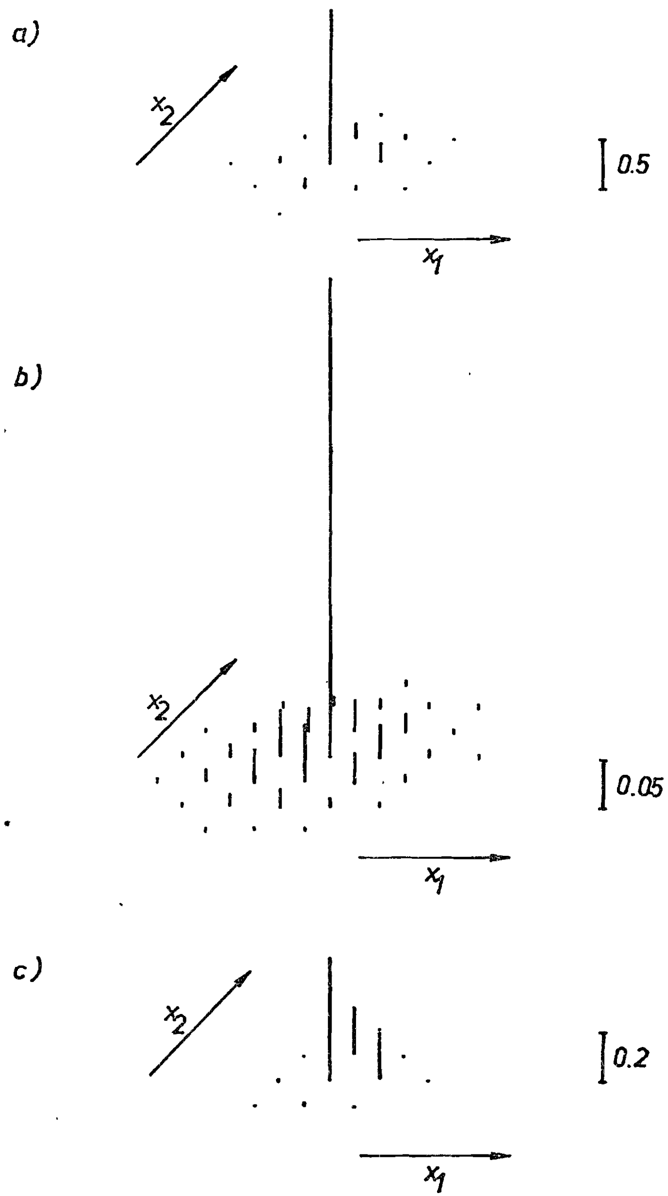

Fig. 6. Dislocation ($\beta_1 = 0.5$)
a) action density, b) topological charge density,
c) local violation of the equations of motion, i.e. the Δ density (cf. Eq. (4.7)).

background topological charge operator Q_B^ε is a suitable means to avoid counting of artificial lattice configurations whose action is considerably less than that of classical vortex configurations. One expects therefore that an immediate topological characterization of equilibrium Monte-Carlo configurations by means of the background charge operator Q_B^ε should give a topological susceptibility less affected by scaling violations. This has been shown to be a remedy in the case of the non-linear O(3) σ-model /22/.

6. Discussion and conclusions

In the present paper we have, inspired by the success in case of the pure non-Abelian gauge theory /3 - 6/, searched for classical topological excitations in the lattice 2D Abelian Higgs model. We have considered the particular case b = 1 , i.e., the border case between type I and type II in the language of phenomenological superconductivity, which is distinguished by the fact that multivortices can exist non-interacting, similarly to the Yang-Mills multiinstantons.

Starting from a lattice transcription of (compact U(1)) gauge and charged matter fields and from a Wilson-like action we have tried to solve the lattice field equations (4.1) in an iterative way. The presence of matter fields makes the appropriate procedure not easy to find. The most suitable "cooling" algorithm seems to be a gentle relaxation with controllable global steps. While the topology encoded in the gauge field is exposed relatively early, the development of the corresponding Higgs field pattern needs some acceleration in order to escape trapping in a local minimum near $\phi = 0$.

The structure of the fully developed, maximally classical field configurations conforms well with expectations based on general properties known for classical continuum fields: quantization of action according to the topological charge, rotational symmetry and exponential approach to the classical vacuum $|\phi|^2 = f^2$ for single vortices according to the scalar mass m_s.

In order eventually to learn about the vacuum of the quantized theory by this technique, the configurations to be cooled are to be taken from a Monte-Carlo generated equilibrium sample. This has been done, actually, with reasonable statistics for $\beta_1 = 0.5$ and $\beta_1 = 1.0$. Unlike the Yang-Mills case, where the coupling constant resides only in the sample to start with, the relaxation procedure and its results in the present case are explicitly dependent on β_1 due to the balance of the gauge field kinetic term and the Higgs potential in the action (Eq. (3.4)). Thus, increasing β_1 makes the unit of action approach π , improves the maximal classicality and the life-time of quasi-stable configurations. Finally, the width of the vortices explicitly depends on β_1. However, so far the quantum continuum limit of this theory is not investigated properly. In particular one does not know how the coupling constant(s) is (are) to be tuned with the lattice spacing. A Monte-Carlo renormalization study seems to be worthwhile. The topological "background charge distribution" ascribed to the Monte-Carlo parent configurations by cooling into the nearest quasi-stable plateau shows characteristic changes with β_1. For the above reasons, however, it is useless so far to define a topological susceptibility and test it for scaling.

All the classical fields "frozen out" finally collapse into narrow, highly non-classical excitations with somewhat smaller action. They are called dislocations as long as they contribute one unit to the respective topological charge definition. Thus, we are in the position to define a minimal action of topologically active excitations which differs for different charge operators. The local one (3.5) gives a minimal action well below the action quantum π_L , while the use of the non-local operator (3.7) brings the minimal action close to π_L . Once it comes to a renormalization group study of the quantized theory, the use of the background charge operator Q_B^{ε} for immediately analysing Mont-Carlo configurations is expected to result in a considerably improved scaling of the topological susceptibility. This experience one should have in mind contemplating on the lattice topology of gauge fields in general.

Vortex- antivortex superpositions do not solve the continuum equations of motion. In our data, however, there are a few cooling histories corresponding to separated vortex-antivortex pairs almost as quasi-stable as multi-vortices are. Less well separated pairs give rise to a peculiar pattern of slowed down relaxation probably due to collective motion and annihilation. These findings support the semi-classical dilute gas picture.

This work is meant as well as a step towards a lattice study of the quantized 2D Abelian Higgs model with emphasis on the topological aspects as it should sharpen the attention for the intrinsic problems of topological analyses on the lattice in general. We consider as particular interesting any evidence for the presence of a vortex-antivortex background structure. Of course, methods like those employed here might be useful, too, in a study of the real time flux-tube dynamics in type II superconductors.

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Вихри в двухмерной абелевой Хиггсовской модели на решетке

Численно найдены вихревые конфигурации в упомянутой модели с помощью релаксации на основе решеточных полей, генерированных методом Монте-Карло. Полученные вихри являются приближенными решениями решеточных уравнений движения, обладают действием $S_L \simeq \pi |Q_L|$ и однозначно определенным топологическим зарядом Q_L. Они экспоненциально спадают к вакуумным значениям полей. Найдены также вихревые-антивихревые суперпозиции в соответствии с картиной разреженного газа. Отдельные вихри, превращаясь в "дислокации", в ходе релаксации исчезают. Нелокальное определение топологического заряда, в отличие от локального, оказывается нечувствительным относительно дислокаций.

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Lattice Vortices in the Two-Dimensional Abelian Higgs Model

We generate and identify multi-vortices of the 2D Abelian Higgs model on a finite lattice by relaxation of Monte-Carlo equilibrium configurations. The lattice vortices have action and a uniquely defined topological charge corresponding to the continuum ones. They exhibit the expected exponential decay behaviour and satisfy approximately the classical equations of motion. Vortex-antivortex superpositions are seen as well, supporting the dilute gas picture. Single vortices finally relax into "dislocations" and dissappear. A background charge construction turns out nearly insensitive with respect to dislocations.

The investigation has been performed at the Laboratory of Computing Techniques and Automation, JINR.

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