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ANALYTIC BREMSSTRAHLUNG INTEGRATION FOR THE PROCESS $e^+e^- \to \mu^+\mu^-\gamma$ IN QED

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1. INTRODUCTION

One of the most important reactions observed in e^+e^- -storage rings is the muon pair creation, or, generally, the creation of two fermions:

$$e^+e^- \to f^+f^- . \qquad (1)$$

This process provides a unique possibility to study the standard electroweak theory[1] over a wide range of energies. Taking into account radiative corrections of order α^3, it is unavoidable to study in parallel the process

$$e^+e^- \to f^+f^-\gamma , \qquad (2)$$

where the fermion pair created is accompanied by a bremsstrahlung photon. The analysis of (1) heavily relies on the differential cross section $d\sigma/d\cos\theta$ with respect to the scattering angle θ. For unpolarized beams usually one determines the total cross section,

$$\sigma_{tot} = \int_{-1}^{+1} d\cos\theta \, \frac{d\sigma}{d\cos\theta} , \qquad (3)$$

and the forward-backward asymmetry,

$$A_{FB} = \frac{1}{\sigma_{tot}} \left[\int_0^1 d\cos\theta \frac{d\sigma}{d\cos\theta} - \int_{-1}^{0} d\cos\theta \, \frac{d\sigma}{d\cos\theta} \right] . \qquad (4)$$

The total cross section σ_{tot} is sensible to the C -even contributions, and the charge asymmetry A_{FB} measures the C -odd terms.

Here, we start a specialized study of the bremsstrahlung contributions (2) to the observables (3, 4) within the standard electroweak theory. We obtain analytic expressions for $d\sigma/d\cos\theta$, A_{FB} and σ_{tot},

where the bremsstrahlung integration has been done over the complete photon phase space, beginning here with the QED corrections most easily being obtained.

There are two extreme approaches to hard bremsstrahlung problems. One is the consequent numeric integration of the squared matrix element by Monte-Carlo (MC) methods as has been highly developed by a large theoretical collaboration[2]. With no doubt, MC-integrated cross sections are of great value for applications due to their flexibility concerning experimental cuts. The value of analytic results (the other extreme) is two-fold. Of course, it is desirable to get analytic results on simple processes even if they are not simply to be obtained. Further, one may use the analytically integrated hard bremsstrahlung and subtract by MC-integration the not needed phase space regions to get a completely independent theoretical prediction for cross sections with realistic cuts. Analytical integrations have been done by several groups. The first result on $d\sigma/d\cos\theta$ is in[3]. Aiming at expressions being applicable over the whole relativistic energy range including the region of the Z -boson pole, here we mention[4] where several distributions have been presented but not the $\cos\theta$ - spectrum, and[5] where the problem raised here has been studied but not succeeding in a compact analytic expression.

In the present article, we derive analytic results on the pure QED bremsstrahlung, Fig.2a, in connection with the Born cross section of the standard electroweak theory, Fig.1. Adding the QED virtual corrections of Fig.2b we get a gauge invariant infrared finite set of diagrams. Inclusion of the fermionic vacuum polarization would complete the QED radiative corrections. The region of applicability of our approximation is defined by the relative magnitude of the two Born diagrams as functions of s. It is well-known that at PETRA-energies the electroweak radiative corrections and the genuine Z -boson exchange Born cross section are quite small with the exception of QED corrections. So, for $s \lesssim 1600$ GeV^2 the dominant contributions are included.

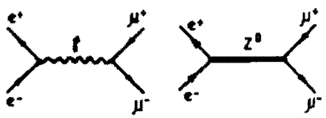


Fig.1. Born diagrams for $e^+e^- \to f^+f^-$ in the electroweak standard theory.

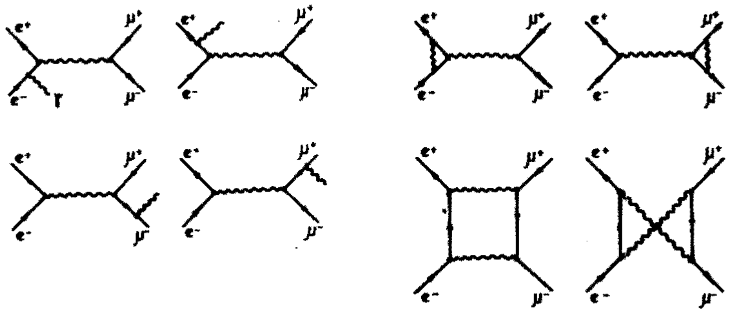


Fig.2. Gauge invariant and infrared finite set of QED bremsstrahlung (a) and one loop diagrams (b).

As already stated above, we also assume that m_e^2, m_f^2 .

The article is organized as follows. In Section 2 we introduce the notation and analytic results, Section 3 contains numerical results, and in Appendices A-C some formal intermediates are presented which are of interest also for the derivation of expressions for interference and pure weak integrated photon Bremsstrahlung.

2. ANALYTIC RESULTS

The cross section for (1) together with (2) is in the adopted approximation:

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{2s}\Big\{ Q^2\Big[1+\cos^2\theta + \frac{\alpha}{\pi}(F_0 + QF_1 + Q^2F_2) + \frac{\alpha}{\pi}F_{VP}\Big] + 2\,\mathrm{Re}\,\chi|Q|\,\big[v_e v(1+\cos^2\theta) + 2a_e a\cos\theta\big] + |\chi|^2\big[(v_e^2+a_e^2)(v^2+a^2)(1+\cos^2\theta) + 8v_e a_e v a\cos\theta\big]\Big\}. \tag{5}$$

Here, $s=4E^2$, and θ is the emission angle of the created fermion f^+ with respect to the e^+ -beam axis in the cms. The Q , v , a are the charge, vector and axial vector couplings, respectively (Q_μ = -1):

$$v = 1-4s_\theta^2|Q|, \qquad a=1. \tag{6}$$

The relative weight of photon and Z -boson exchange is

$$\chi = \frac{1}{16 s_\theta^2 c_\theta^2}\frac{s}{s-M^2} = \frac{1}{8\pi\alpha}\frac{G_\mu}{\sqrt{2}}\big[1-\Delta r + O(\alpha^2)\big]\frac{M^2 s}{s-M^2}, \tag{7a}$$

$$\chi \simeq \frac{1}{8\pi\alpha}\frac{G_\mu}{\sqrt{2}}\frac{M^2 s}{s-M^2}, \tag{7b}$$

where $\Delta r = X/4\pi$ is taken from [6,7] . The definition of χ deserves some comment. We use the on mass shell renormalization framework of Sirlin [7,8] , where

$$s_\theta^2 = 1-c_\theta^2 = \sin^2\theta_W = 1-M_W^2/M_Z^2 . \tag{8}$$

As has been discussed extensively in the literature [9] , the definition (7b) is preferred by eating some large virtual radiative corrections and will be used here. The complex parameter M^2 ,

$$M^2 = M_Z^2 - i\Gamma_Z M_Z, \tag{9}$$

contains the physical mass and width of the Z -boson. In the framework chosen, M_Z is determined experimentally while Γ_Z is predicted by the theory [10] . To be definite, in the following we use M_Z =93 GeV, the t-quark mass m_t = 40 GeV, and Higgs boson mass M_H = 100 GeV. This, together with the feinstructure constant α and the muon decay Fermi constant G_μ allows to calculate M_W = 82.0 GeV, $\sin^2\theta_W$ = = 0.222, Γ_Z = 2.17 GeV + $\Gamma_Z(t)$, $\Gamma_Z(t)$ being the partial width for the t -quark channel. The integrated bremsstrahlung and virtual QED correction are contained in the F_i :

$$F_0 = F_V(m_e) + F_{br}^{i}, \tag{10}$$

$$F_1 = F_{box} + F_{br}^{int}, \tag{11}$$

$$F_2 = F_V(m_f) + F_{br}^{f}. \tag{12}$$

The virtual corrections are well-known (see App.A). The initial (F_{br}^i), interference (F_{br}^{int}), and final (F_{br}^f) bremsstrahlung contributions are derived in Appendices B and C. The compact final result is:

$$F_{0,2} = f_{0,2}(\cos\theta) + f_{0,2}(-\cos\theta), \tag{13}$$

$$F_1 = f_1(\cos\theta) - f_1(-\cos\theta), \tag{14}$$

$$\begin{aligned} f_0 = {} & \tfrac{2}{3}\left[\tfrac{5}{3} + 2L_e - L_f - L_- + 4C_- L_-\right] \\ & + C_-^2\left[\tfrac{10}{3} - \tfrac{13}{3}L_e + \pi^2 + 2(1-L_e)(L_+ + L_-) - L_i^2\right] \\ & + \tfrac{1}{C_-}\left[-\tfrac{19}{9} + \tfrac{5}{3}L_e + \tfrac{2}{3}L_f + \tfrac{4}{3}L_- + 2L_+\right] \\ & + \tfrac{1}{C_-^2}\left[(L_e-2)L_+ + \tfrac{1}{2}L_+^2 - \Phi(C_-)\right], \end{aligned} \tag{15}$$

$$\begin{aligned} f_1 = {} & 3\left[4C_-^2 L_i + 3L_+ - 1/C_- - L_+/C_-^2\right] \\ & + C_-\left[6 + \pi^2/3 + L_+ + L_- - 2L_+L_-\right], \end{aligned} \tag{16}$$

$$f_2 = 1 - \tfrac{3}{2}C_+^2. \tag{17}$$

The following abbreviations are used:

$$\begin{aligned} & L_e = \ln S/m_e^2, \quad L_f = \ln S/m_f^2, \\ & C_+ = \tfrac{1}{2}(1+\cos\theta) = \cos^2\tfrac{\theta}{2}, \quad C_- = \tfrac{1}{2}(1-\cos\theta) = \sin^2\tfrac{\theta}{2}, \\ & L_+ = \ln\cos^2\tfrac{\theta}{2}, \quad L_- = \ln\sin^2\tfrac{\theta}{2}, \quad L_i = \ln\tan^2\tfrac{\theta}{2}, \\ & \Phi(x) = -\int_0^1 \frac{dt}{t}\ln(1-xt). \end{aligned} \tag{18}$$

Finiteness and integrability of (10-12) are ensured by the following modification near the end points cos θ = ± 1 (see App.B):

$$\cos\theta \longrightarrow \cos\left(\theta^2 + 4m_e^2/s\right)^{1/2}. \tag{19}$$

The initial radiative correction F_0 is dependent on s, m_e, m_f and θ, while F_1 and F_2 are functions of the scattering angle only. Integration over cos θ allows to obtain analytic expressions for σ_{tot} and A_{FB}:

$$\begin{aligned} \sigma_{tot} = {} & \tfrac{4}{3}\pi\frac{\alpha^2}{s}\left[Q^2\left(1+\tfrac{\alpha}{\pi}F_0^{tot} + \tfrac{\alpha}{\pi}Q^2F_2^{tot} + \tfrac{3}{8}\tfrac{\alpha}{\pi}F_{vp}\right) + 2\,\mathrm{Re}\,\chi|Q|V_eV \right. \\ & \left. + |\chi|^2(V_e^2+a_e^2)(V^2+a^2)\right], \end{aligned} \tag{20}$$

$$A_{FB} = \frac{1}{\sigma_{tot}}\,\tfrac{4}{3}\pi\frac{\alpha^2}{s}\left[Q^3\tfrac{\alpha}{\pi}F_1^{tot} + \tfrac{3}{2}\,\mathrm{Re}\,\chi|Q|a_ea + 3|\chi|^2V_eVa_ea\right], \tag{21}$$

$$F_{0,2}^{tot} = \tfrac{3}{8}\int_{-1}^{+1}dc\; F_{0,2}, \tag{22}$$

$$F_1^{tot} = \tfrac{3}{8}\left[\int_0^1 dc\,F_1 - \int_{-1}^0 dc\,F_1\right], \qquad c \equiv \cos\theta, \tag{23}$$

$$F_0^{tot} = \tfrac{2}{3} - \tfrac{7}{6}L_e + L_eL_f - L_f + \tfrac{\pi^2}{3}, \tag{24}$$

$$F_1^{tot} = \tfrac{3}{2} - \tfrac{\pi^2}{8} + \tfrac{3}{4}\ln^2 2 - \tfrac{15}{2}\ln 2 \;=\; -4.572, \tag{25}$$

$$F_2^{tot} = 3/4. \tag{26}$$

The $F_{0,2}^{tot}$ are first derived in [2] . Concerning (15-17), we took advantage from a contact with the authors of [7] for an understanding of the partial disagreement with their eq.(14).

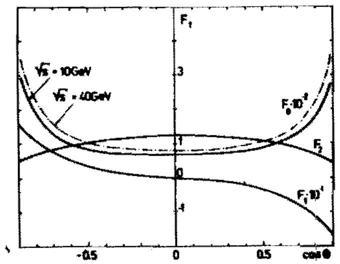
3. NUMERICAL RESULTS

Based on the parameters chosen, it is for S in GeV^2

$$\chi = -4.49 \times 10^{-5} S/(1 - S/8649). \tag{27}$$

The width of the Z -boson may be neglected in the energy range of interest here. As long as $S \lesssim$ 1600 GeV^2, the Born interference contribution doesn't exceed 15% of the photon exchange Born cross section and the pure weak contribution is quite small (although in the numerics we will include it).

The pure QED radiative corrections (with exception of the vacuum polarization) are contained in the F_i , eqs.(10-12). The dependence of the F_i on the scattering angle is shown in Fig. 3. The F_1 and F_2 are smooth over the complete kinematical region, whearess F_0 (initial state radiation) has sharp peaks of order S/m_e^2 times logarithms at the end points, cos θ = 1-2 m_e^2/S. These peaks are due to the well-known fermion mass singularities. Fig.4 shows the differential cross section. The QED Born cross section is symmetric and, in the normalization chosen, independent of the energy. The Born cross section of the GWS-theory is asymmetric in cos θ (shown here for S = 1600 GeV^2). The inclusion of the total Bremsstrahlung discussed here leads to a considerable modification, especially at the end points.



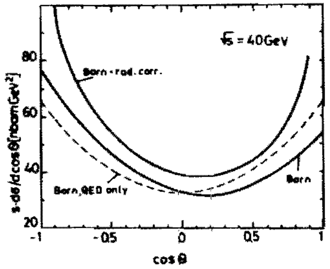
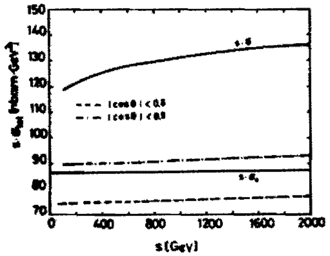


Fig.3. The QED α^3 corrections of Fig.2 as defined in (10-12) as functions of $\cos\theta$ with parameter s .

Fig.4. The differential cross section $d\sigma/d\cos\theta$.

It is well-known that the total cross section σ^0_{tot} of the GWS-theory in Born approximation is very near to σ_{tot} of QED for mixing angles around 0.25 since then the vector couplings of leptons become small: $V_e = V_\mu \approx 0.01$ for $\sin^2\theta = 0.222$ as chosen here. This may be seen in Fig.5. The QED bremsstrahlung correction to σ_{tot} is considerable due to F_0 and becomes much smaller if a cut on $\cos\theta$ is applied.

The integrated forward-backward asymmetry A_{FB} as function of s is shown in Fig.6. In Born approximation, A^0_{FB} is negative and nearly linearly rising with s in the depicted energy range.



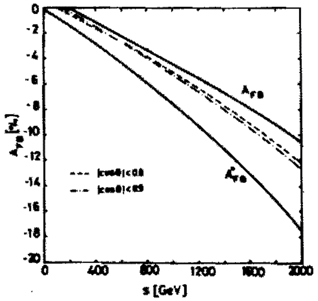


Fig.5. The total cross section σ_{tot} as function of s (σ_0-Born cross section; the slashed and slashed-dotted lines are radiatively corrected with cuts as explained in the text).

Fig.6. The integrated forward-backward asymmetry A_{FB} as function of s .

The bremsstrahlung correction $Q_\mu F_1^{tot}$ is positive and constant. Correspondingly, its relative influence diminishes with rising s . In real experiments a cut is applied on the scattering angle, e.g. $|\cos\theta| < 0.8 \div 0.9$. The result of those cuts is shown, too. Like for σ_{tot}, the correction becomes much smaller. Interestingly, the corrected A_{FB} is closer to its Born value for $|\cos\theta| < 0.9$ than for $|\cos\theta| < 0.8$. This is due to a mismatch of the tendency of F_1 to add larger contributions to A_{FB} from scattering angles near $|\cos\theta| = 1$ against the peaking of F_0 in the same points influencing the denominator of A_{FB} in opposite direction.

We are very much obliged to Dr. D.Yu.Bardin for stimulating and fruitful discussions and for cooperation in the first stage of the work. We would like to thank Prof. F. Kaschluhn for interest into our work and support and the organizers of the IX Warsaw Symposium on Elementary Particle Physics in Kazimierce (1986) where we had the opportunity to present our results.

APPENDIX A

The QED vertex correction is in the approximation of small fermion masses and in the normalization adopted here:

$$F_V(m_f) = -2(L_f-1)\left(P_{IR} - \ln\frac{\eta}{m_f}\right) - 2 + \frac{2}{3}\pi^2 + \frac{3}{2}L_f - \frac{1}{2}L_f^2, \qquad (A.1)$$

the $F_V(m_e)$ being defined analogously. The contribution of the two QED box diagrams is:

$$F_{Box} = f_{box}(\cos\theta) - f_{box}(-\cos\theta), \qquad (A.2)$$

$$f_{box} = 8(1+\cos^2\theta)\,L_-\cdot\left(P_{IR} - \ln\frac{\eta}{m_f}\right) + 4C_-^2 L_e L_f + 2C_- L_+ - 2C_-(L_-^2+L_+^2). \qquad (A.3)$$

The vacuum polarization is

$$F_{VP} = \sum_{leptons} Q_l^2 F_{VP}(m_l) + F_{VP}^{hadrons}, \qquad (A.4)$$

$$F_{VP}(m) = -\frac{10}{9} - \frac{8}{3}\frac{m^2}{s} + \frac{2\sqrt{\lambda}}{3s}\left(1+\frac{2m^2}{s}\right)\ln\frac{s+\sqrt{\lambda}}{s-\sqrt{\lambda}}, \qquad (A.5)$$

$$F_{VP}(m) \approx -\frac{10}{9} + \frac{2}{3}L_m \qquad (m^2 \ll s), \qquad (A.6)$$

$$\lambda = s^2 - 4m^2 s.$$

In the numerical results we do not include the F_{VP} . This causes a shift for σ_{tot} compared to a complete QED one loop calculation.

APPENDIX B

Here we scetch our derivation of a soft photon contribution to the bremsstrahlung cross section:

$$e^-(k_1) + e^+(k_2) \longrightarrow f^-(p_1) + f^+(p_2) + \gamma(p). \tag{B.1}$$

We start with

$$M_\beta^{IR} = M_0 \cdot \varphi_\beta^{IR}, \tag{B.2}$$

$$M_0 = \frac{i}{s} Q\, \bar{u}(k_2)\gamma^\mu u(k_1)\ \bar{u}(p_1)\gamma_\mu u(p_2) \tag{B.3}$$

$$\varphi_\beta^{IR} = \left(\frac{2k_{2\beta}}{\bar{z}} - \frac{2k_{1\beta}}{z}\right) + Q\left(\frac{2p_{1\beta}}{v} - \frac{2p_{2\beta}}{\bar{v}}\right). \tag{B.4}$$

The z, $\bar{z}$ and v, $\bar{v}$ are the fermion propagators,

$$\overset{(-)}{z} = -2k_{1(2)}\cdot p, \qquad \overset{(-)}{v} = -2p_{1(2)}\cdot p. \tag{B.5}$$

Formally neglecting the photon momentum everywhere but in the denominators of φ_{IR}, we introduce the

$$\frac{1}{64}\sum_{spins}|M_\beta^{IR}|^2 \longrightarrow T^A(x,t)\cdot B^{IR}(t,z,\bar{z},v,\bar{v}), \tag{B.6}$$

$$T^A(x,t) = Q^2\frac{t^2+(s-t)^2}{s^2},$$

$$B^{IR} = \left(-\frac{m_e^2}{z^2} - \frac{m_e^2}{\bar{z}^2} + \frac{s}{z\cdot\bar{z}}\right) + Q\left(\frac{t}{\bar{z}\cdot\bar{v}} + \frac{t}{z\cdot v} - \frac{s-t}{\bar{z}\cdot v} - \frac{s-t}{z\cdot\bar{v}}\right) + Q^2\left(-\frac{m_f^2}{\bar{v}^2} - \frac{m_f^2}{v^2} + \frac{s}{v\cdot\bar{v}}\right). \tag{B.7}$$

Here,

$$t = -2k_2p_2 = \frac{x}{2} - \frac{1}{2s}\sqrt{s^2-4m_e^2s}\,\sqrt{x^2-4m_f^2s}\,\cos\theta, \tag{B.8}$$

$$t \simeq x\cdot C_-, \tag{B.9}$$

$$x = -2p_2(k_1+k_2) = 2\sqrt{s}\,p_2^0, \tag{B.10}$$

where p_2^0 is the energy of f^+ in the cms: $x \in (0, s)$. For scattering angles with cos θ very close to ±1, the approximation (B.9) is too crude and has to be replaced by

$$t \simeq \frac{x}{2}\left[1-\left(1-2\frac{m_e^2}{s}\right)\cos\theta\right] \simeq \frac{x}{2}\left[1-\cos\left(\theta^2+m_e^2/E^2\right)^{1/2}\right]. \tag{B.11}$$

The integration over the photon phase space has been performed with the method developed in[11], i.e. using dimensional regularization and the R_γ -system, the rest system of $(f^-\gamma)$: $\vec{p}_1^{\,R} + \vec{p}^{\,R} = 0$. We write the soft bremsstrahlung contribution to (10-12) as follows:

$$\frac{d\sigma^S}{d\cos\theta} = \frac{d\sigma^{Born}}{d\cos\theta}\,\frac{\alpha}{\pi}\,\delta_{soft}, \tag{B.12}$$

$$\delta_{soft} = \int_0^{\bar{\omega}} d\omega\, I^n(\omega) \tag{B.13}$$

$$I^n(\omega) = \frac{2}{(2\pi)^{n-4}\,\Gamma(\frac{n}{2}-1)}\,\frac{1}{\omega}\left(\frac{\omega}{2m_f}\right)^{n-4}\int_0^1 d\alpha\int_{-1}^{+1} d\xi\,(1-\xi^2)^{(n-4)/2}\,B(\alpha,\xi), \tag{B.14}$$

$$B(\alpha,\xi) = \left[-\frac{m_e^2}{k_{10}^2(1-\beta_1\xi)^2} - \frac{m_e^2}{k_{20}^2(1-\beta_2\xi)^2} + \frac{s}{k_{30}^2(1-\beta_3\xi)^2}\right] + Q\left[\frac{t}{p_{t0}^2(1-\beta_t\xi)^2}\right. \tag{B.15}$$
$$\left.+\frac{t}{m_f\cdot k_{10}\cdot(1-\beta_1\xi)} - \frac{s-t}{p_{st_0}^2(1-\beta_{st}\xi)^2} - \frac{s-t}{m_f\cdot k_{20}\cdot(1-\beta_2\xi)}\right] + Q^2\left[-\frac{m_f^2}{p_{20}^2(1-\beta_{p_2}\xi)^2} + \frac{s}{m_f\cdot p_{20}(1-\beta_{p_2}\cdot\xi)}\cdot -1\right],$$

$$\xi = \cos\theta_\gamma^R, \qquad p_t = k_2\cdot\alpha + p_2\cdot(1-\alpha),$$
$$k_3 = k_1\cdot\alpha + k_2\cdot(1-\alpha), \qquad p_{st} = k_1\cdot\alpha + p_2\cdot(1-\alpha), \tag{B.16}$$

and β_i are the velocities corresponding to p_i, k_i in the R_γ -system. In deriving (B.12-B.15) one takes advantage of the fact that $T^A(x,t)$ does not depend on the photon angles and that all momenta used may be chosen to depend on only one of the photon angles, cos θ_γ^R. The Feynman parameter integral over α has been introduced to simplify the dependence of numerators on the photon momentum. Eqs.(B13-B.14) have been obtained after integrating over (n -3) angles in the n - dimensional space-time and after restricting the photon momentum p_0^R to be smaller than the infinitesimal parameter $\bar{\omega}$:

$$\bar{\omega} \ll m_e, m_f \ll s. \tag{B.17}$$

This, in term, leads to the restriction for x to the interval $x \in (s - 2m_f\bar{\omega},\ s)$, containing at $x = s$ the infrared singularity and allows to separate the Born cross section factor. In fact the integral over ω is the limited by $\bar{\omega}$ x-integral (see Appendix C).

Some more details on the method used may be taken from [11-13]. The kinematics is the same as in [13]. By straightforward integration one gets

$$\delta_{soft} = \left(P_{IR} + \ln\frac{2\varpi}{\eta} \right) \cdot \delta_0^S + \delta_1^S,$$

$$\delta_0^S = -2(1-L_e) + 4QL_i - 2Q^2(1-L_f),$$

$$\delta_1^S = \left[-\frac{\pi^2}{6} + (1-L_e)\cdot(L_e+L_f+L_+ +L_-) + \frac{1}{2}L_e^2 - \frac{1}{2}L_i^2 \right]$$

$$-2QL_fL_i + Q^2\left[1+L_f-L_f^2-\frac{\pi^2}{6}\right], \qquad \text{(B.18)}$$

$$P_{IR} = \frac{1}{n-4} + \frac{\gamma_E}{2} - \ln(2\sqrt{\pi}).$$

APPENDIX C

The matrix element M_β^{br} corresponding to Fig.2 is the same as in [13] and has been integrated as follows:

$$\frac{d\sigma^{br}}{d\cos\theta} = \frac{\alpha^3}{\pi^2 s}\int d\Gamma \sum_{spins} |M_\beta^{br}|^2, \qquad \text{(C.1)}$$

$$\int d\Gamma = \frac{\pi^2}{4s}\int_0^s x\,dx\, \frac{s-x}{4\pi(s-x+m_f^2)} \int_{-1}^{+1} d\cos\theta_\gamma^R \int_0^{2\pi} d\varphi_\gamma^R . \qquad \text{(C.2)}$$

Because of the infrared singularity we cut the x-integral into two parts: $i_1 = (0,\, s-2m_f\varpi)$, $i_2 = (s-2m_f\varpi,\, s)$.
In App. B we split up the singular part of the squared matrix element and integrated it over i_2. What remains is

$$\int_{(i_1+i_2)} d\Gamma \left[\sum_{spins} |M_\beta^{br}|^2 - 64\cdot T^A(x,t)\cdot B^{IR} \right] + \int_{(i_1)} d\Gamma \left[64\cdot T^A(x,t)\cdot B^{IR} \right], \qquad \text{(C.3)}$$

where T^A and B^{IR} are defined in (B.7). The first integral over the unrestricted phase space is finite. The second one is finit too and even well-defined in the limit $\varpi \to 0$ with the exception of terms containing the numerator $(s-x)$ for which the exclusion of i_2 is important:

$$\int_0^{s-2m_f\varpi} \frac{dx}{s-x} = \ln\frac{s}{2m_f\varpi} . \qquad \text{(C.4)}$$

Effectively, one may use (C.1-C.2) and take into account (C.4) whenever necessary. This has been done using the system of analytic manipulation SCHOONSCHIP [14], heavily relying on approximations of calculations and tables of integrals described in [12]. The result is

$$\frac{d\sigma^{br}}{d\cos\theta} = \frac{d\sigma_{Born}}{d\cos\theta}\,\frac{\alpha}{\pi}\,\delta_{soft} + \frac{d\sigma^{hard}}{d\cos\theta}, \qquad \text{(C.5)}$$

$$\frac{d\sigma^{hard}}{d\cos\theta} = \frac{\pi\alpha^2}{2s}\,\frac{\alpha}{\pi}\,\delta_{hard}, \qquad \text{(C.6)}$$

$$\delta_{hard} = \delta_{hard}^{i} + Q\,\delta_{hard}^{int} + Q^2\,\delta_{hard}^{f}, \qquad \text{(C.7)}$$

$$\delta_{hard}^{i}(\cos\theta) = \Big\{ \frac{10}{9} + \frac{4}{9}L_e + (L_e-1)(1+\cos^2\theta)\left(-\frac{11}{6} - \ln\frac{2\varpi}{m_f} + L_f\right) \qquad \text{(C.8)}$$
$$-\frac{2}{3}L_f + \frac{8}{3}c_-L_- + \frac{1}{c_-^2}\left[L_+L_e - 2L_+ - \Phi(c_-)\right] - \frac{2}{3}L_- + \frac{2}{c_+}L_-$$
$$+\frac{1}{2c_+^2}L_-^2 + \frac{1}{c_-}\left[\frac{4}{3}L_- - \frac{19}{9} + \frac{5}{3}L_e + \frac{2}{3}L_f\right]\Big\} + [\cos\theta \leftrightarrow -\cos\theta],$$

$$\delta_{hard}^{int}(\cos\theta) = \Big\{ 2(1+\cos^2\theta)L_-\cdot\left(-2\ln\frac{2\varpi}{m_f} + 2L_f - 3\right) + \frac{3}{c_-} + \frac{3}{c_-^2}L_+ \qquad \text{(C.9)}$$
$$+ 8L_- + \cos\theta\cdot\left(3 + \frac{\pi^2}{6} + \frac{1}{2}L_+^2\right) \Big\} - [\cos\theta \leftrightarrow -\cos\theta],$$

$$\delta_{hard}^{f}(\cos\theta) = \Big\{ 1 + (1+\cos^2\theta)\cdot\left[(1-L_f)\ln\frac{2\varpi}{m_f} + \frac{1}{8} - \frac{5}{4}L_f + \frac{3}{4}L_f^2 - \frac{\pi^2}{4}\right] \Big\} \qquad \text{(C.10)}$$
$$+ [\cos\theta \leftrightarrow -\cos\theta].$$

from the sum of δ_{soft} (with Born factor) and δ_{hard} one gets the cut-off independent F_{br}^a, $a = i$, int, f, the IR-singularity of which will be compensated by the corresponding virtual corrections.

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Аналитическое вычисление диаграмм тормозного излучения в процессе $e^+e^-\to\mu^+\mu^-\gamma$ в рамках КЭД

Аналитически вычислены дифференциальное сечение диаграмм тормозного излучения фотона и зарядовая асимметрия A_{FB} в процессах e^+e^--аннигиляции в два фермиона и обсуждаются в рамках электрослабой стандартной теории.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Fedorenko O.M., Riemann T. E2-86-596

Analytic Bremsstrahlung Integration for the Process $e^+e^-\to\mu^+\mu^-\gamma$ in QED

The photon bremsstrahlung correction to the differential cross section and to the charge asymmetry A_{FB} of e^+e^--annihilation into two fermions has been integrated analytically over the complete photon phase space and is discussed in the context of the electroweak standard theory.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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