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**THE EMC-EFFECT AND STRUCTURE  
OF NUCLEI**

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Recent data on lepton deep inelastic scattering (DIS) on nuclei<sup>1,2/</sup> have turned out to be in disagreement with theoretical expectations<sup>3,4/</sup> based on the traditional idea that a nucleus is a system of almost free nucleons (EMC-effect). This attracts attention to theoretical investigations of the quark structure of nuclei (for latest reviews, see, e.g.,<sup>5,6/</sup>). Yet it was shown<sup>7/</sup> lately that earlier calculations did not take into account the binding of nucleons, an important nuclear property. The calculations indicated that the account of binding of nucleons as well as their Fermi motion can explain the EMC-effect at  $x \geq 0.3$  ( $x = Q^2/2mq_0$  is the scaling variable of a nucleon). Though general ideas of the role of nucleon binding are doubtless, the model used in the earlier numerical calculations<sup>7/</sup> (i.e., the Fermi-gas of nucleons moving in the attractive potential) needs further improvement\*. That is why in this paper we present results of the calculations of structure-function (SF) ratio  $R(x) = 2F_2^A(x)/AF_2^d(x)$  using a realistic model for the nuclear structure.

As it has already been shown<sup>7/</sup>, the single-nucleon contribution to the nuclear SF can be written as follows

$$F_2^{A(N)}(x) = \int \frac{d^4p}{(2\pi)^4} S(p) F_2^N(x_p), \quad (1)$$

where  $x_p = Q^2/2pq$  is the scaling variable of a bound nucleon;  $F_2^N$  is the nucleon SF averaged over the spin and isospin:

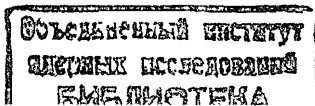
$$S(p) = \sum_{\lambda} |\phi_{\lambda}(\vec{p})|^2 2\pi \delta(m + \epsilon_{\lambda} - p_0), \quad (2)$$

$S(p)$  is the four-momentum distribution of nucleons in a nucleus (the spectral function);  $m + \epsilon_{\lambda} = E_0(A) - E_{\lambda}(A-1)$ ,  $E_0(A)$  and  $E_{\lambda}(A-1)$  are the g.s. energy and energy of the residual-nucleus excited state  $|(A-1)_{\lambda}\rangle$ , respectively. The detection probability of a given excited state is determined by

$$\phi_{\lambda}(\vec{p}) = \int d\vec{r} e^{-i\vec{p}\vec{r}} \phi_{\lambda}(\vec{r}); \quad \phi_{\lambda}(\vec{r}) = \langle (A-1)_{\lambda} | \psi(\vec{r}) | A \rangle,$$

where  $\psi(\vec{r})$  is the nucleon field operator. The functions  $\phi_{\lambda}(\vec{r})$  obey the following normalization condition  $\sum_{\lambda} \int d\vec{r} |\phi_{\lambda}(\vec{r})|^2 = A$ .

\* Results of harmonic oscillator wave functions are presented in the recent paper<sup>18/</sup>.



and the Dyson equation <sup>8/</sup>

$$(\epsilon_\lambda + \nabla^2/2m) \phi_\lambda(\vec{r}) = \int d\vec{r}' M(\vec{r}, \vec{r}', \epsilon_\lambda) \phi_\lambda(\vec{r}'), \quad (3)$$

where  $M$  is the nucleon mass operator in a nucleus. We emphasize that states  $|(A-1)_\lambda\rangle$  have a complex many-particle structure. The contribution to the spectral function from many-particle states can be effectively allowed for in the following way <sup>9/</sup>

Let us consider the equation

$$(-\nabla^2/2m) \phi_n(\vec{r}, \epsilon) + \int d\vec{r}' M(\vec{r}, \vec{r}', \epsilon) \phi_n(\vec{r}', \epsilon) = \mathcal{E}_n(\epsilon) \phi_n(\vec{r}, \epsilon) \quad (4)$$

and assume that the whole orthonormalized set of functions  $\{\phi_n(\vec{r}, \epsilon)\}$  with eigenvalues  $\mathcal{E}_n(\epsilon)$  (these are complex values,  $\mathcal{E}_n(\epsilon) = E_n(\epsilon) - i\Gamma_n(\epsilon)/2$  in the region  $\epsilon < \mu$ ,  $\mu$  being the nucleus binding energy per nucleon) obeys this equation. Using eq.(4) and the representation of the nucleon propagator in terms of functions  $\{\phi_n(\vec{r}, \epsilon)\}$  one can obtain the following expression

$$S(p) = \sum_n |\phi_n(\vec{p}, \epsilon)|^2 \frac{\Gamma_n(\epsilon)}{(\epsilon - E_n(\epsilon))^2 + \Gamma_n^2(\epsilon)/4} \quad \epsilon = p_0 - m. \quad (5)$$

The physical meaning of eq.(5) is obvious. Namely, it means that the hole excitation spectrum of nuclei is a set of resonances; centres of the resonances correspond to one-particle levels of the shell model. Note that such a pattern is clearly shown by the  $(e, e'p)$ -data, and these data enable us to determine  $S(p)$ , in principle.

Via eq.(5), eq.(1) is transformed into

$$F_2^A(x) = \sum_n \int \frac{d^3p}{(2\pi)^3} \int \frac{d\epsilon}{2\pi} |\phi_n(p, \epsilon)|^2 \frac{\Gamma_n(\epsilon)}{(\epsilon - E_n(\epsilon))^2 + \Gamma_n^2(\epsilon)/4} F_2^N\left(\frac{mx}{m + \epsilon - p_n}\right). \quad (6)$$

The same expression was used when calculating  $R(x)$  (see Fig.1). The mass operator was chosen in the form

$$M(\vec{r}, \vec{r}', \epsilon) = \delta(\vec{r} - \vec{r}') \{ (1 + a(\epsilon)) V_0(r) + V_c(r) + V(r) + i\beta(\epsilon) V_0(r)/V_0(0) \}, \quad (7)$$

where  $V_0(r) = V_{s.o.}(r) + V_c(r)$  is a shell-model potential, e.g., the Saxon-Woods one with generally accepted parameters <sup>11/</sup> ( $V_{s.o.}$  and  $V_c$  are the spin-orbit and Coulomb parts, respectively). For  $a(\epsilon)$  and  $\beta(\epsilon)$  we use the following parametrization (for details, see <sup>9/</sup>)

$$a(\epsilon) = a(\epsilon - \mu)/V_0(0), \quad a = 0.525, \quad (7a)$$

$$\beta(\epsilon) = 16.6 [1 - \exp(\gamma(\epsilon - \mu))] \text{ MeV}, \quad \gamma = 0.027 \text{ MeV}^{-1}$$

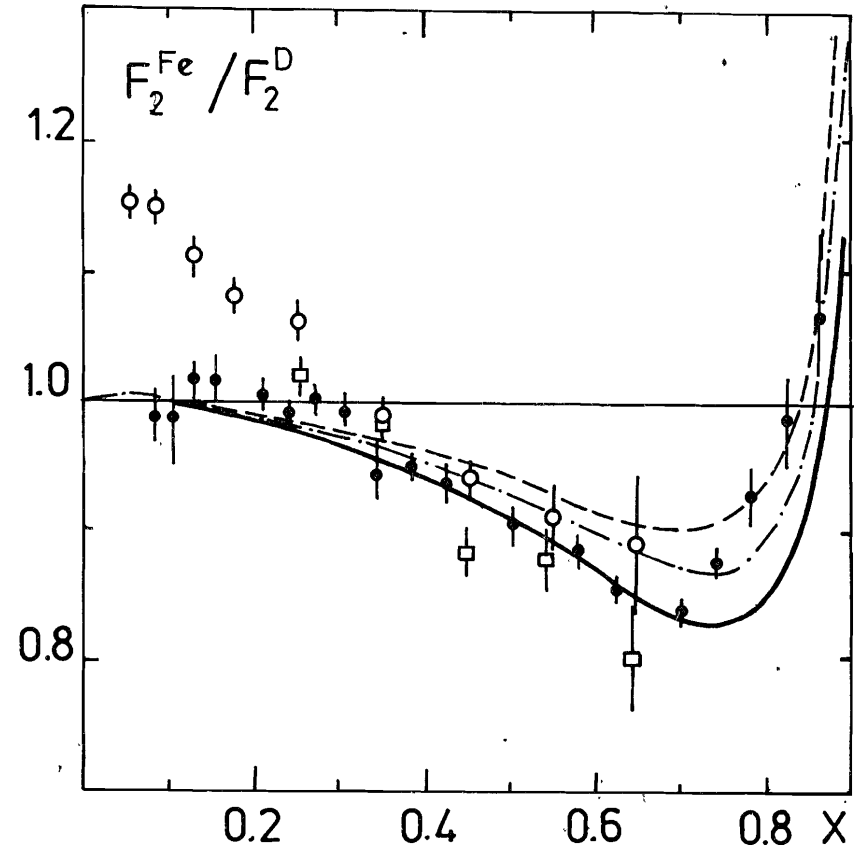


Fig.1. The ratios of SF calculated with the set of parameters from eq.(7a) (—) with  $a = 0.525$ ,  $\beta \neq 0$ ; (---)  $a = 0.525$ ,  $\beta = 0$ ; (- -)  $a = 0$ ,  $\beta = 0$  for parametrization (8a) on the nucleon SF. The experimental data from o - EMC, • - SLAC and □ - BCDMS for  $^{56}\text{Fe}$ .

The calculations were performed for the range of nuclei with the following nucleon SF <sup>12/</sup>

$$F_2^N(x) = 0.59 \sqrt{x} (1-x)^{2.8} + 0.33(1-x)^{3.8} + 0.49(1-x)^8 \quad (8)$$

and other parametrizations used in early papers

$$F_2^N(x) = \sqrt{x} (1-x)^2 + 0.15(1-x)^4, \quad (8a)$$

$$F_2^N(x) = 4.229 x^{0.52} (1-x)^{3.08} + 0.6 (1-x)^{4.55} \quad (8b)$$

In Fig.1 we also present, for comparison, quantities  $R_{\text{shell}}(x)$  calculated with  $a=\beta=0^*$  and the nucleon SF (8a). It is evident that the curve  $R_{\text{shell}}(x)$  lies above the curves given by eq.(6). This circumstance can be understood from considering the following expression <sup>7/</sup>

$$R^A(x) \approx F^N\left(\frac{mx}{m + \langle \epsilon \rangle}\right) / F_2^N(x). \quad (9)$$

This equation is correct in the region  $x \ll 1$  (in fact, up to  $x \approx 0.6$ ) and can be obtained by expanding  $F_2^N$  in eq.(6) around point  $\epsilon = \langle \epsilon \rangle$ , where  $\langle \epsilon \rangle$  is the average nucleon-separation energy

$$\langle \epsilon \rangle = \int \frac{d^4 p}{(2\pi)^4} \epsilon S(p) = \sum_n \int_{-\infty}^{\mu} \frac{d\epsilon}{2\pi} \frac{\epsilon \Gamma_n(\epsilon)}{(\epsilon - E_n(\epsilon))^2 + \Gamma_n^2(\epsilon)/4}. \quad (10)$$

From the fact that  $F_2^N(x)$  decreases for  $x > 0.01$  it follows that  $R(x)$  drops with increasing  $|\langle \epsilon \rangle|$ . From eq.(10) it follows that the inclusion of the  $\epsilon$ -dependence and of the imaginary part of the mass operator leads to the increase of  $|\langle \epsilon \rangle|$  as compared with the shell-model value  $|\langle \epsilon \rangle_{\text{sh}}|$ . For completeness, in the table we give values  $\langle \epsilon \rangle$  and  $\langle \epsilon \rangle_{\text{sh}}$ .

Table

The average excitation and kinetic energies (in MeV/A)

Nucleus	$-\langle \epsilon \rangle$		$\langle T \rangle$	
	$a=0$ $\beta=0$	$a=0.525$ $\beta=0$	$a=0$ $\beta=0$	$a=0.525$ $\beta=0$
$^{14}\text{N}$	20.1	26.1	16.4	17.5
$^{56}\text{Fe}$	20.5	29.4	18.4	19.5

In this connection we would like to emphasize that the shell-model calculation gives a too small nuclear binding energy, at the same time the consideration of the  $\epsilon$ -dependent complex nucleon mass operator leads to a satisfactory description of the nuclear binding energy (for details see <sup>9/</sup>).

\* The shell-model calculations of the single-nucleon contribution to the nuclear SF using the results of ref.<sup>4/</sup> were also made in ref.<sup>13/</sup>.

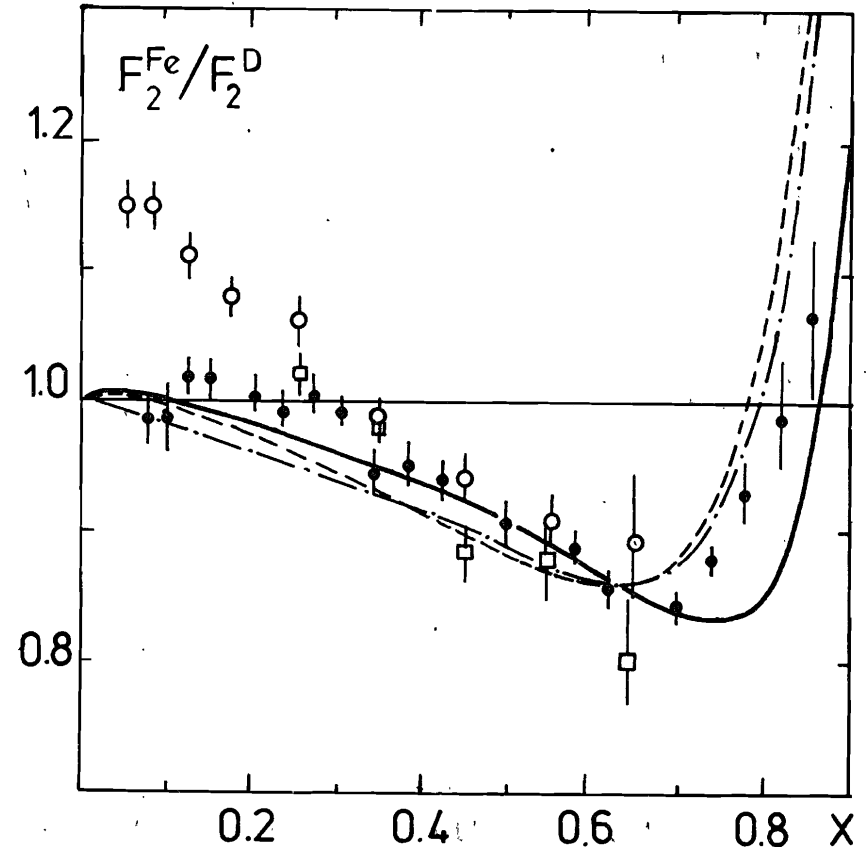


Fig.2. The ratio  $R(x)$  for different parametrization (8) (- . -), (8a) (—) and (8b) (- - -) of the nucleon SF with the set of parameters from eq. (7a) for  $^{56}\text{Fe}$ .

The figures show satisfactory agreement with the data in the region  $x \geq 0.3$ . Note, however, that in the region  $x \rightarrow 1$  the ratio  $R(x)$  is dependent on the choice of nucleon SF (see Figs.2,3).

Let us discuss the validity of eq.(1) and possible corrections to it. First of all, from comparison of the virtual-photon interaction time ( $\tau_{\text{int}} \sim (mx)^{-1}$ ) with the mean internucleon distance ( $\sim m_{\pi}^{-1}$ ) one can expect that the corrections to eq.(1) are negligible in the region  $\tau_{\text{int}} \ll m_{\pi}^{-1}$  or  $x \gg m_{\pi}/m \approx 0.15$ . This expectation is confirmed a posteriori by the calculations that agree with data in the region  $x \geq 0.3$ . However, in the region  $x \leq 0.15$  (and for  $x > 1$ ) some significant corrections to eq.(1)

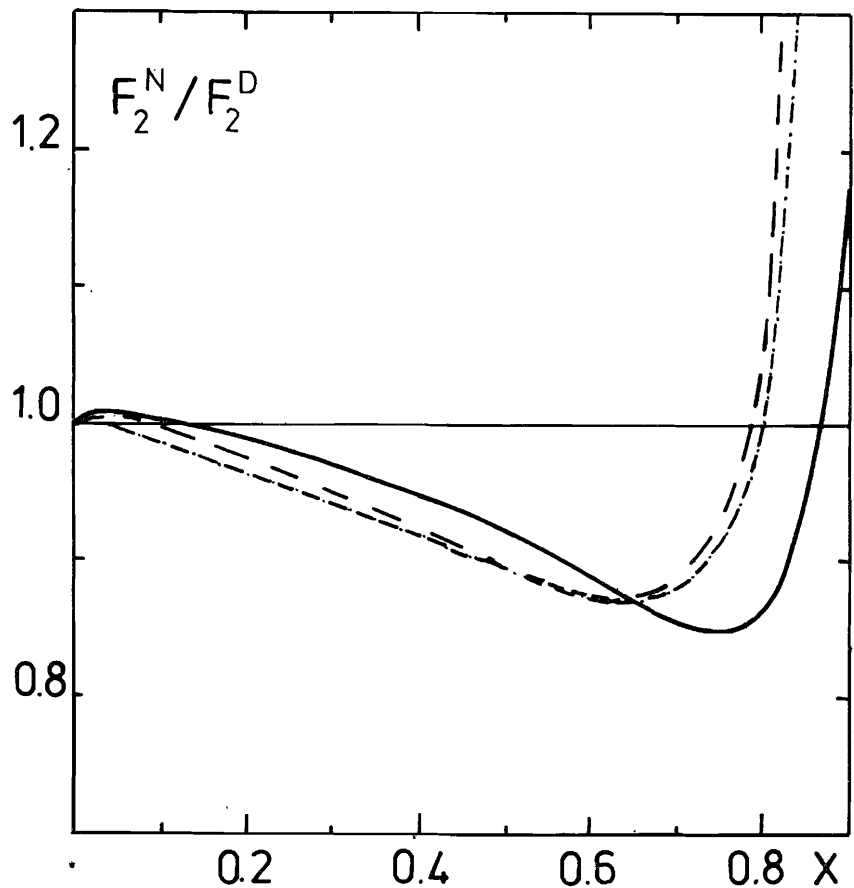


Fig.3. Same as in fig.2, but for  $^{14}\text{N}$ .

are possible. To estimate their total value, let us write the nucleon SF as follows

$$F_2^A = F_2^{A(N)} + \delta F_2^A. \quad (11)$$

The value  $\delta F_2^A$  is restricted by the QCD energy sum rule stating that in the limit  $Q^2 \rightarrow \infty$  the first moments of  $F_2(x)$  of different targets

$$\langle x \rangle_{\text{target}} = \frac{\mu_t/m}{\int_0^1 dx F_2(x)}$$

behave like masses of the corresponding targets<sup>14/</sup>, namely

$$\langle x \rangle_A / \langle x \rangle_N = \frac{M_A}{m} = A(1 + \mu/m). \quad (12)$$

The nucleon contribution to  $\langle x \rangle_A$  is obtained by the integration over  $x$  of both sides of eq.(1),<sup>A</sup> and the result can be found from eq.(12) by the substitution  $\mu \rightarrow \langle \epsilon \rangle$ . Thus, we have

$$\frac{\langle \delta x \rangle_A}{\langle x \rangle_N} = \frac{\mu - \langle \epsilon \rangle}{m}, \quad \langle \delta x \rangle_A = \int_0^1 dx \delta F_2^A(x). \quad (13)$$

We note that both quantities  $\mu$  and  $\langle \epsilon \rangle$ , can be obtained from experiments.

Among different types of contributions to  $\delta F_2^A(x)$  at small  $x$  one should specify the reaction on exchanged mesons. The contribution to the nuclear SF from pion exchange diagram (Fig.4a) was calculated in refs.<sup>15,16/</sup>; in papers<sup>17/</sup> the mesonic contribution to SF is constructed in accordance with the corresponding contribution to the nuclear binding energy.

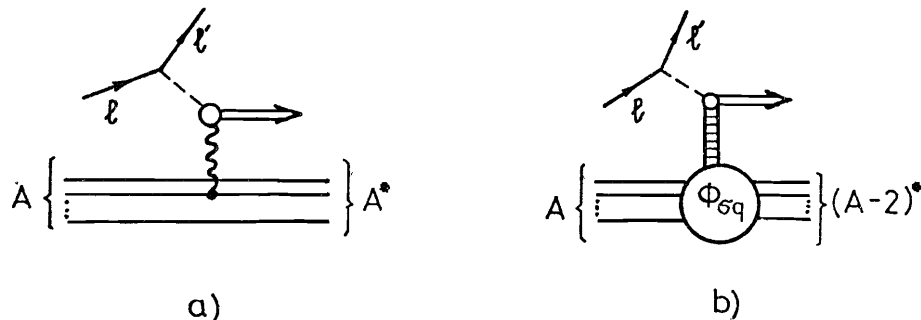


Fig.4. a) An example of DIS off the exchanged meson. b) DIS off the virtual  $6q$ -configuration.

In the region  $x > 1$  one can expect, for example, the contribution from the DIS off virtual multi-quark configurations (see, e.g., ref.<sup>17/</sup>). We note that the calculation of such contributions is difficult due to the absence of information on the probability to detect such configurations in a nucleus (the vertex  $\Phi_{6q}$  in Fig.4b). From this point of view the nuclear DIS-experiment is an ideal tool for obtaining such an information. We should also emphasize that the nucleon momentum distribution in a nucleus  $N(\vec{p}) = \int \frac{d^3p_0}{2\pi} S(\vec{p})$  has a "long tail", this is proved by (ee'p)-data<sup>10/</sup>. Its existence leads to the fact that  $F_2^{A(N)}(x > 1) \neq 0$  in the region  $x-1 \gg |\langle \epsilon \rangle|/m$ , as it follows from eq.(1). Thus, the extraction of the information on possible exotic configurations from the data must be accompanied by an accurate subtraction of a high-momentum-nucleon contribution.

In conclusion, we would like to point out that the use of a more realistic model for the nuclear structure within the approach of ref.<sup>7/</sup> made the EMC-effect description better.

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Акулиничев С.В. и др.  
EMC-эффект и структура ядер

E2-86-61

Мы представляем результаты новых расчетов одночастичного вклада в ядерную структурную функцию при корректном учете ядерной структуры. Расчеты показали, что учет связи нуклонов, а также их Ферми-движения, может объяснить EMC-эффект при  $x \geq 0,3$ . Обсуждается также область применимости однонуклонного приближения и возможные поправки к этому приближению. Среди различных видов вклада в EMC-эффект при малых  $x$  можно выделить реакции на обменных мезонах. В области  $x > 1$  можно ожидать, что окажется существенным вклад от глубоконеупругого рассеяния на многокварковых конфигурациях.

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Akulichev S.V. et al.  
The EMC-Effect and Structure of Nuclei

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We present the results of new calculations of a single-nucleon contribution to the nuclear structure functions with a correct account of the nuclear structure. The calculations indicate that the account of binding of nucleons as well as their Fermi motion can explain the EMC-effect at  $x \geq 0.3$ . We also discuss applicability of the single-nucleon approximation and possible corrections to it. Among different types of contributions to EMC-effect at small  $x$  one should specify the reaction on exchanged mesons. In the region  $x > 1$  one can expect the contribution from the virtual multi-quark configurations.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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