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V.K.Mitrjushkin, A.M.Zadorozhny

TEMPERATURE PHASE TRANSTIONS IN SU(2) - GAUGE THEORY

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1. As is known, at sufficiently high temperatures gauge systems undergo phase transition (PT) from the low-temperature confined phase to high-temperature quark-gluon phase[1,2]. The matter state with high temperature may be realized in relativistic nucleus-nucleus head-on collisions at an energy accessible at present accelerators.

As far as the PT temperature θ_c has a nonperturbative origin, it is natural to use the lattice formulation. From a point of view of lattice gauge theories the study of temperature transitions is very essential for investigating a continuum limit. The reason is that θ_c is a physical observable independent of the cut-off and UV-divergences, and the determination of the θ_c gives a good possibility to find the onset of the scaling behaviour. Motivated by these arguments, several numerical studies deal with $SU(2)$-gauge theory (see, e.g., papers[3-11]).

In this paper we study the SU(2) gauge theory and calculate the temperature string $\langle L \rangle$ and susceptibility χ using distribution functions of the order parameter.

Let us consider a nonsymmetric lattice in a four-dimensional Euclidean space with the number of sites $N_t \times N_s^3$ and periodic boundary conditions.

The partition function Z is defined in a standard manner

$$Z = \int [dU] \exp\{-S\}, \quad [dU] \equiv \prod_{links} dU_\ell , \tag{1}$$

where S is the Wilson action

$$S \equiv 4/g^2 \sum_{\square} (1 - 1/2 \, \mathrm{Tr}\, U_\square). \tag{2}$$

The temperature θ is defined as an inverse size of the lattice in the "time" direction

$$\theta = 1/aN_t$$

In the weak coupling limit we can use the renormalization-group predictions. Then, we obtain for θ :

$$\theta/\Lambda_L = \frac{1}{N_t} f(g) \equiv \frac{1}{N_t} \left(\frac{11 g^2}{24\pi^2}\right)^{51/121} \cdot e^{\frac{12\pi^2}{11 g^2}} \left(1 + O(g^2)\right). \tag{3}$$

As is known, in the SU(N)-gauge theory the temperature PT is related with spontaneus breakdown of the global Z_N-symmetry. Some universality arguments[12] and model considerations[13] allow us to conclude that a pure gauge SU(2) theory at nonzero temperature belongs to the class of universality that includes 3d-Ising model. If so, all the critical exponents in the Ising model must coincide with the critical exponents in the SU(2)-gauge theory.

Some recent papers[3-10] have dealt with the study of the order parameter $\langle L \rangle$ (the temperature string):

$$\langle L \rangle \equiv \left\langle \frac{1}{2} \mathrm{Tr} \prod_{t=1}^{N_t} U_{(x,t);\mu=4} \right\rangle .$$

In a vicinity of the critical point the order parameter $\langle L \rangle$ behaves as follows:

$$\langle L \rangle = \begin{cases} 0 , & \theta \le \theta_c \;\; (4/g^2 \le 4/g_c^2) \\ \sim A \left| \frac{4}{g^2} - \frac{4}{g_c^2} \right|^{\beta}, & \theta \gtrsim \theta_c . \end{cases}$$

In the 3d-Ising model the critical exponent β is known to be[14]: $\beta_{3d\,Ising} \simeq 0.31$. The values obtained for β in SU(2) theory are in good agreement with this value.

Before expounding our results, we should make the following important comments:

i) the thermodynamic limit requires the validity of the conditions

$$N_t \gg 1 , \quad N_s \gg 1 , \quad N_s/N_t \gg 1 .$$

So, the choice N_t = 2 seems to be not very good, and from this point of view the choice N_t = 4 looks better. At the same time the choice N_s/N_t = 2 (as in our work) is reasonable. Indeed, though N_s/N_t = 2 is not very large, the position of the critical point may be corrected by the finite-size scaling procedure.

ii) With increasing N_t the critical value $4/g_c^2$ increases too. At N_t = 2;3 values of $4/g_c^2$ are much less than those at which the asymptotical scaling is expected to set in.

In other words, in this case the two-loop formula (3) is not valid, and it is impossible to calculate θ_c. This is another argument in favour of the use of $N_t \geqslant 4$.

2. As it was noted above, the order parameter $\langle L\rangle$ must be zero at $4/g^2 \leqslant 4/g_c^2$, and nonzero at $4/g^2 > 4/g_c^2$. But the difficulty in determination of the critical point is connected with the existence in the confinement phase two symmetric states: with $\langle L\rangle > 0$ and with $\langle L\rangle < 0$. Due to a finite size of the lattice the transitions from one state to another may occur **("tunneling")**. If averaging is made over a large number of iterations, L will change in sign several times, and positive and negative values of L in the course of averaging will compensate each other so that the average value $\langle L\rangle$ gets zero.

In practice this phenomenon is significant at $g^2 \sim g_c^2$, which would lead to a wrong determination of g_c^2. That is why very often the critical point is determined with the use of $\langle |L| \rangle$ for which the above-described problem does not arise. But the usage of the $\langle |L| \rangle$ is connected with another difficulty. Indeed, very close to θ_c spurious nonzero estimates for magnetization result from finite lattices, because $\langle |L| \rangle$ then differs from zero appreciably due to a broad width of the distribution.

In this paper another method is used for determining g_c^2 which is based on sampling of the distribution functions of the order parameter. These distribution functions can be used to get improved estimates of quantities often obtained by a standard Monte-Carlo analysis in a different way, e.g., the order parameter, susceptibility, etc.. This method turns out to be rather effective in studying properties of the Ising model/15/.

Let us define the current "magnetization" of the lattice as follows

$$\bar{L} = \frac{1}{N_s^3} \sum_{\vec{x}} L_{\vec{x}} , \tag{4}$$

where summation runs over all $\vec{x}$. Now we can define the distribution function of the order parameter:

$$P(\bar{L}) \equiv Z^{-1} \int [dU]\, e^{-S} \delta\Big(\bar{L} - \frac{1}{N_s^3} \sum_{\vec{x}} L_{\vec{x}}\Big) . \tag{5}$$

By definition the order parameter $\langle L\rangle$ equals

$$\langle L\rangle = \int d\bar{L} \cdot \bar{L} \cdot P(\bar{L}) . \tag{6}$$

Below θ_c we can define the susceptibility in the following way:

$$\chi(\theta) = \lim_{N_s \to \infty} \chi_{N_s}(\theta) ,$$

$$\theta \chi_{N_s}(\theta) \equiv 1/N_s^3 \sum_{\vec{x},\vec{y}} \langle L_{\vec{x}} L_{\vec{y}} \rangle \equiv N_s^3 \langle \bar{L}^2 \rangle ; \quad \theta < \theta_c . \tag{7}$$

Like in the Ising model, for values of N_s much larger than the correlation length ξ ($N_s \gg \xi$) the difference between χ and χ_{N_s} must be small/15/

$$\chi = \chi_{N_s} + a/N_s .$$

In this phase the distribution function $P(\bar{L})$ is even $(P(\bar{L}) = P(-\bar{L}))$ and hence $\langle L\rangle$ = 0. The knowledge of the distribution function gives us the possibility to determine higher moments and cumulants of the distribution. As expected, away from the critical point all cumulants become negligible for large N_s, and $P_{N_s}(\bar{L})$ is a gaussian/15/

$$P_{N_s}(\bar{L}) \sim exp\left\{-\bar{L}^2 N_s^3 / 2\theta \chi_{N_s}\right\} ; \quad \theta < \theta_c . \tag{8}$$

For temperatures above θ_c a spontaneous average magnetization $\langle L\rangle = \pm M \neq 0$ appears in the thermodynamic limit, and hence the symmetry property of $P(\bar{L})$ no longer takes place ($P(-\bar{L}) \neq P(L)$). Rather, we must distinguish the probability $P^{(+)}(\bar{L})$

for positive magnetization from the probability $P^{(-)}(\bar{L})$ for negative magnetization, and $P^{(+)}(-\bar{L}) = P^{(-)}(\bar{L})$. The distribution $P^{(+)}(P^{(-)})$ is sharply peaked at $\bar{L} = \bar{L}_{max}$ $(\bar{L} = -\bar{L}_{max})$. The susceptibility χ can be defined as follows

$$\chi = \lim_{N_s \to \infty} \chi_{N_s}$$
$$\theta \chi_{N_s} = N_s^3 \left(\langle \bar{L}^2 \rangle_{N_s} - (\bar{L}_{max})^2 \right) . \qquad (9)$$

It must be noted that distributions $P^{(\pm)}$ are no longer gaussians. Indeed, at $|\bar{L}| < L_{max}$ distributions will be dominated by configurations corresponding to the two-phase **coexistence** ("tunneling") which results in deviations from the gaussian. Certainly with increasing N_s the **tunneling** effect will be suppressed, and the distributions $P^{(\pm)}(\bar{L})$ will approach the Gaussian ones:

$$P^{(\pm)}(\bar{L}) \sim exp\left\{ - \frac{(\bar{L} \mp L_{max})^2 N_s^3}{2\theta \chi_{N_s}} \right\} . \qquad (10)$$

At the same time, at large values of magnetization $|\bar{L}| > L_{max}$ tunneling effects are not large, and as a result the right wing of the distribution $P^{(+)}(\bar{L})$(left wing of the distribution $P^{(-)}(\bar{L})$) should agree with the gaussian behavior (10). This allows us to get rid of the tunneling effects due to the lattice volume being finite: it is sufficient to make the distribution $P^{(+)}$ ($P^{(-)}$) symmetric with respect to a vertical axis with $\bar{L} = L_{max}$ ($\bar{L} = -L_{max}$). Then formula (10) permits us to determine the susceptibility χ_{N_s}. This procedure of determining the order parameter $\langle L \rangle$ $(\equiv L_{max})$ and susceptibility allows a more reliable and accurate calculation for these quantities without enormous increasing the statistics.

3. In our Monte-Carlo calculations we have used a heat-bath method, and the sequence of **renewing** of variables on links and at sites was chosen in a random way (stochastic sweeps). Statistical errors were determined by a standard bunching-method /16/. A typical statistics per point was 7000-8000 iterations. For thermalization first 1500-2000 iterations were employed, over others averaging was carried out. A most part of the calculations was made on a lattice $4{\times}8^3$ making use of the periodic boundary **conditions.**

In Fig.1 distributions $P(\bar{L})$ are shown, obtained by the Monte-Carlo method at various values of $4/g^2$. It is seen that at $4/g^2$ = 2.25 the maximum of the distribution coincides with zero (Fig.1a), and the distribution is with a good accuracy described by a guassian distribution. At $4/g^2$ = 2.28 the distribution maximum is shifted from zero (Fig. 1b), the right wing of the distribution being well described by a Gaussian one, while at $4/g^2$ = 2.26 the distribution is certainly non-gaussian though its maximum is close to zero (Fig. 1c). The study of such distributions gives for the phase transition point the following value:

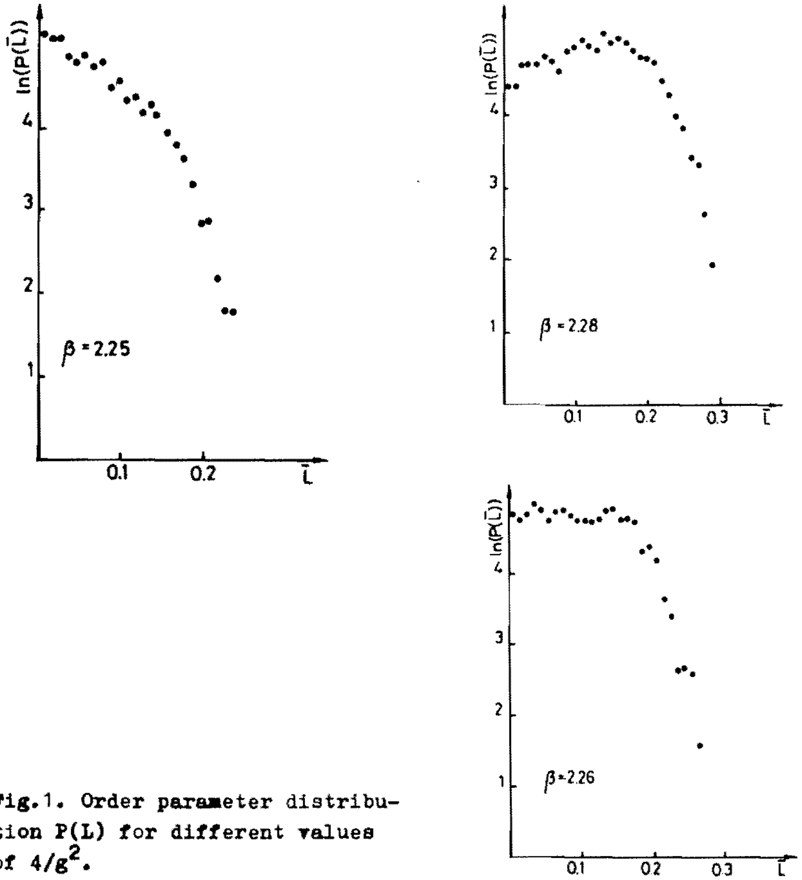

Fig.1. Order parameter distribution P(L) for different values of $4/g^2$.

$$4/g_c^2 = 2.26 \pm 0.05.$$

The dependence of the average of temperature string $\langle L \rangle$ $(\equiv L_{max})$ on $4/g^2$ is shown by crosses in Fig. 2. The corresponding values of $\langle L \rangle$ are collected in Table 1. The corresponding critical temperature equals

$$\theta_c/\Lambda_L \simeq 38.2$$

and the critical exponent is

$$\beta \simeq 0.33.$$

For comparison, in Fig.2 black points show the values obtained by the Monte-Carlo method for $\langle |L| \rangle$ (see also Table 1). Note that within errors these values for $\langle |L| \rangle$ are in agreement with the values for $\langle |L| \rangle$ drawn in the graph of ref. /9/ for the lattice size 4×12^3. Note that the use of these data much complicates the determination of the phase transition point.

A still more striking discrepancy results from calculating the susceptibility χ in different ways. In Fig.3 crosses stand for values of χ $(\equiv \chi_{fit})$ obtained by analysing the width of distribution $P(L)$, i.e., by formulas (8), (10), whereas black pointes denote the values of χ $(\equiv \chi_{MC})$ found by a direct computation by the Monte-Carlo method (formulas (7), (9)). The corresponding values of susceptibility are written in Table 2. It is seen that the maximum position for χ_{MC} is to a great extent shifted to the right ($4/g^2 > 2.3$) and does not coincide with the position of the critical point determined by calculating $P(L)$. We suppose that a reliable determination of the susceptibility by a direct Monte-Carlo computation of the averages

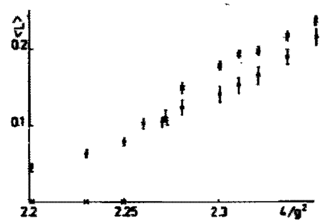

Fig.2. The dependence of $\langle L \rangle$ on $4/g^2$ is shown by crosses. Black point show the values obtained by the Monte Carlo method for $\langle |L| \rangle$.

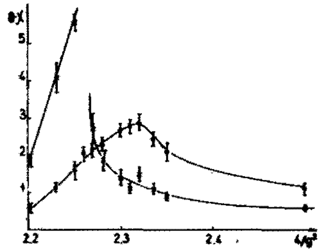

Fig.3. Crosses stand for values of $\theta\chi_{fit}$ obtained by analysing the width of distribution P(L). Black points denote the values of $\theta\chi_{MC}$ found by a computation by the Monte Carlo method.

TABLE 1.

$4/g^2$	L_{max}	$\langle \lvert L \rvert \rangle$	$4/g^2$	L_{max}	$\langle \lvert L \rvert \rangle$
2.2	0	0.046±0.03	2.32	0.20±0.005	0.168±0.01
2.23	0	0.066±0.006	2.335	0.22±0.003	0.192±0.01
2.25	0	0.08 ±0.006	2.35	0.24±0.003	0.217±0.005
2.26	0	0.104±0.006			
2.27	0.11 ±0.01	0.106±0.006	2.5	0.336±0.003	0.324±0.005
2.28	0.15 ±0.005	0.126±0.01	2.55	0.363±0.003	0.344±0.005
			3.0	0.48 ±0.003	0.465±0.004
2.3	0.18 ±0.005	0.142±0.01	3.1	0.495±0.002	0.484±0.003
2.31	0.195±0.003	0.154±0.01	3.2		0.50 ±0.004

TABLE 2.

$4/g^2$	$\theta\chi_{fit}$	$\theta\chi_{MC}$	$4/g^2$	$\theta\chi_{fit}$	$\theta\chi_{MC}$
2.2	1.9±0.15	0.61±0.1	2.32	1.5±0.2	2.9±0.25
2.23	4.1±0.4	1.16±0.1	2.335	1.1±0.1	2.46±0.2
2.25	5.6±0.3	1.58±0.2	2.35	0.88±0.08	2.1±0.25
2.26	-	2.05±0.15			
2.27	2.75±0.4	2.13±0.15	2.5	0.6±0.05	1.1±0.15
2.28	1.9± 0.3	2.3 ±0.15	2.55	0.4±0.03	1.24±0.3
			3.0	0.3±0.02	0.91±0.25
2.3	1.45±0.2	2.7± 0.2	3.1	0.3+0.02	0.76±0.15
2.31	1.1 ±0.1	2.8 ±0.2	3.2		0.86±0.2

$\langle \sum_{\vec{x}\vec{y}} L_{\vec{x}} L_{\vec{y}} \rangle$ requires an essential increase in the statistics, which signifies that the determination with the use of the distributions is more effective and reliable.

We think that precisely this problem involved in the study of distribution functions $P(L)$ allows the investigation of various correlators and a correct calculation of critical exponents, in the first place, γ (for susceptibility) and α (for specific heat). This will be published elsewhere.

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Задорожный А.М., Митрюшкин В.К. E2-86-575

Температурные фазовые переходы
в SU(2)-калибровочной теории

Используется метод определения критической температуры θ_c, а также "намагниченности" $< L >$, восприимчивости χ, основанный на определении функций распределения параметра порядка. Этот метод позволяет добиться более высокой точности при вычислениях вблизи точки фазового перехода. С помощью этого метода вычисляются θ_c, $< L >$ и χ в SU(2)-калибровочной теории на решетке размером $4{\cdot}8^3$.

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Mitrjushkin V.K., Zadorozhny A.M. E2-86-575

Temperature Phase Transitions in SU(2)-Gauge Theory

The distribution function P(L) of the order parameter is studied for pure SU(2) gauge theory on the lattice $4{\cdot}8^3$. It is shown that the study of P(L) gives a possibility of determining the thermal string $< L >$, critical temperature θ_c, susceptibility χ, etc., with a good accuracy.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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