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HIGH-QUALITY PION FORM-FACTOR DATA FROM e^+e^- VERSUS THAT FROM INVERSE ELECTROPRODUCTION: A COMPATIBILITY TEST

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The pion form factor is the only representative among vertex functions describing the electromagnetic structure of hadrons, which is in principle measurable in the whole region of its kinematical variable, t. However, not all of the processes where $|F_\pi(t)|$ is measured, are easy to analyze. The reactions of pion electroproduction on nucleons [1] (space-like region) and the inverse electroproduction [2] (time-like region in a vicinity of the elastic threshold) are contributed by several diagrams and the pion form factor is extracted from the measured cross sections under certain plausible but not firmly grounded assumptions. A question of reliability of these data is therefore in order. This is in contrast to the reliable measurement of electron-positron annihilation into two pions where the cross section is directly proportional to the modulus squared of $|F_\pi(t)|$ and where a plenty of precise data points do exist nowadays.

When questioning the reliability of the direct and inverse electroproduction data, one is tempted to use somehow the precise experimental information from e^+e^- annihilation. A viable way is then to relate both sets of data via analyticity through the dispersion integral

$$F_\pi(Q^2) = \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} \frac{\mathrm{Im}\, F_\pi(t)}{t + Q^2}\, dt. \qquad (1)$$

Now, one is faced with the problem that the experiment doesn't offer $\mathrm{Im}\, F_\pi(t)$ directly. That this is not a serious problem has been demonstrated in [3]. Indeed, we have been able to develop a simple and efficient procedure to determine the quality of each individual data point from electroproduction [1]. The application of a slightly modified version of that approach for determining the reliability of $F_\pi(t)$ data from inverse electroproduction is a subject of the present note.

A central point in our analysis is to extract the reliable information on $\mathrm{Im}\, F_\pi(t)$ in the elastic region from the e^+e^- cross section:

$$\sigma_{exp}(t) = \frac{\pi\alpha^2\beta_\pi^3}{3t} \left| F_\pi(t) + \xi e^{i\phi} \frac{m_\omega^2}{m_\omega^2 - t - i m_\omega \Gamma_\omega} \right|^2 , \qquad (2)$$

where α is the fine-structure constant and $\beta_\pi = (1 - \frac{4m_\pi^2}{t})^{1/2}$ is the CMS velocity of an outgoing pion. The second term in (2) represents the contribution of the ω-meson to a two-pion final state. This effect is induced by the well-known $\rho-\omega$ mixing due to G-parity nonconserving interactions with the strength of electromagnetism [4-6]. Parameter ξ can be related to the ratio of the differences of light quark masses [6,7] or is equivalently given in terms of measured widths according to

$$\xi = \frac{6}{\alpha m_\omega}\left(\frac{m_\omega^2}{m_\omega^2 - 4m_\pi^2}\right)^{3/2} [\Gamma(\omega \to e^+e^-)\cdot\Gamma(\omega \to \pi^+\pi^-)]^{1/2}, \tag{3}$$

ϕ is the interference phase essentially equal to the hadronic part of the ρ-ω mixing phase:

$$\phi = \mathrm{arctg}\frac{m_\rho \Gamma_\rho}{m_\rho^2 - m_\omega^2}. \tag{4}$$

Now, one can make use of the elastic unitarity condition valid for $4m_\pi^2 < t < t_{\pi^\circ\omega}$, which guarantess the identity between the pion form factor phase and the P-wave isovector phase shift $\delta_1^1(t)$ in this region, to obtain the quadratic equation [8]

$$|F_\pi|^2 + 2|F_\pi|Z(t) + \left\{\frac{\xi^2 m_\omega^4}{(m_\omega^2 - t)^2 + m_\omega^2\Gamma_\omega^2} - \left[\frac{3t}{\pi\alpha^2\beta_\pi^3}\sigma_{exp}(t)\right]^2\right\} = 0 \tag{5}$$

whose physical solution gives for $|F_\pi(t)|$ the expression

$$|F_\pi(t)| = -Z(t) + \left\{Z^2(t) + \left[\frac{3t}{\pi\alpha^2\beta_\pi^3}\sigma_{exp}(t)\right]^2 - \frac{\xi^2 m_\omega^4}{(m_\omega^2 - t)^2 + m_\omega^2\Gamma_\omega^2}\right\}^{1/2} \tag{6}$$

with

$$Z(t) = \frac{\xi m_\omega^2}{(m_\omega^2 - t)^2 + m_\omega^2\Gamma_\omega^2}\{(m_\omega^2 - t)\cos(\phi - \delta_1^1) - m_\omega\Gamma_\omega \sin(\phi - \delta_1^1)\}. \tag{7}$$

One can take measured numbers of $\Gamma(\omega \to e^+e^-)$, $\Gamma(\omega \to \pi^+\pi^-)$ and standard values for $m_\omega, m_\rho, \Gamma_\rho$ to determine the quantities ξ and ϕ. The experimental information on $\delta_1^1(t)$ is very precisely taken into account by means of a Padè-type approximation [9]

Writing $F_\pi(t) = |F_\pi(t)|e^{i\delta_1^1(t)}$ we obtain the experimental data on $\mathrm{Im}F_\pi(t)$ in the elastic region. This part of integration in (1) already saturates the normelized value $F_\pi(0) = 1$ to 93% and even slightly more for $Q^2 \equiv -t < 0$.

To make correct predictions on $F_\pi(Q^2)$ values at experimentally measured points, it is necessary to evaluate the second integral in the decomposition

$$F_\pi(Q^2) = \frac{1}{\pi}\int_{4m_\pi^2}^{t_{\pi^\circ\omega}} \frac{\mathrm{Im}^E F_\pi(t)}{t + Q^2}dt + \frac{1}{\pi}\int_{t_{\pi^\circ\omega}}^{\infty} \frac{\mathrm{Im}^A F_\pi(t)}{t + Q^2}dt. \tag{8}$$

As is shown in [10] by means of the perturbative QCD result, $\mathrm{Im}F_\pi(Q^2)$ has at least one zero and it approaches asymptotically zero from negative values. The following parametrization reflects these properties:

$$\mathrm{Im}^A F_\pi(t) = \frac{a(t_0 - t)}{(t - b)^M}. \tag{9}$$

The parameters a, t_0, b and M can be determined from the requirements of smooth junction between both parts of $\mathrm{Im}F_\pi(t)$ at the point $t_{\pi^\circ\omega}$

$$\mathrm{Im}F_\pi(t_{\pi^\circ\omega}) = \frac{a(t_0 - t_{\pi^\circ\omega})}{(t_{\pi^\circ\omega} - b)^M}, \tag{10a}$$

$$\frac{d}{dt}\mathrm{Im}^E F_\pi(t_{\pi^\circ\omega}) = \frac{d}{dt}\frac{a(t_0 - t)}{(t - b)^M}\Big|_{t = t_{\pi^\circ\omega}}, \tag{10b}$$

$$\frac{d^2}{dt^2}\mathrm{Im}F_\pi(t)\Big|_{t = t_{\pi^\circ\omega}} = \frac{d^2}{dt^2}\frac{a(t_0 - t)}{(t - b)^M}\Big|_{t = t_{\pi^\circ\omega}}, \tag{10c}$$

and from the sum rule

$$1 - \frac{1}{\pi}\int_{4m_\pi^2}^{t_{\pi^\circ\omega}} \frac{\mathrm{Im}F_\pi(t)}{t}dt = \frac{a}{\pi}\int_{t_{\pi^\circ\omega}}^{\infty} \frac{(t_0 - t)}{t(t - b)^M}dt \tag{10d}$$

following from eq.(8) and the normalization $F_\pi(0) = 1$. The solution of the nonlinear system (10) is found numerically:

$$a = 0.41557, \quad t_0 = 165.5762, \quad b = 27.6907, \quad M = 1.3193 \quad (m_\pi^2 = 1) \tag{11}$$

Now, we are ready to compare predictions of eq.(8) supplemented by parametrization (9) with values of a, t_0, b and M given in (11), with nine data points obtained in three measurements of the reaction $\pi^- p \to e^+e^- n$ at Dubna in 1972, 1973 and 1976 [2]. Three of the points are situated just under the two-pion threshold and the calculation of corresponding integrals (8) can be easily done by standard program GQUAD from the CERN library. The singular integration for six data points above the elastic threshold yields, by using the program CAUCHY, corresponding real parts of $F_\pi(t)$ which have to be combined with $\mathrm{Im}^E F_\pi(t)$ discribed above. Forming the χ^2 for each data point we have obtained the results summarized in the Table. The average of all nine partial χ^2 is 3.47 indicating relatively poor global quality of the data. However, the removal of five points at $t =$ = (0.059, 0.075, 0.080, 0.088, 0.090) GeV^2 gives for the remaining four points (asterisked in the Table) the average χ^2-value of 1.16, which is already acceptable. These points can be therefore considered to be reliable.

Table

t [GeV^2]	F_π^{exp}	ΔF_π^{exp}	F_π^{theory}	χ^2
0.059	0.90	0.09	1.130	6.52
0.068*	1.10	0.07	1.154	0.58
0.075	1.00	0.08	1.173	4.68
0.080*	1.07	0.17	1.263	1.29
0.088	1.14	0.06	1.270	4.69
0.090	1.06	0.09	1.274	5.65
0.106	1.12	0.06	1.322	5.02
0.116*	1.30	0.07	1.363	0.80
0.122*	1.26	0.08	1.373	2.01

Noting that all nine measured data lie under the curve predicted on the basis of high-quality e^+e^- data, we can conclude that the results of three inverse electroproduction experiments [2] are affected by a systematic error not taken into account by the authors.

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Надежные данные по пионному формфактору из e^+e^- в сравнении с данными из обратного электророждения: проверка совместимости

Сделана простая проверка совместимости экспериментальных данных по пионному формфактору, полученных из обратного электророждения, с точной экспериментальной информацией по электрон-но-позитронной аннигиляции с использованием аналитического продления последней в область упругого порога. Из всех девяти точек из обратного электророждения четыре оказываются надежными.

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Martinivič Ľ. E2-86-581

High-Quality Pion Form-Factor Data from e^+e^- Versus that from Inverse Electroproduction: a Compatibility Test

A simple compatibility check between pion form factor data obtained in inverse electroproduction and electron-positron annihilation is made by using the precise experimental information on the latter, analytically continued to the elastic threshold. Four points from nine inverse electroproduction data are shown to be reliable.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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