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ADLER RELATION AND NEUTRINO-PRODUCTION OF SINGLE HADRONS

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1. Introduction

A partial conservation of axial-vector current (PCAC) /1/ is a very profound principle reflecting a non-trivial structure of vacuum. In contrast to conservation of vector current which is ensured by the equality of nucleon masses, special requirements are needed in the case of axial current even in the chiral limit when quark masses are equal to zero. Existence of a massless Goldstone meson (pion) and fulfilment of definite relations between axial meson and pion contributions to the weak hadron current are required. Therefore, within the 4-momentum transfer $Q^2 \to 0$ PCAC makes it possible to express coupling constants or amplitudes of the axial meson interaction with hadron via analogous values for a pion. One example is the Goldberger-Treiman /2/ relation, another one is the Adler relation /3/, which connects the amplitude of the neutrino-hadron inelastic interaction (e.g. $\nu N \to \mu F$, where $F \neq N$) at $Q^2 = 0$ with the pion interaction amplitude ($\pi N \to F$).

In the paper by Piketty-Stodolsky /4/ one finds a detailed analysis of hadron neutrino-production (π, A_1, ρ, ...). The main problem is the Q^2 extrapolation from the region of PCAC applicability to the values of $Q^2 \approx 1$ $(GeV/c)^2$. When the Adler relation was "matched" with the contribution of the intermediate A_1-meson state in paper /4/, a peculiar result was obtained, namely the cross-section of elastic pion scattering was equal to the cross section of diffractive dissociation ($\pi \to A_1$), while experimentally these cross-sections differ by more than an order of magnitude. The reason for this paradox and the problem of extrapolation over Q^2 are considered in section 2 of this paper.

In Lackner's paper /5/ the process of coherent single pion neutrino-production off nuclei is considered. However, the dependence on Q^2 was taken into account only in the form of the nuclear formfactor dependence on the longitudinal momentum transfer. The absence in /5/ of Q^2-dependence of the amplitude of neutrino-production off nucleon results in the incorrect dependences of neutrino-production cross-sections on the neutrino energy ($\propto E$) and on the nucleus atomic number ($\propto A^{1/3}$). These problems are considered in Sec. 3.

The coherent single pion neutrino-production was also regarded in Rein-Sehgal's paper /6/. The amplitude of neutrino-production off a nucleon as a function of Q^2 was taken in the pole form with the axial mass $M_A \approx 1$ GeV. However, this approximation was not grounded and the problem /4/ of extrapolation over Q^2 was not discussed at all. Besides, a completely incorrect formula contradicting Quantum Mechanics was used for the amplitude of elastic πA-scattering. These problems are discussed in Sec. 3.

In the present paper the dispersion analysis of the single pion neutrino-production amplitude is performed (Sec. 2). Results of the analysis are used to obtain the cross-sections of coherent (Sec. 3) and incoherent (Sec. 4) single pion neutrino-production off nuclei. In Sec. 5 the results of calculations are compared with the existing experimental data. The experimental selection of coherent events is also discussed. Sec. 6 deals with the process of single A_1-meson neutrino-production.

2. Dispersion Generalization of the Adler Relation

Let us consider pion production in the charged current:

$$\nu N \to \mu^- \pi^+ N. \tag{1}$$

The process will be characterized by the following kinematic variables: E is the incident neutrino energy; E' is the muon energy; $\nu \equiv E-E'$; $q^2 \equiv -Q^2$ is the square of the 4-momentum transfer from neutrino to the muon; $k_{T,L}$ are the transversal or longitudinal (with respect to the $\vec{q}$-momentum) momentum components of the recoil nucleon; $k_L \approx (m_\pi^2 + Q^2)/2\nu$. The variables k_T and k_L appear to be more convenient than the nucleon 4-momentum transfer squared $t \approx -k_T^2-k_L^2$.

Let us consider the contribution of axial current to the cross-section of reaction (1). At $Q^2 = 0$ the PCAC hypothesis gives a very simple expression known as the Adler relation /3/ for a contribution of axial current to the cross-section of reaction (1):

$$\left.\frac{d\sigma(\nu N\to\mu\pi N)}{d\nu\, dQ^2\, dk_T^2}\right|_{Q^2=0} = \frac{G^2}{2\pi^2}\,\frac{E'}{\nu E}\, f_\pi^2\, \frac{d\sigma_{el}^{\pi N}}{dk_T^2}, \tag{2}$$

where $G = 10^{-5}\, m_N^{-2}$ is the universal constant of the weak interaction; $f_\pi = 0.93\, m_\pi$ is the $\pi\to\mu\nu$ decay constant; $d\sigma_{el}^{\pi N}/dk_T^2$ is the differential cross-section of elastic πN-scattering at the pion energy ν.

If one puts down the dispersion relation over Q^2 for the axial part of the process (1) amplitude, then one could naively expect at small values of Q^2 the dominance of the pion pole. However, it is not true, since the pion pole contribution is suppressed by the pion mass squared /3,4/. A contribution of the same order appears from more distant singularities, i.e. poles (A_1, A_3, ...) and a cut at $M^2 = 9\, m_\pi^2$. This contribution is connected with the pion pole by the Adler relation.

It has been shown in ref. /4/ that if one takes into account only the contribution of A_1-meson to the cross-section of single pion neutrino-production

$$\frac{d\sigma(\nu N\to\mu\pi N)}{d\nu\, dQ^2\, dk_T^2} = \frac{G^2}{\pi^2}\,\frac{1}{4E^2}\, f_A^2\, \frac{Q^2}{(Q^2+m_A^2)^2}\,\frac{4EE'-Q^2}{2|\vec{q}|}\,\frac{d\sigma(\pi N\to A_1 N)}{dk_T^2}, \tag{3}$$

then at $Q^2 \to 0$ it follows from PCAC that

$$\frac{f_A^2}{m_A^4}\lim_{Q^2\to 0} Q^2\,\frac{d\sigma(\pi N\to A_1 N)}{dk_T^2} = f_\pi^2\,\frac{d\sigma_{el}^{\pi N}}{dk_T^2}, \tag{4}$$

where $d\sigma(\pi N\to A_1 N)/dk_T^2$ is the reaction $\pi N\to A_1 N$ cross-section dependent on the pion virtuality Q^2; f_A is the $A_1\to\mu\nu$ decay constant connected with the $\rho^0\to e^+e^-$ decay constant f_ρ by the second Weinberg sum rule /7/ $f_A = f_\rho$.

Relation (4) ensures transition to Adler relation (2) at $Q^2\to 0$. However, a problem appears how to extrapolate relation (2) to the region of large Q^2. If, one substitutes the experimental value $\sigma(\pi N\to A_1 N)$ to (3), then at large $Q^2 \approx m_A^2$ one finds the neutrino production cross section to be an order of magnitude smaller than the result of extrapolation of the Adler relation (2) with the axial formfactor

$$F_N^{pole}(Q^2) = (1+Q^2/m_A^2)^{-1}. \tag{5}$$

The paradox is caused by the fact that in contrast to the vector current, for which the dominance of vector mesons takes place, a contribution from the A_1-meson to the axial current is not dominant. It is known that the cross-section $\sigma(\pi N\to A_1 N)$ is only about 10% of the total cross-section of pion diffractive dissociation. Therefore, in the dispersion relation for the amplitude of reaction (1) one should expect a dominance of the cut connected with multiparticle final states in the diffractive dissociation reaction $\pi N\to X N$. As is well-known (see, for instance /8/), the main contribution to the non-resonant background in this reaction is connected with the ρ-π pair production, which is well described by the Deck model /9/. In this case the dependence of the spectral function of the dispersion relation on M^2 ($\rho\pi$-system effective mass squared) is defined by the factor $(M^2 - m_\pi^2)^{-1}$. Thus the cut contribution to the axial formfactor (normalized per unity at $Q^2 = 0$) is presented by the following expression:

$$F_N^{cut}(Q^2) = (m_\rho + m_\pi)^2 \int\limits_{(m_\rho+m_\pi)^2}^{\infty} \frac{dM^2}{(Q^2+M^2)(M^2-m_\pi^2)} =$$

$$= \frac{(m_\rho+m_\pi)^2}{Q^2+m_\pi^2} \ln\left[1+\frac{Q^2+m_\pi^2}{(m_\rho+m_\pi)^2}\right]. \qquad (6)$$

Power series expansion of this expression at small Q^2, has a form: $F_N^{cut}(Q^2) \approx 1 - Q^2/2(m_\rho + m_\pi)^2$. It follows from the first Weinberg sum rule /7/ derived in the chiral limits $m_\pi = 0$ that $2m_\rho^2 = m_A^2$ (note that relation $2(m_\rho + m_\pi)^2 = m_A^2$ is fulfilled numerically much better).

Thus Q^2-dependences of A_1-pole and $\rho\pi$-cut contributions in the axial formfactor of reaction (1) coincide at small Q^2. From comparison of (5) and (6) one can easily see that numerical similarity of these expressions at $Q^2 \gtrsim m_A^2$ is also held with a high accuracy. The fact that the $\rho\pi$-cut "center of mass" coincides with the A_1-pole position seems quite non-trivial. Similarity of the $\rho\pi$-system mass spectrum in the reaction $\pi N \to \rho\pi N$ and the Breit-Wigner curve describing A_1-meson, results from the superposition of a decrease of the Deck amplitude with M^2 and a threshold increase of the $\rho\pi$-system phase space volume. The phase space volume is not included in integral (6), therefore the imitation of the pole contribution by the non-resonant background in these two cases seems to be caused by different reasons.

Since Q^2-dependences of formfactors (5) and (6) are close and the cross-section normalization at $Q^2 = 0$ is fixed by Adler relation (2), the relative values of pole and cut contributions are irrelevant. The cross-section is extrapolated to the region of $Q^2 \neq 0$ by multiplying expression (2) by $F_N^2(Q^2)$, where the formfactor $F_N(Q^2)$ is taken further on in form (5).

3. Coherent Neutrino-Production of Pions off Nuclei

Let us consider the process $\nu A \to \mu\pi A$, where a nucleus remains in the initial state. The process is of interest, since here the diffractive mechanism contribution is enhanced. Indeed, the vector current contribution in this case is not only suppressed by the factor $1/\nu$, but also forbidden by the quantum number selection rules in the coherent process. The axial current contribution in the coherent neutrino production cross-section can be calculated using the Adler relation:

$$\frac{d\sigma(\nu A \to \mu\pi A)}{dQ^2\, d\nu\, dk_T^2} = \frac{G^2}{2\pi^2}\,\frac{E'}{\nu E}\, f_\pi^2\, \frac{d\sigma_{el}^{\pi A}}{dk_T^2}\left|F_A(Q^2)\right|^2. \qquad (7)$$

Here

$$\frac{d\sigma_{el}^{\pi A}}{dk_T^2} = \frac{1}{4\pi}\left|f_{el}^{\pi A}(k_T)\right|^2, \qquad (8)$$

$$f_{el}^{\pi A}(k_T) = i\int d^2b\, \exp(i\vec{k}\vec{b})\left\{1 - \exp\left[i f_{el}^{\pi N}(0)\, T(b)\right]\right\}, \qquad (9)$$

where $T(b) = \int_{-\infty}^{\infty} dz\, \tilde{\rho}_A(z,b)$ is the nuclear profile function;

$$\tilde{\rho}_A(z,b) = \int d^2b'\, \rho_A(z,b')\, \frac{1}{2\pi B_{\pi N}} \exp\left[-\frac{(\vec{b}-\vec{b}')^2}{2B_{\pi N}}\right], \qquad (10)$$

$\rho_A(z,b)$ is the single particle nuclear density dependent on the longitudinal coordinate z and the impact parameter b; $B_{\pi N}$ is the slope parameter of elastic πN-scattering. All formulae in this section are written in the form of optical approximation for the sake of visual proof. More accurate formulae obtained with the substitution of the following type $\exp(-\sigma T) \to \left[1 - \frac{\sigma T}{A}\right]^A$ are used for numerical calculations in section 5, which allows introducing essential corrections for light nuclei.

For heavy nuclei $T(b)$ in the $B_{\pi N} \ll R_A^2/2$ approximation has a simpler form: $T(b) \approx \int_{-\infty}^{\infty} dz\, \rho_A(b,z)$. The amplitude $f_{el}^{\pi N}(k_T)$ in (9) is normalized by the requirement $f_{el}^{\pi N}(0) = i/2\,(1-\rho)\sigma_{tot}^{\pi N}$, where $\rho = \mathrm{Re} f_{el}^{\pi N}(0)/\mathrm{Im} f_{el}^{\pi N}(0)$.

At small values of $k_T^2 \ll 1/R_A^2$ differential cross-section (8) can be presented in the following form:

$$\frac{d\sigma_{el}^{\pi A}}{dk_T^2} = \frac{(\sigma_{tot}^{\pi A})^2}{16\pi} \exp(-B_T k_T^2), \qquad (11)$$

where

$$\sigma_{tot}^{\pi A} = 2\int d^2b \left\{1 - \exp\left[-\tfrac{1}{2}\sigma_{tot}^{\pi N}\, T(b)\right] \cos\left[\tfrac{1}{2}\rho\,\sigma_{tot}^{\pi N}\, T(b)\right]\right\}, \qquad (12)$$

$$B_T = \frac{1}{2}\langle b^2\rangle = \frac{1}{\sigma_{tot}^{\pi A}}\int d^2b\, b^2\left\{1-\exp\left[-\frac{1}{2}\sigma_{tot}^{\pi N}T(b)\right]\right\}. \tag{13}$$

As in the case of the neutrino-production on a nucleon, the amplitude is extrapolated to the region of values of $Q^2 \neq 0$ in (7) using the formfactor $F_A(Q^2)$ determined from dispersion relations. Since the amplitude of neutrino-production on nucleus is proportional to the nucleon amplitude, it can be easily shown that $F_A(Q^2)$ is factorized in the following form:

$$F_A(Q^2) = F_N(Q^2)\,\tilde{F}_A(Q^2). \tag{14}$$

The formfactor $F_N(Q^2)$ was introduced in the previous section and Q^2-dependence of the nuclear formfactor $\tilde{F}_A(Q^2)$ is determined by the anomalous thresholds /10/ close to $Q^2 = 0$ and causing a strong Q^2-dependence of the $\tilde{F}_A(Q^2)$. However, an accurate calculation of the anomalous cut contribution is a complicated problem even for light nuclei /11/. Let us use an adequate method of calculation by the Glauber formalism:

$$\tilde{F}_A(Q^2) = \frac{\sigma_{tot}^{\pi N}}{\sigma_{tot}^{\pi A}}\int d^2b\int_{-\infty}^{\infty} dz\, \exp(ik_L z)\tilde{\rho}(b,z)\,\times$$
$$\times \exp\left[-\frac{1}{2}\sigma_{tot}^{\pi N}\int_z^{\infty} dz'\,\tilde{\rho}(b,z')\right]. \tag{15}$$

At $Q^2 \ll 2\nu/R_A$ this expression can be simplified if $\tilde{F}_A$ is written in the following form:

$$\tilde{F}_A(Q^2) \approx \exp\left(-\frac{1}{2}B_L k_L^2\right). \tag{16}$$

where, as follows from (15),

$$B_L = \langle z^2\rangle - \langle z\rangle^2. \tag{17}$$

The averaging is done by the formula

$$\langle z^n\rangle = \frac{\sigma_{tot}^{\pi N}}{\sigma_{tot}^{\pi A}}\int d^2b\int_{-\infty}^{\infty} dz\, z^n\tilde{\rho}(b,z)\,\times$$
$$\times \exp\left[-\frac{1}{2}\sigma_{tot}^{\pi N}\int_z^{\infty} dz'\,\tilde{\rho}(b,z')\right]. \tag{18}$$

Though expressions (11) and (16) are only valid for low values of k_T and k_L, we shall use calculations in the total range to simplify them. Expressions (8) and (15) are more accurate.

One should also note that we have not taken into account the inelastic screening /12,13/ corrections to $\sigma_{tot}^{\pi A}$. At small values of $\nu \lesssim 10$ GeV, which give the main contribution to the cross-section in the existing neutrino experiments, these corrections are negligibly small. These corrections grew with ν up to the value of an order of 10%. Their contribution to the total cross-section is negative and can be estimated by the formula /13/:

$$\Delta_{in}\sigma_{tot}^{\pi A} = 4\pi\int d^2b\int dM^2 \left.\frac{d\sigma_{dd}^{\pi N}}{dM^2 dq_T^2}\right|_{q_T=0} \exp\left[-\frac{1}{2}\sigma_{tot}^{\pi N}T(b)\right]\,\times$$
$$\times\left|F(q_L,b)\right|^2. \tag{19}$$

Here $d\sigma_{dd}^{\pi N}/dM^2dq_T^2$ is the differential cross-section of reaction $\pi N \to XN$, where M is the effective mass of the X beam; $q_L \approx \approx (M^2-m_\pi^2)/2\nu$ and q_T are longitudinal and transversal momentum transfer components;

$$F(q_L,b) = \int_{-\infty}^{\infty} dz\,\rho_A(b,z)\exp(iq_L z) \tag{20}$$

is the longitudinal formfactor of a nucleus.

Finally, we small calculate the k_L-dependence of cross-section (7).

$$\frac{d\sigma(\nu A\to\mu\pi A)}{dk_L^2} = \int d\nu\, dQ^2 dk_T^2\,\delta\left[k_L^2-\left(\frac{Q^2+m_\pi^2}{2\nu}\right)^2\right]\frac{d\sigma(\nu A\to\mu\pi A)}{dQ^2 d\nu\, dk_T^2} =$$
$$= \left(\frac{Gf_\pi}{\pi m_A}\right)^2\frac{\sigma_{el}^{\pi A}E^2}{(\tau_A+\gamma_A)^3}\left[\tau_A-\ln(1+\tau_A)\right]\exp\left[-\frac{B_L m_A^4}{4E^2}(\tau_A+\gamma_A)^2\right], \tag{21}$$

where $\tau_A = 2Ek_L/m_A^2-\gamma$; $\gamma_A = m_\pi^2/m_A^2$.

At high energies, when $B_L m_A^4\gamma_A^2/E^2 \ll 1$, expression (21) has its maximum at $k_L \approx \frac{3}{2}m_\pi^2/E$, which is equal to the value

$$\left.\frac{d\sigma(\nu A\to\mu\pi A)}{dk_L^2}\right|_{max} = \left(\frac{Gf_\pi}{\pi m_A}\right)^2\frac{2}{27}\sigma_{el}^{\pi A}E^2.$$

When k_L increases, cross-section (21) decreases as $1/k_L$. Therefore, at a high neutrino energy the axial formfactor of a nucleon causes a maximum in the distribution of cross-section over k_L^2. The width of this maximum is much smaller than the value $\Delta k_L^2 \approx 1/B_L$, which is determined by the nuclear formfactor (this phenomenon resembles the reaction of Coulomb hadron production off nuclei). However, one should not misidentify this peak as a contribution of the coherent events /14/ which forms a peak $\sim 1/B_L$ wide.

It follows that at high energies the total cross-section for coherent neutrino-production of pions increases as $\ln E$ and depends on the atomic number of a nucleus as $\sigma_{el}^{\pi A} \approx A^{2/3}$. At the same time at low energies the cross-section grows linearly with E and increases as $A^{1/3}$ with the atomic number of a nucleus. The results of numerical calculations are presented in section 5.

4. Incoherent Neutrino-Production of Pions off Nuclei

A process followed by the nuclear destruction appears to be the background to the coherent neutrino-production of pions. This process is analogous to the quasielastic hadron scattering on nuclei. Its cross-section denoted by $\sigma(\nu A\to\mu\pi X)$ is equal to:

$$\frac{d\sigma(\nu A\to\mu\pi X)}{d\nu\, dQ^2 dk_T^2} = \frac{d\sigma(\nu N\to\mu\pi N)}{d\nu\, dQ^2 dk_T^2}\,\frac{\sigma_{abs}^{\pi A}}{\sigma_{in}^{\pi N}}, \tag{22}$$

where $\sigma_{abs}^{\pi A} = \sigma_{tot}^{\pi A} - \sigma_{el}^{\pi A} - \sigma_{qel}^{\pi A}$ is the absorption cross-section of pions on nuclei:

$$\sigma_{abs}^{\pi A} = \int d^2b \left\{ 1 - \exp\left[-\sigma_{in}^{\pi N} T(b)\right]\right\}. \tag{23}$$

The cross-section of neutrino-production of pions off a nucleon included in (22) has contributions of axial current as well as of vector current, which can be calculated in the ρ-dominance approximation /4/. As a result, the cross-section has the following form:

$$\frac{d\sigma(\nu N\to\mu\pi N)}{d\nu\, dQ^2 dk_T^2} = \frac{G^2}{2\pi^2}\frac{E'}{\nu E}\left\{ f_\pi^2 (1+Q^2/m_A^2)^{-2}\frac{d\sigma_{el}^{\pi N}}{dk_T^2} + \right.$$
$$\left. + f_\rho^2 \frac{Q^2}{m_\rho^4}(1+Q^2/m_\rho^2)^{-2}\frac{d\sigma_T(\pi N\to\rho N)}{dk_T^2}\right\}, \tag{24}$$

where $f_\rho = \sqrt{2}m_\rho^2/g_\rho$; g_ρ is the dimensionless universal coupling constant ($\rho\pi\pi$, $\rho NN, \dots$), $g_\rho^2/4\pi \approx 2.1$.

Only the transversal component of $\sigma_T(\pi N\to\rho N)$ cross-section remains in (24). Actually, the data /15/ fit on this reaction cross-section in the energy range 2 GeV $< \sqrt{S} <$ 10 GeV using expression

$$\sigma_T(\pi N\to\rho N) = \sigma_0^{\pi\rho}\nu^{\beta}, \tag{25}$$

where ν is the energy in the lab system in GeV, yields the values of $\sigma_0 = 8.27 \pm 0.5$ mb, $\beta = -1.85 \pm 0.03$. The value β indicates that in this energy range the helicity flip π-exchange dominates.

Let us also calculate the k_L- distribution of the cross-section for incoherent neutrino-production. To simplify the integration, let us fix $\beta = -2$.

$$\frac{d\sigma(\nu A\to\mu\pi X)}{dk_L^2} = \frac{G^2}{\pi^2}\frac{\sigma_{abs}^{\pi A}}{\sigma_{in}^{\pi N}}\left\{ f_\pi^2\sigma_{el}^{\pi N}\frac{E^2}{m_A^2}\left[\frac{\tau_A - \ln(1+\tau_A)}{(\tau_A+\gamma_A)^3}\right]\right.$$
$$\left. - \frac{f_\rho^2}{4m_\rho^2}\sigma_0^{\pi\rho}\frac{2\tau_\rho(1-\gamma_\rho) + (\tau_\rho + 2\gamma_\rho + \tau_\rho\gamma_\rho)\ln[\gamma_\rho(\tau_\rho+1)/(\tau_\rho+\gamma_\rho)]}{(\tau_\rho+\gamma_\rho)(1-\gamma_\rho)^3}\right\}, \tag{26}$$

where $\gamma_\rho = m_\pi^2/m_\rho^2$; $\tau_\rho = 2Ek_L/m_\rho^2 - \gamma_\rho$.

5. Calculations

Calculating the $\sigma_{tot}^{\pi A}$ cross-section by formula (12) and the slope parameters B_T and B_L by formulae (13), (17) and (18) we used the Woods-Sakson nuclear density distribution:

$$\rho(r) = \rho_0\left[1 + \exp\left(\frac{r-R}{a}\right)\right]^{-1},$$
$$\rho_0 = \frac{3A}{4\pi R^3}\left(1 + \frac{\pi^2 a^2}{R^2}\right)^{-1}. \tag{27}$$

The values of parameters R and a taken from ref. /17/ for different nuclei are listed in the Table. The radius of the nucleon charge distribution was taken into account /18/. The values of B_T, B_L, $\sigma_{tot}^{\pi A}$

calculated at $\sigma_{tot}^{\pi N} = 24$ mb and the values of $\sigma_{abs}^{\pi A}$ calculated at $\sigma_{in}^{\pi N} = 20$ mb are also presented in the Table. The value of ρ for the t-channel isoscalar amplitude is approximately independent of the energy /19/ and was fixed by $\rho = 0.2$.

It worth noting that the expression for $\sigma_{tot}^{\pi A}$, which was used in ref. /16/, is erroneous and contradicts the diffractive origin of the π-A scattering. When absorption of pions by nuclear matter increases the cross-section $\sigma_{tot}^{\pi A}$ should also increase. In the case of diffractive scattering on a "black disk" the total cross-section attains the maximum value $\sigma_{tot}^{\pi A} = 2\pi R^2$, while the cross-section calculated in ref. /6/ tends to zero. The use of this erroneous formula leads to some uncontrolled errors in calculations and makes the comparison with experimental data senseless. Note that results of calculation of $\sigma_{tot}^{\pi A}$ adduced in the Table have uncertainty only about 1% /18/.

Table. Parameters of Woods-Sakson density (27) and the results of calculation of $\sigma_{tot}^{\pi A}$, $\sigma_{abs}^{\pi A}$, B_T and B_L.

A	R fm	a fm	$\sigma_{tot}^{\pi A}$ fm^2	$\sigma_{abs}^{\pi A}$ fm^2	B_T fm^2	B_L fm^2
20	2.80	0.571	38.1	25.5	3.49	2.85
27	2.84	0.569	49.3	29.8	3.68	2.85
40	3.39	0.612	65.6	42.1	4.87	3.76
64	4.20	0.569	91.5	57.0	5.96	4.31
110	5.33	0.535	138.2	87.7	8.39	6.01
150	5.72	0.650	181.3	111.8	10.6	7.24
184	6.51	0.535	230.8	135.0	12.0	8.09
207	6.62	0.546	250.2	150.7	12.6	8.15
238	6.80	0.605	284.3	168.2	13.5	8.98

As at a low energy ν the cross-section $\sigma_{tot}^{\pi N}(\nu)$ increases, we also calculate the values of $\sigma_{tot}^{\pi A}$, B_T and B_L for different values of $\sigma_{tot}^{\pi N}$. It turns out that the effects of simultaneous growth of $\sigma_{tot}^{\pi N}$ and B_T, B_L cancell each other. This justifies the above-adopted approximation with $\sigma_{tot}^{\pi N}$ independent of ν. Nevertheless it is desirable to exclude in the experiment the region of small ν, which gives a negligible contribution to the coherent neutrino-production cross-section.

The k_L^2 distributions of coherent and incoherent contributions to the cross-section of single pion neutrino-production off a neon nucleus at E = 40 GeV is given in Fig. 1. One can see that the inco-

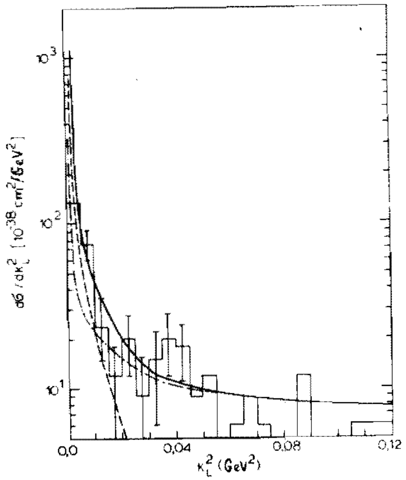

Fig. 1. The k_L^2-distribution of the coherent (the dashed line) and incoherent (the dashed-dotted line) contributions to the single pion neutrino-production off neon. The experimental histogram from /14/ is normalized for the theoretical cross-section at $k_L^2 < 0.015$ GeV^2. The solid line is a sum of the coherent and incoherent contributions.

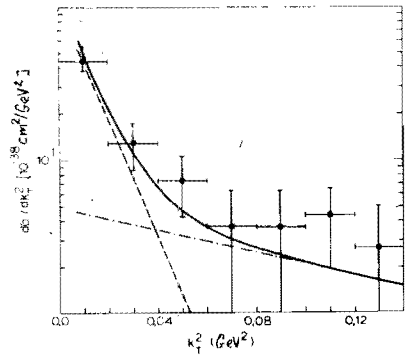

Fig. 2. The k_T^2-distribution of the coherent (the dashed line) and incoherent (the dashed-dotted line) contributions to the single pion neutrino-production off neon for the events with $k_L^2 < 0.015$ GeV^2. Experimental data are taken from /14/. The solid line is a sum of the coherent and incoherent contributions.

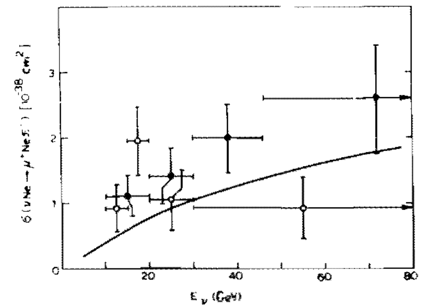

Fig. 3. The total cross-section of the reaction $\bar{\nu} Ne \to \mu^+ \pi^- Ne$. Experimental points o-are from /14/; ● -are from /20/. The theoretical curve is calculated with allowance for the cuts used in the experiment.

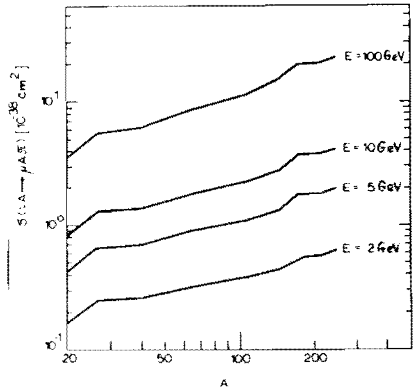

Fig. 4. The cross-section of single pion coherent neutrino-production vs the atomic number of a target nucleus at different neutrino energies.

herent contribution, as the coherent one has a maximum at small k_L^2. As mentioned above, this is connected with the nucleon formfactor behaviour in the axial current (see formula (26)). The vector current contribution at small values of k_L^2 is suppressed due to the factor Q^2 in formula (24). As is seen in Fig. 1, the contribution of incoherent events under the peak is large and cannot be separated by simple extrapolation from the region of large k_L^2.

To select coherent events, it is necessary to fit the experimental distribution over k_T^2 with two Gaussian exponents. One of them corresponds to the coherent contribution, another to the incoherent one. Their relative values are determined by the k_L^2 cut. Thus in /14/ events with $k_L^2 < 0.015$ GeV^2 were selected. The corresponding data are shown in Fig. 2. If one describes this distribution by one exponent /14/, a considerable contribution of the incoherent events will result in a too smooth distribution with the $B_T = 28^{+7}_{-6}$ GeV^{-2} slope for the events from $k_L^2 < 0.015$ GeV^2. In Fig. 2 one can see also the results of our calculations for the coherent and incoherent contributions to the cross-section integrated over k_L^2 in the region of $k_L^2 <$ 0.015 GeV^2. Comparison of data and calculations proves that events with $k_T^2 > 0.05$ GeV^2 are incoherent.

In Fig. 3 the results of calculations of the total cross-section of coherent single pion neutrino-production are compared with the experimental data. In Fig. 4 the cross-section of coherent neutrino-production of pions is shown as a function of the atomic number at various neutrino energies.

6. Single A_1-Meson Neutrino-Production

The Adler relation connects the cross-section for neutrino-production of single A_1-mesons at $Q^2 \approx 0$ with the cross-section $\sigma(\pi N \to A_1 N)$, which, as mentioned above, is very small at high energies ν. Therefore, a noticeable contribution to the cross-section of the coherent process arises only from the large $Q^2 \sim m_A^2$. It is described by the expression /4/:

$$\frac{d\sigma(\nu A \to \mu A_1 A)}{dQ^2\, d\nu\, d k_T^2} = \frac{G^2}{4\pi^2} f_A^2 \frac{Q^2}{(Q^2+m_A^2)^2} \times \tag{28}$$

$$\times \left(\frac{\nu}{E^2} + \frac{4E'}{\nu E}\right) \widetilde{F}_A^2(Q^2)\, \frac{d\sigma_{el}^{A_1 A}}{d k_T^2}$$

$\widetilde{F}_A(Q^2)$ is given by expression (16), where $k_L \approx (m_A^2 + Q^2)/2\nu$. In this expression the cross-section of elastic scattering for transversally and longitudinally polarized A_1-mesons were for simplicity supposed to be equal. The distribution of the cross-section over k_L^2 has the following form:

$$\frac{d\sigma(\nu A \to \mu A_1 A)}{d k_L^2} = \frac{2E^2}{m_A^4}\left(\frac{G f_A}{2\pi}\right)^2 \frac{(\sigma_{tot}^{\pi A})^2}{16\pi B_T} \exp(-B_L k_L^2)\, I_3 \tag{29}$$

$$I_3 = -(\tau_A+\gamma_A)^{-2}\left[\left(1-\frac{1}{\tau_A+\gamma_A}\right)\left(15+\frac{1}{\tau_A+\gamma_A}\right) + 8\left(1+\frac{1}{\tau_A+\gamma_A}\right)\ln(\tau_A+\gamma_A)\right] ,$$

where for numerical calculations it is assumed that $\sigma_{tot}^{A_1 N} = \sigma_{tot}^{\pi N}$. The constant of A_1-meson coupling to the weak lepton charged current is found from the second Weinberg sum rule /7/ $f_A = f_\rho = \sqrt{2}\, m_\rho^2 / g_\rho$. The results of calculations of the differential cross-section (29) for a neon nucleus at E = 40 GeV are shown in Fig. 5.

In the same figure the distribution over k_L^2 for the incoherent A_1-meson production is presented. The distribution was calculated using the formula:

$$\frac{d\sigma(\nu A \to \mu A_1 X)}{d k_L^2} = \left(\frac{G}{2\pi}\right)^2 \left[f_A^2 \frac{(\sigma_{tot}^{\pi N})^2}{16\pi B_T^{\pi N}} \frac{2E^2}{m_A^4} I_3 + \frac{f_\rho^2}{4 m_\rho^2} \sigma_0^{\rho \to A_1} I_4 \right] \frac{\sigma_{abs}^{\pi A}}{\sigma_{in}^{\pi N}} ;$$

$$I_4 = 4(\tilde{\tau}_A+\tilde{\gamma}_\rho)^{-2}(\tilde{\gamma}_\rho-1)^{-3}\Big\{(\gamma_\rho-1)^3 \ln(\tilde{\tau}_\rho+1) - \tag{30}$$

$$- 4(\tilde{\tau}_\rho+\tilde{\gamma}_\rho)(3\tilde{\gamma}_\rho+2\tilde{\tau}_\rho-1)\tilde{\gamma}_\rho \ln\left|\frac{\tilde{\gamma}_\rho(\tilde{\tau}_\rho+1)}{\tilde{\tau}_\rho+\tilde{\gamma}_\rho}\right| -$$

$$- \frac{\tilde{\tau}_\rho(1-\tilde{\gamma}_\rho)}{\tilde{\tau}_\rho+1}\Big[4(\tilde{\tau}_\rho+\tilde{\gamma}_\rho)\big(\tilde{\gamma}_\rho(\tilde{\tau}_\rho+2\tilde{\gamma}_\rho-1)+\tilde{\tau}_\rho+1\big) -$$

$$- (1-\tilde{\gamma}_\rho)^2(4\tilde{\tau}_\rho+6\tilde{\gamma}_\rho-1)\Big]\Big\}$$

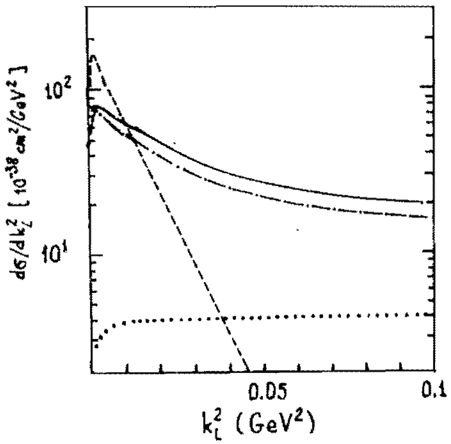

Fig. 5. Distributions over k_L^2 for different contributions to the single A_1-meson neutrino-production cross-section. The dashed line is a coherent cross-section. The dot-dashed line is a contribution of axial current to the incoherent process; the dotted line is a contribution of vector current to the incoherent neutrino-production; the solid line is a sum of all contributions. Calculations have been performed for a neon nucleus at the neutrino energy E = 40 GeV.

where $\tilde{\tau}_\rho = (2k_L E - m_A^2)/m_\rho^2$; $\gamma_\rho = m_A^2/m_\rho^2$. The second term in (30) presents the contribution of vector current, which is large at small energies of ν . The cross-section $\sigma(\rho N \to A_1 N)$ is described by the diagram of a single pion exchange and is parametrized in the following form:

$$\sigma(\rho N \to A_1 N) = \sigma_0^{\rho\to A_1} / \nu^2 . \tag{31}$$

To estimate the parameter $\sigma_0^{\rho\to A_1}$ one can use a generalization of the effective chiral Lagrangian method in the case of vector and axial-vector currents /22/. Using chiral-invariant Lagrangians of $A_1\rho\pi$ and πN interactions in the minimal form and taking into account the formfactor of πN -vertex in the exponential form with the slope parameter of $B_\pi = 5$ GeV^{-2}, we obtain for $\sigma(\rho N \to A_1 N)$:

$$\sigma(\rho N \to A_1 N) \approx \frac{1}{96\pi m_N^2} \, \frac{g_{\pi N}^2 \, g_{A\rho\pi}^2}{\nu^2} \left[1 + \frac{(m_A^2 + m_\rho^2)^2}{2 m_\rho^2 m_A^2}\right] \times$$

$$\times \left\{ \left(1 - 2m_\pi^2 B_\pi\right)\left[m_\pi^2 B_\pi - C - \ln(2 m_\pi^2 B_\pi)\right] + 1 \right\} \approx \frac{30}{\nu^2}\ mb , \tag{32}$$

where ν is in GeV; $g_{\pi N} = 13.6$ is the πN -coupling constant; the constant of minimal $A_1\rho\pi$ -interaction is connected with the decay $A_1 \to \rho\pi$ width by the relation:

$$\Gamma_{A\rho\pi} = \frac{g_{A\rho\pi}^2}{4\pi} R_2 , \qquad R_2 = 0.226\ GeV^{-1}.$$

Since the amplitude of elastic $A_1 N$-scattering is almost imaginary and the π-exchange amplitude is real, the interference of the axial and vector current contributions to the differential cross-section $\sigma(\nu A \to \mu A_1 X)$ can be neglected. The contributions of the first and second terms to the cross-section on the neon-target and their sum are shown separately in Fig.5.

It is seen that the axial current contribution is dominant. It follows from the expressions (29),(30) and (21), (26) that relation of the coherent cross section to the incoherent one in the case of A_1 production is the same as in the caseof pion production. But A_1 incoherent production cross section is not enhanced at small values of k_L . The observed peak in the differential coherent cross section is determined mainly by the formfactor exp(-B k_L^2). Thus one can extract the coherent ivents by the fitting procedure analogously to the one done above in k_T distribution.

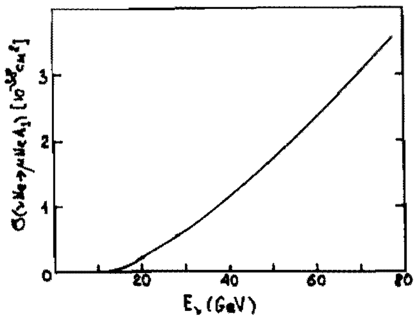

Fig. 6. Total cross section of coherent A_1-meson neutrino-production on neon nucleus.

It should be noted that neutrino-production of single A_1-mesons is not sensitive to the PCAC predictions, since the region of small Q^2, determined by the Adler relation, gives a negligible contribution to the total cross-section. The interest to this process seems to be connected with the possibility to study A_1-interactions with nucleons. It turned to be impossible to obtain similar information in the process of diffractive A_1-meson production by pions off nuclei /23/ due to necessity of model allowance for inelastic corrections, making a contribution of an order of 100% to this process. On the other hand, as noted in section 3, a contribution of inelastic corrections to the A_1-meson elastic scattering cross-section is negligibly small. Therefore, the value of $\sigma_{tot}^{A_1 N}$ can be reliably obtained from the analysis of experimental data using expressions (29) and (30).

Total cross section of coherent A_1 neutrino-production on neon nucleus calculated at different energies using (29) is shown in fig.6. It is seen that the cross section rises with the energy faster than the pion neutrino-production cross-section and exeeds it at the energies above 40 GeV.

7. Discussion of Results

A study of single pion neutrino-production in the coherent process on nuclei allows separating the contribution of axial current, which is generally determined by the PCAC requirement. Nevertheless, the accurate and carefull calculation of this process gives a possibility for the experimental verification of the PCAC effects.

Let us summarize the main results of the present paper.

1. The extrapolation of the Adler relation to the region of $Q^2 \neq 0$ has been obtained. It has been shown that the main contribution to the dispersion integral for the axial current is made by a cut connected with the $\rho\pi$-production in the intermediate state and not by the A_1-pole. Nevertheless, it turned out that the $\rho\pi$-cut "center of mass" coincides with the A_1-meson mass $m_A = 1.3$ GeV. Therefore, the formfactor in the pole form (5) can be used, though at high energies, when the large values of Q^2 give a noticeable contribution, expression (6) is more exact.

2. With the coherent neutrino-production of pions off nuclei an additional dependence on Q^2 appears. It is related to the complex structure of a nucleus. Simple expression (16) has been obtained for it.

3. Expression (21) for the dependence of the cross-section of single pion neutrino-production on longitudinal momentum transfer has been obtained. It has been shown that at small k_L^2 the experimentally observed peak is mainly due to the axial formfactor of a nucleon, but not of a nucleus. Therefore, small k_L^2 cannot be the only criterion for selection of coherent events.

4. The cross-section of incoherent neutrino-production of pions off nuclei has been calculated with allowance for the contribution of vector current. It has been shown that this process is also characterized by the narrow peak in the distribution over k_L^2.

5. The cross-section for the coherent and incoherent neutrino-production of A_1-mesons has been calculated. It is shown that these processes are insensitive to the PCAC effects. However, they allow obtaining unique information on the A_1-meson interaction with nucleon.

Finally, we would like to emphasize once more the necessity of more accurate selection of coherent events from the experimental data. The selected events with small values of $k_L^2 \lesssim 1/B_L$ should be distributed over the variable $k_T^2 \approx |t-t_{min}|$ (not over t, as is done in /20/). The next fit of k_T^2-dependence of the cross-section by two Gaussian exponents should lead to the $B \approx B_T$ values of slopes for the coherent peak and $B \approx B_{e\ell}^{\pi N}$ values for the incoherent substrate. This is the usual procedure in the hadronic experiments. Comparison of the results of fitting of the incoherent background with the distribution over k_T^2 for the events with visible protons appears to be an additional test of the calculations.

An independent selection of coherent events is also possible in the distribution over k_L^2. In this case it is necessary to add the weight factor $(1+Q^2/m_A^2)^2$, neutralizing the influence of the nucleon axial formfactor to each event. After that the coherent peak should have a nuclear slope $B \approx B_L$ and the incoherent background should become almost k_L-independent.

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Бельков А.А., Копелиович Б.З. E2-86-595

Соотношение Адлера и нейтринорождение одиночных адронов

Рассмотрены процессы нейтринорождения одиночных адронов /реакция $\nu A \to \mu h A'$, где $h = \pi$, A_1.../ с точки зрения экспериментальной проверки следствий PCAC. Получено дисперсионное обобщение соотношения Адлера для нейтринных реакций в области физических значений переданного 4-импульса $Q^2 \neq 0$, которое затем используется для вычислений сечений нейтринорождения одиночных пионов в когерентных и некогерентных процессах на ядрах. Результаты расчетов сравниваются с экспериментальными данными по реакции $\bar{\nu}Ne \to \mu^+\pi^- Ne$. Обсуждается процедура выделения когерентных событий в эксперименте. Рассмотрен процесс нейтринорождения одиночных A_1-мезонов на ядрах.

Работа выполнена в Лаборатории ядерных проблем ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1986

Belkov A.A., Kopeliovich B.Z. E2-86-595

Adler Relation and Neutrino-Production of Single Hadrons

The processes of single hadron neutrino-production (reaction $\nu A \to \mu h A'$, where $h = \pi$, A_1,...) are considered from the viewpoint of experimental verification of PCAC predictions. Dispersion generalization of the Adler relation for neutrino reactions in the range of physical values of 4-momentum transfer $Q^2 \neq 0$ is obtained. This generalization is used to calculate the cross-sections for neutrino-production of single pions in coherent and incoherent processes on nuclei. The results of calculations are compared with experimental data on the reaction $\bar{\nu}Ne \to \mu^+\pi^- Ne$. A procedure of the coherent event selection in the experiment is discussed. A process of single A_1-meson neutrino-production is considered.

The investigation has been performed at the Laboratory of Nuclear Problems, JINR.

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