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QCD-ANALYSIS OF SINGLET STRUCTURE FUNCTIONS USING JACOBI POLYNOMIALS.

THE DESCRIPTION OF THE METHOD

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I. Introduction

At present there exist three basic approaches to compare data on deep inelastic processes with the QCD predictions that are calculated by perturbation methods up to the second order in "running" coupling constant $\alpha_s(Q^2)$ /1-4/.

The first approach is based on the analysis of the moments defined by the following integrals

$$M_i(n,Q^2)=\int_0^1 dx\, x^{n-2} F_i(x,Q^2)$$

$$i=1,2,3; \quad n=2,3,\dots, \tag{1}$$

where $F_i(x,Q^2)$ are the inelastic structure functions (SF), measured experimentally. The advantage of this method is that theoretical expressions for the moments $M^{QCD}(n,Q^2)$ are written in an explicit analytical form, which makes the procedure of data to theory comparison very straightforward. Difficulties in using this method arise due to the absence of the data on the SF in the range $x \leq 0.05$ and $x \geq 0.65$. Therefore, to calculate the experimental values of the moments, the SF's are to be extrapolated into unmeasured regions of x. This extrapolation can introduce a bias in the final results of the analysis.

Analogous problems arise in the framework of the second approach to the QCD-analysis based on the application of the integro-differential evolution equations for the SF derived previously in the framework of the Abelian quantum field theory /5,6/ and then in the QCD /7-9/. For instance, for the nonsinglet structure function $F_{NS}(x,t)$, $t=\ln(Q^2/\Lambda^2)$ this equation

$$3\pi\frac{\partial F_{NS}(x,t)}{\partial t}=\alpha_s(Q^2)\Big\{\big[3+4\ln(1-x)\big]F_{NS}(x,t)+$$
$$+\int_x^1\frac{dy}{1-y}\Big[(1+y^2)F_{NS}\Big(\frac{x}{y},t\Big)-2F_{NS}(x,t)\Big]\Big\} \tag{2}$$

connects the derivative and the integral of the SF taken over the argument $z = x/y$ inside the interval $x \leq z \leq 1$. As has been shown in ref.[10], the character of the SF's scaling violation in the region x = 0.3-0.5 depends essentially on the behaviour of integrand $F_{NS}(x,t)$ at $z \geqslant$ 0.65. To find the numerical solutions of (2), one should again extrapolate $F_{NS}(z,t)$ into the unmeasured region $x \geqslant$ 0.65.

The third approach to the QCD analysis is based on the method of the SF expansion over orthogonal with weight polynomials. For the first time Bernstein polynomials were used[11]; later this idea was developed in papers[12,13] and applied by the EMC for the analysis of $F_2(x,Q^2)$ measured from the iron target[14]. The Laguerre polynomials are suggested for this purpose in paper[15] and used by the CHARM collaboration for the analysis of the data on $F_2(x,Q^2)$ and $xF_3(x,Q^2)$ in the leading order (LO) in $\alpha_s(Q^2)$ [16]. The Jacobi polynomials are also proposed[17] and used for the data analysis of the nonsinglet SF[18-20]. A detailed comparison of the expansion method with other methods of the QCD-analysis can be found in papers[21-23].

The present paper is devoted to further development of the Jacobi polynomials expansion method both for the nonsinglet and singlet structure functions QCD-analysis (section 2). The estimation of the precision of methods and the stability of the results of the analysis with respect to the form of the weight function and the number of expansion terms is performed using different parametrizations of the SF's (section 3). The relation between the QCD theoretical moments and moments of the expanded structure functions is discussed (section 4) and the application of the method to the real data is illustrated using the EMC structure functions $F_2(x,Q^2)$ measured from an iron target (section 5). The results are summarized in section 6.

2. Application of the SF Expansion over the Jacobi Polynomials to the QCD-Analysis

An expansion of the SF over the Jacobi polynomials[17] $\Theta_i^{\alpha\beta}(x)$ satisfying the orthogonality condition (in the notation of refs.[18-20])

$$\int_0^1 dx\, w^{\alpha\beta}(x)\, \Theta_k^{\alpha\beta}(x)\, \Theta_e^{\alpha\beta} = \delta_{ke} \tag{3}$$

with the weight function $w^{\alpha\beta}(x) = x^\alpha(1-x)^\beta$ has the form

$$F_i(x,Q^2) = x^\alpha(1-x)^\beta \sum_{k=0}^{\infty} a_k(Q^2)\, \Theta_k^{\alpha\beta}(x). \tag{4}$$

Using relation (3) we can write down the coefficients as:

$$a_k(Q^2) = \int_0^1 dx\, F_i(x,Q^2)\, \Theta_k^{\alpha\beta}(x). \tag{5}$$

Substitution into eq. (5) of expansion of the Jacobi polynomials in powers of x^j

$$\Theta_k^{\alpha\beta}(x) = \sum_{j=0}^{k} c_j^{(k)}(\alpha,\beta)\, x^j \tag{6}$$

leads to the formula

$$a_k(Q^2) = \sum_{j=0}^{k} c_j^{(k)}(\alpha,\beta)\, M(j+2,Q^2), \tag{7}$$

where $M(j+2,Q^2)$ are the moments determined by relation (1). Combining expres. (4) and (7) we finally get the expression for $F_i(x,Q^2)$:

$$F_i(x,Q^2) = x^\alpha(1-x)^\beta \sum_{k=0}^{\infty} \Theta_k^{\alpha\beta}(x) \sum_{j=0}^{k} c_j^{(k)}(\alpha,\beta)\, M(j+2,Q^2), \tag{8}$$

where the Q^2-dependence of $F_i(x,Q^2)$ is defined by the Q^2-dependence of the moments $M(j+2,Q^2)$. Formula (8) is valid both for singlet and nonsinglet SF.

Prior to description of our further development we briefly comment on papers[18-20]. The authors used formula (4) for expanding nonsinglet structure functions $xF_3(x,Q^2)$ and studied their

Q^2 -evolution with the help of the evolution equation for the coefficients $a_k(Q^2)$

$$a_k(Q^2)=\sum_{j=0}^{k}\sum_{i=0}^{j}\left[\frac{\ln(Q^2/\Lambda^2)}{\ln(Q_0^2/\Lambda^2)}\right]^{d_{NS}^{j+2}} c_j^{(k)}(\alpha,\beta)\, d_i^{(j)}(\alpha,\beta), \tag{9}$$

where d_{NS}^{j+2} are anomalous dimension and $d_i^{(j)}(\alpha,\beta)$ are matrix elements entering into the expansion

$$x^j=\sum_{i=0}^{j} d_i^{(j)}(\alpha,\beta)\,\Theta_i^{\alpha\beta}(x) \tag{10}$$

which is inverse to expr. (6). Equation (9) is the result of substitution into expr. (8) of the solution of the evolution equation for nonsinglet moments

$$M_{NS}^{QCD}(n,Q^2)\equiv\frac{1}{6}\langle\Delta(Q^2)\rangle_n=\left[\frac{\ln(Q^2/\Lambda^2)}{\ln(Q_0^2/\Lambda^2)}\right]^{d_{NS}^{n}}\cdot\frac{1}{6}\langle\Delta(Q_0^2)\rangle_n \tag{11}$$

found in perturbation theory, and of the expression for $M(j+2,Q_0^2)$ through the integral (1) of $x F_3(x,Q_0^2)$ with the change of the term x^{j+2} in (1) by expansion (10). Performing the nonsinglet analysis with the help of expansion (4) the authors have retained the first three terms of the series and treated the coefficients $a_0(Q^2)$, $a_1(Q^2)$ and $a_2(Q^2)$ together with the scale parameter Λ as free parameters. It should be noted that such a method being very useful in the nonsinglet case does not admit a generalization to the case of singlet structure functions since the relation between the singlet moments at Q_0^2 and Q^2 is of a more complex nature than eq. (11).

Our approach to the QCD-analysis of the singlet and nonsinglet SF follows the primary idea of work[17]. In contrast to [18-20] we shall substitute into the right-hand side of ref (8) instead of $M(j+2,Q^2)$ the theoretical expressions for the singlet or nonsinglet moments $M^{QCD}_{S(NS)}(j+2,Q^2)$ found in the QCD perturbation

theory. In the obtained expression (8) for a practical use, one naturally has to employ only a finite number of the series terms, i.e. the finite sum

$$F_i(x,Q^2)=x^{\alpha}(1-x)^{\beta}\sum_{k=0}^{N_{Max}}\Theta_k^{\alpha\beta}(x)\sum_{j=0}^{k}c_j^{(k)}(\alpha,\beta)\,M^{QCD}_{S(NS)}(j+2,Q^2) \tag{12}$$

is used for fitting the experimental data on $F_i(x,Q^2)$. For this fit we will use the practical procedure developed in papers[25,26] and applied for the analysis of the experimental data on singlet moments of the SLAC and BCDMS collaborations. The QCD-parameters (Λ and others) entering into the theoretical expressions for the moments are varied. In the nonsinglet case the expr. (11) is used for M^{QCD}_{NS} , while for the singlet moments we shall use the expression found in $\overline{MS}$-scheme up to the second order in perturbation theory[3,4] (in the notation of ref.[24]):

$$M_S(n,Q^2)=\frac{5}{18}\left[\langle\Sigma(Q^2)\rangle_n\left(1+\frac{\bar{g}^2(Q^2)}{16\pi^2}B_{2,n}^{\psi}\right)+\frac{\bar{g}^2(Q^2)}{16\pi^2}B_{2,n}\langle G(Q^2)\rangle_n\right], \tag{13}$$

where the function

$$\langle\Sigma(Q^2)\rangle_n=\left[(1-\alpha_n)\langle\Sigma(Q_0^2)\rangle_n-\tilde{\alpha}_n\langle G(Q_0^2)\rangle_n\right]\left[\frac{\bar{g}^2(Q^2)}{\bar{g}^2(Q_0^2)}\right]^{d_+^n}H^n_{+\psi}(Q^2,Q_0^2)+ \tag{14}$$
$$+\left[\alpha_n\langle\Sigma(Q_0^2)\rangle_n+\tilde{\alpha}_n\langle G(Q_0^2)\rangle_n\right]\left[\frac{\bar{g}^2(Q^2)}{\bar{g}^2(Q_0^2)}\right]^{d_-^n}H^n_{-\psi}(Q^2,Q_0^2)$$

determines Q^2-evolution of the moment of a singlet combination of quark distributions given at a reference point Q_0^2 , and

$$\langle G(Q^2)\rangle_n=\left[\alpha_n\langle G(Q_0^2)\rangle_n-\varepsilon_n\langle\Sigma(Q_0^2)\rangle_n\right]\left[\frac{\bar{g}^2(Q^2)}{\bar{g}^2(Q_0^2)}\right]^{d_+^n}H^n_{+G}(Q^2,Q_0^2)+$$

$$+\left[(1-\alpha_n)\langle G(Q_0^2)\rangle_n+\varepsilon_n\langle\Sigma(Q_0^2)\rangle_n\right]\left[\frac{\bar{g}^2(Q^2)}{\bar{g}^2(Q_0^2)}\right]^{d_-^n} H_{-G}(Q^2,Q_0^2) \tag{15}$$

determines the Q^2 -evolution of the moments of gluon distribution. The normalization condition for the total nucleon momentum leads to the relation/3-4/

$$\langle\Sigma(Q_0^2)\rangle_2+\langle G(Q_0^2)\rangle_2=1\,. \tag{16}$$

The quantities $\langle\Delta(Q_0^2)\rangle_n$, $\langle\Sigma(Q_0^2)\rangle_n$ and $\langle G(Q_0^2)\rangle_n$ entering into expr. (11) and (13-15) are the moments of unknown non-singlet, singlet and gluon distributions at the reference point Q_0^2 :

$$\begin{aligned}
\langle\Delta(Q_0^2)\rangle_n&=\int_0^1 dx\, x^{n-2}\, x\,\Delta_{NS}(x,Q_0^2)\\
\langle\Sigma(Q_0^2)\rangle_n&=\int_0^1 dx\, x^{n-2}\, x\,\Sigma(x,Q_0^2)\\
\langle G(Q_0^2)\rangle_n&=\int_0^1 dx\, x^{n-2}\, x\,G(x,Q_0^2).
\end{aligned} \tag{17}$$

They are considered as free parameters to be determined from the data analysis/1-4,24/. The number of these parameters is $N_{Max}-1$ and $2N_{Max}-2$ for the NS and S cases, respectively.

To reduce the number of free parameters when working with a sufficiently large number of terms of series (12), one can use a particular parametrization for quark and gluon distributions at Q_0^2 for example,

$$\begin{aligned}
x\Sigma(x,Q_0^2)&=C_v\, x^{\alpha_v}(1-x)^{\beta_v}+C_s(1-x)^{\beta_s}\\
xG(x,Q_0^2)&=C_g\,(1-x)^{\beta_g}(1+\gamma_g x).
\end{aligned} \tag{18}$$

Such parametrizations allow us to use a finite number of parameters C_i , α_i and β_i for calculation of an arbitrary large number of the moments (17) and to study, first, the accuracy of the SF reconstruction with the help of the series, second, the influence

of the weight function on the convergence of the series and, finally, the stability of the fit results on N_{Max} , i.e. on the number of the series terms included into the data analysis.

3. Estimation of the Accuracy of the SF Reconstruction

3.1. Required accuracy

Since the main goal of the QCD-analysis is the determination of the parameter Λ , we should know up to what accuracy the SF should be reconstructed by the fitting procedure in order to get an optimal precision on the Λ . For this purpose we shall estimate the sensitivity of SF's and moments values to the change of Λ .

Let us consider the quantity characterising the changes of moments with changing Λ by $\Delta\Lambda$:

$$\Delta M_s\,\% = \frac{|M_s(n,Q^2,\Lambda)-M_s(n,Q^2,\Lambda+\Delta\Lambda)|}{M_s(n,Q^2,\Lambda)}\cdot 100\%\,. \tag{19}$$

We shall choose for quark and gluon distributions entering into formulae (17) the typical form (18), where the parameters C_i, α_i, β_i ($i = v, s$) are taken from the analysis of the EMC iron-target data/27/.

The results on $\Delta M_s\%$ for Λ =200 MeV and $\Delta\Lambda$ = 50, 100 MeV at Q_0^2 = 27.5 GeV^2 and n= 2,4,6,8 are shown in Table I.

Table 1

The values of $\Delta M_s(n,Q^2,\Lambda-\Delta\Lambda)\%$

	$Q^2[GeV^2]=50$		$Q^2[GeV^2]=90$		$Q^2[GeV^2]=120$		$Q^2[GeV^2]=160$	
n	$\Delta\Lambda=50$	$\Delta\Lambda=100$	$\Delta\Lambda=50$	$\Delta\Lambda=100$	$\Delta\Lambda=50$	$\Delta\Lambda=100$	$\Delta\Lambda=50$	$\Delta\Lambda=100$
2	0.09	0.20	0.16	0.36	0.19	0.42	0.21	0.48
4	0.63	1.16	0.98	2.16	1.19	2.61	1.39	3.03
6	0.73	1.68	1.34	2.94	1.61	3,54	1.86	4.11
8	1.57	1.86	1.88	3.46	2.19	4.17	2.40	4.84
10	1.94	2.07	1.75	3.85	2.10	4.69	2.43	6.40

As is seen, the change of the Λ by 25% leads to unsignificant changes (from 0.1% to 2%) in the values of the moments.

From the estimation of the SF sensitivity to the Λ values we will use the Buras-Gaemers parametrization [28]

$$F_{BG}(x,Q^2) = x^{\alpha_0+\alpha_1 S}\,(1-x)^{\beta_0+\beta_1 S}$$

$$S = -\ln\left[\frac{\ln(Q^2/\Lambda^2)}{\ln(Q_0^2/\Lambda^2)}\right] \qquad (20)$$

with $Q_0^2 = 27.5\ \mathrm{GeV}^2$, $\Lambda = 200$ MeV and parameters α_i and β_i obtained from the EMC data fit [27]. The values of interest

$$\Delta F_{BG}\,\% = \frac{|F_{BG}(x,Q^2,\Lambda) - F_{BG}(x,Q^2,\Lambda-\Delta\Lambda)|}{F_{BG}(x,Q^2,\Lambda)}\ 100\%$$

for typical intervals of x and Q^2 are shown in Table 2

Table 2

		$\Delta F_{BG}(x,Q^2,\Lambda)\%$				$\Delta F_{BG}(x,Q^2,\Lambda)\%$	
x	$Q^2[\mathrm{GeV}^2]$	$\Delta\Lambda=50$	$\Delta\Lambda=100$	x	$Q^2[\mathrm{GeV}^2]$	$\Delta\Lambda=50$	$\Delta\Lambda=100$
0.25	35	0.09	0.20	0.55	35	0.44	0.96
	50	0.22	0.48		50	1.05	2.29
	90	0.40	0.79		90	1.94	4.27
	120	0.49	1.06		120	2.33	5.15
	160	0.56	1.22		160	2.70	5.98
	200	0.62	1.35		200	2.96	6.60
0.35	335	0.20	0.43	0.65	35	0.60	1.31
	50	0.47	1.02		50	1.43	3.13
	90	0.86	1.86		90	2.65	5.85
	120	1.03	2.27		120	3.18	7.07
	160	1.19	2.63		160	3.63	8.22
	200	1.31	2.89		200	4.05	9.08

from which it is seen that the change of the Λ by 25% chauses 1-4% change in SF's at $x = 0.65$; at other values of x the change is even smaller.

Thus, from Tables 1 and 2 it follows that to determine the Λ

with an accuracy better than 10% from the analysis of the ideal SF, one should keep in the SF expansion (12) as many terms as required for the reconstruction of the function with an accuracy better than dozens of per sent. For real SF's test of the results stability on a number of terms should be performed.

3.2. Approximation of a Singlet SF by the Jacobi Polynomials

It is known [29,30] that for a uniform convergence of the Jacobi-polynomial series to a function F given in the interval (0,1) this function should satisfy the following conditions:

a) $\|F(Q^2)\| = \int_0^1 dx\; x^\alpha (1-x)^\beta\, F(x,Q^2) < \infty$

b) $\int_0^1 dx\; x^\alpha (1-x)^\beta \left[(F(x,Q^2) - F(z,Q^2))/(x-z)\right]^2 < \infty$

at any point $z \in (0,1)$.

One can easily see that at $\alpha > -1$ and $\beta > -1$ the conditions are fulfilled for the SF which is limited, differentiable and non-negative in the interval $0 < x < 1$.

How rapidly does the Jacobi-polynomial series converge to $F(x,Q^2)$ if it is the singlet SF. To answer this question, we shall consider an expansion in series (12) of the typical function:

$$f(x) \equiv x\Sigma(x,Q_0^2) = 2.67\,x^{0.25}(1-x)^{3.0} + 0.48(1-x)^8 \qquad (21)$$

the first term of which is responsible for valence and the second one for sea quark distributions. As the measure of the convergence rate we will use

$$\Delta^{\alpha\beta}_{N_{Max}} = \int_0^1 |f(x) - f_{N_{Max}}(x)|\,dx\,, \qquad (22)$$

where $f_{N_{Max}}(x)$ is the finite sum (12).

The analysis of the behaviour of this integral in the allowed region of the parameters α and β has shown that it has two minima, positions of which are shown in fig. 1 by dashed areas. In the first area (around $\beta \approx 3$, $\alpha = 0.25$) the integral (22) is about $10^{-4} \div 10^{-3}$ while in the second one ($\beta = -0.9 \div +3.0$ and $\alpha = -(0.9 \div 0.8)$) it is much less: $10^{-7} \div 10^{-5}$. The dependence of $\Delta^{\alpha\beta}_{N_{Max}}$ on α at the fixed value of $\beta = 3$ for differendt values of N_{Max}, presented in fig. 2, shows that the convergence rate saturates at the values of $N_{Max} \geq 13$. At the values of α and β corresponding to the minima of integral (22) the convergence rate is faster at a negative value of α (see fig.3).

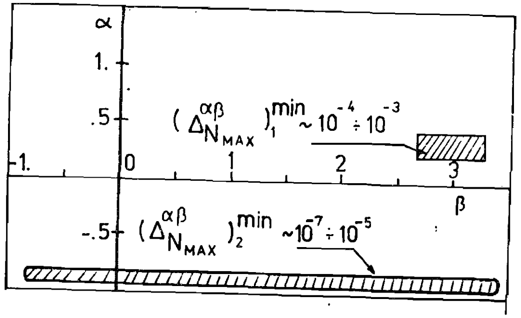


Fig.1. Regions (dashed) of the weight function parameters α and β where the integral $\Delta^{\alpha\beta}_{N_{max}}$ is minimal.

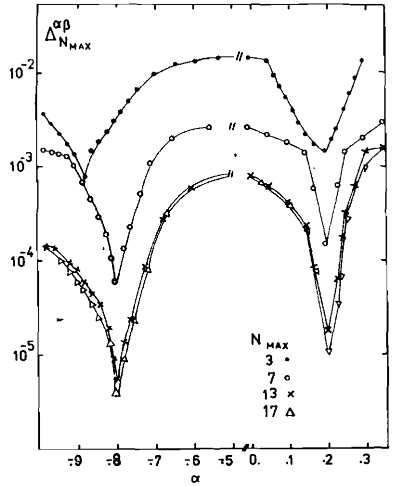


Fig.2. Dependence of integral $\Delta^{\alpha\beta}_{N_{max}}$ on the parameter α at fixed values N_{max} and $\beta = 3$.

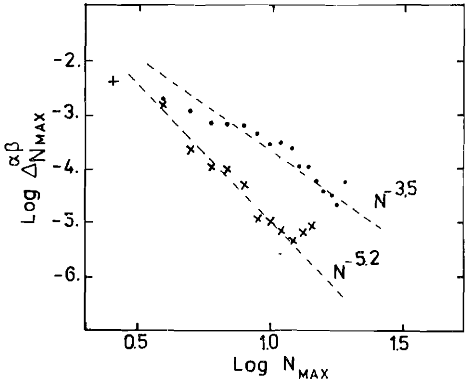


Fig.3. Dependence of the integral $\Delta^{\alpha\beta}_{N_{max}}$ on N_{max} : points are at $\alpha = 0.25$, $\beta = 3$ (first minimum), crosses are at $\alpha = -0.85$, $\beta = 3$ (second minimum). Dashed lines are linear parametrizations.

To estimate the accuracy of the function reconstruction, it is important to know not only the integral (22), but also the difference: $\varepsilon_{N_{max}} = f(x) - f_{N_{max}}(x)$. This difference at the second minimum of the $\Delta^{\alpha\beta}_{N_{max}}$ ($\beta = 3$, $\alpha = -0.85$) is shown in Table 3 for various x and N_{max} . It is seen that $\varepsilon_{N_{max}}$ rapidly decreases with increasing N_{max} , and at $N_{max} \geq 8$ the reconstruction accuracy becomes better than 10^{-4} in the whole region of x .

Note that series (12) for function (21) satisfies the Cauchy convergence test /29/

$$\lim_{k\to\infty} \left| a_k \Theta^{\alpha\beta}_k(x) \right|^{1/k} = C < 1 . \tag{23}$$

For the values of $\beta = 3$ and $\alpha = -0.85$ providing the best accuracy in the reconstruction of the function, we have $c \leq 0.65$ for $3 \leq K \leq 12$.

We have performed the similar analysis for other than (21) parametrizations of $x\Sigma(x,Q_0^2)$ -functions describing the EMC/27/ and CDHS/31/ data, and found the similar to the above results for the convergence and accuracy of reconstruction.

Table 3. Values of the difference $\varepsilon_{N_{Max}}$ as a function of x for various N_{Max}

x	$\varepsilon(x)=f(x) - f_{N_{Max}}(x)$		
	$N_{max} = 4$	$N_{max} = 8$	$N_{max} = 12$
0.1	$5.49\ 10^{-3}$	$2.05\ 10^{-4}$	$-2.30\ 10^{-5}$
0.2	$-4.05\ 10^{-3}$	$-1.23\ 10^{-4}$	$1.21\ 10^{-5}$
0.3	$-2.48\ 10^{-3}$	$7.01\ 10^{-5}$	$-4.93\ 10^{-6}$
0.4	$4.04\ 10^{-4}$	$3.16\ 10^{-6}$	$1.88\ 10^{-6}$
0.5	$1.27\ 10^{-3}$	$-2.80\ 10^{-5}$	$-4.41\ 10^{-7}$
0.6	$6.50\ 10^{-4}$	$7.52\ 10^{-6}$	$-9.04\ 10^{-8}$
0.7	$-9.33\ 10^{-5}$	$7.6\ 10^{-6}$	$1.52\ 10^{-7}$
0.8	$-2.72\ 10^{-4}$	$-5.03\ 10^{-6}$	$-7.23\ 10^{-8}$
0.9	$-8.32\ 10^{-5}$	$8.59\ 10^{-7}$	$4.47\ 10^{-9}$

In conclusion of this section we can say that the accuracy of the SF reconstruction with the help of a finite number of terms of its expansion in the series of Jacobi polynomials depends on the choice of the weight function parameters. At the best choice of these parameters ($\beta = 3$, $\alpha = -0.85$) the SF could be reconstructed in the whole kinematical region with an accuracy better than 10^{-3} with the number of series terms $N_{max} \geqslant 4$.

3.3. Estimation of the Gluon Contribution

In the previous section we have considered the reconstruction of the SF as a function of the x-variable at the reference point Q_0^2 where, as is seen from expr. (14), in the leading order (LO) the gluon contribution $\langle G(Q_0^2)\rangle_n$ cancels out. To estimate the gluon contribution to the expression for the moment at $Q^2 \neq Q_0^2$, we shall rewrite the LO formula in the form:

$$\langle\Sigma(Q^2)\rangle_n = \Big\{ \langle\Sigma(Q_0^2)\rangle_n \left[1 - \alpha_n b(Q^2,Q_0^2)\right] - \langle G(Q_0^2)\rangle_n \tilde{\alpha}_n b(Q^2,Q_0^2)\Big\} \left[\frac{\bar{g}^2(Q^2)}{\bar{g}^2(Q_0^2)}\right]^{d_-^n}, \qquad (24)$$

where

$$b(Q^2,Q_0^2) = \left[\frac{\bar{g}^2(Q^2)}{\bar{g}(Q_0^2)}\right]^{d_+^n - d_-^n} - 1, \qquad (25)$$

and $H_\varphi, H_G = 0$ at LO.

From the Tables of anomalous dimensions $d_\pm^n$ /3,4,24/ it follows that the difference $d_+^n - d_-^n$ grows with n from 0.75 to 1.62 for the first 8 moments, so the average value is $\langle(d_+^n - d_-^n)\rangle \approx 1.2$. Taking into account a weak logarithmic Q^2-dependence of $\bar{g}^2(Q^2)$ one may conclude that $[\bar{g}^2(Q^2)/\bar{g}^2(Q_0^2)]^{d_+^n - d_-^n} \approx 1$ up to $Q^2 = 200$ GeV^2. Therefore, factor (25) and, consequently, the second term in expr. (24) are small. From here as well as from the fact that the coefficients $\tilde{\alpha}_n$ decrease with increasing n from $\tilde{\alpha}_2 = 0.429$ down to $\tilde{\alpha}_{10} = 0.045$ while α_n increase with increasing n from $\alpha_2 = \tilde{\alpha}_2 = 0.429$ up to $\alpha_{10} = 0.999$/24/ we conclude that the gluon contribution to $M_{2S}(n,Q^2)$ (supposing that $\langle G(Q_0^2)\rangle_n$ is of the same order as $\langle\Sigma(Q_0^2)\rangle_n$) in the leading order is really small. Numerical calculations show that the gluon contribution to expr. (24) (when Q^2 changes from $Q_0^2 = 27.5$ GeV^2 up to $Q^2 = 140$ GeV^2) is less than 6.5% of the quark contribution for n=2, it is smaller than 1.5% for n=4, smaller than 0.6% for n=6 and even smaller for larger n. Thus, in the data fitting by theoretical formulae the gluon contribution will produce an effective influence on the results of the LO singlet QCD - analysis only in the case when the accuracy of the function reconstruction at point Q_0^2 is much higher than small gluon contributions to the quark terms. As is shown in section 3.2, with the help of Jacobi polynomials we can get the required accuracy at $N_{MAX} \sim 8$.

4. Relation between the QCD Moments and the Moments of the Expanded Structure Functions

The usual procedure to check the accuracy of the numerical solutions of evolution equations for the SF consists in the calculation of moments (1) of the found functions and in the comparison with theoretical expressions for $M^{QCD}(n, Q^2)$ /10/. This procedure is not required for out method because the moments of the SF expressed in terms of the finite series (12)

$$M^{Jacobi}(n+2,Q^2) = \int_0^1 dx\, x^n\, F_{N_{max}}(x,Q^2) =$$

$$= \int_0^1 dx\, x^n \Big\{ w^{\alpha\beta}(x) \sum_{\ell=0}^{N_{max}} \Theta_\ell^{\alpha\beta}(x) \sum_{j=0}^{\ell} c_j^{(\ell)}(\alpha,\beta)\, M^{QCD}(j+2,Q^2) \Big\} \tag{26}$$

exactly coincide with $M^{QCD}(j+2,Q^2)$ defined by (11) or (14-15) and taken at the same values of QCD parameters (Λ and others). Indeed, after substitution of expansion (10) for x^n into (26), one gets:

$$M^{Jacobi}(n+2,Q^2) = \int_0^1 dx \Big\{ \sum_{k=0}^{n} d_k^{(n)}(\alpha,\beta)\, \Theta_k^{\alpha\beta}(x) \Big\}\, w^{\alpha\beta}(x)\, \times$$

$$\times \sum_{\ell=0}^{N_{max}} \Theta_\ell^{\alpha\beta}(x) \sum_{j=0}^{\ell} c_j^{(\ell)}(\alpha,\beta)\, M^{QCD}(j+2,Q^2)\,. \tag{27}$$

The integral over x can be taken with the help of the orthogonality relation (3) resulting in:

$$M^{Jacobi}(n+2,Q^2) = \sum_{j=0}^{m} \sum_{k\geq j}^{m} d_k^{(m)}(\alpha,\beta)\, c_j^{(k)}(\alpha,\beta)\, M^{QCD}(j+2,Q^2)\,, \tag{28}$$

where a common upper limit $m = \min(n, N_{max})$ has appeared. It is also due to the orthogonality relation (3) because the summation over k and ℓ in (27) goes effectively only up to the number m which is minimal of two values n and N_{max}. Further, from expr. (26) with the help of the orthogonality relation for coefficients $d_\ell^{(j)}$ and $c_i^{(\ell)}$

$$\sum_{\ell\geq j}^{i} d_\ell^{(i)}(\alpha,\beta)\, c_j^{(\ell)}(\alpha,\beta) = \delta_{ij} \tag{29}$$

that can be obtained by the substitution of (10) into (6), we get the following important equality

$$M^{Jacobi}(n,Q^2) = M^{QCD}(m,Q^2)\Big|_{m=\min(n,N_{max})} \tag{30}$$

guaranteeing at large enough N_{max} and $n \leq N_{max}$ the exact coincidence of moments $M^{Jacobi}(n,Q^2)$ with the theoretical moments $M^{QCD}(n,Q^2)$. All higher moments with $n \gg N_{max}$ are equal (due to relations (3) and (28) to the moment $M^{QCD}(N_{max},Q^2)$ i.e. the last in the series (13):

$$M^{Jacobi}(n,Q^2)\Big|_{n>N_{max}} = M^{QCD}(N_{max},Q^2)\,. \tag{31}$$

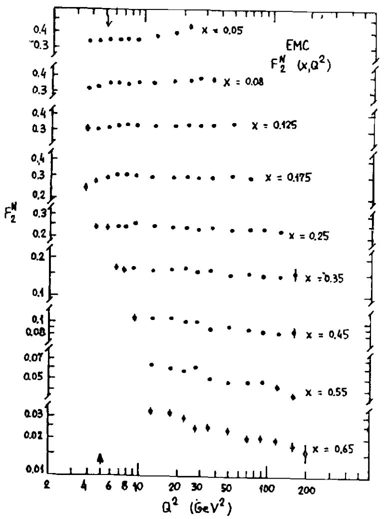
Relation (31) represents a constraint of the method. But this constraint is not very important for large N_{max} since the moments are rapidly decreasing functions of n.

Equation (30) allows us to suggest the finite sum of Jacobi polynomials (12) as a better QCD - parametrization for structure functions than the Buras - Gaemers parametrization (19). The last one requires a two- step comparison of data to the theory: 1) optimization of parameters; 2) consistency check of the found and theoretical moments. In our case the second step is obsolute due to rel. (30).

5. Analysis of the EMC Data

To illustrate the work of the proposed method, we have analysed the EMC data on $F_2(x,Q^2)$ measured from an iron target /27/. (See fig. 4).

Fig.4. Structure functions $F_2^N(x,Q^2)$ measured by the EMC from muon deep inelastic scattering on an iron target.



5.1. Results of the Nonsinglet Analysis

For this type of analysis we have applied the following cuts to the data

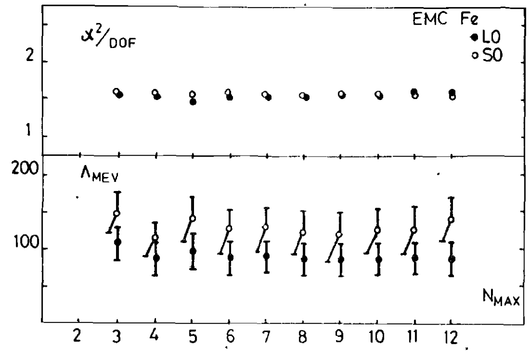
$$x \geqslant 0.3, \qquad Q^2 \geqslant 5 \text{ GeV}^2, \qquad W \geqslant 11 \text{ GeV}.$$

The quark distribution at $Q_0^2 = 27.5$ GeV^2 is taken in the form

$$F_{NS}(x,Q^2) = C_V\, x^{\alpha_V} (1-x)^{\beta_V} (1-\gamma_V x).$$

During the fit, parameters $\alpha_V, \beta_V, \gamma_V, C_V, \Lambda$, α and β of the weight function are considered to be free. The fit results for Λ and corresponding χ^2_{DF} are shown in fig. 5a as a function of N_{max} both for the leading (LO) and next to the leading $(SO, \overline{MS})$ order in $\alpha_s(Q^2)$ cases.

Fig.5a. Results of the nonsinglet analysis of the EMC data on N_{max} in the leading (LO) and next to leading (SO) order in the running constant.



The stable Λ is obtained at $N_{max} \geqslant 4$. The average values for $N_{max} = 4 \div 12$ are:

$$\Lambda_{LO} = 86.5 \pm 21 \text{ MeV}, \qquad \Lambda_{SO} = 125.5 \pm 25 \text{ MeV},$$
$$\Lambda_{SO}/\Lambda_{LO} = 1.4$$

These results are in a very good agreement with the recent[32] QCD analysis performed by the EMC using the evolution equation method [33].

5.2. Results of the singlet analysis

For this type of analysis we have applied the following cuts: $Q^2 \geqslant 5$ GeV^2, $x \geqslant 0.05$ and excluded also four experimental points introducing a large (more than 15) contribution into χ^2. We have investigated: (in the LO):

i) the influence of the weight function parameters α and β and the number of the Jacobi polynomials N_{max} on the stability of the results;

ii) the dependence of the fit values of Λ on the reference point Q_0^2.

This study was performed within the following assumption ("standardt fit"):

a) the $F_2(x,Q^2)$ was considered to be a pure singlet,

b) the forms of quark and gluon distributions at Q_0^2 = 27.5 GeV^2 were taken as given by exprs. (18), where the parameters β_s = 8, β_g = 5.9 and γ_g = 3.5 are fixed (the dependence of the fit Λ on β_g, see below), and c_g was defined from the normalization condition (16);

c) the weight function parameters α and β were fixed in steps within intervals $\alpha = -0.96 \div 0.3$ and $\beta = 0 \div 3$.

So, in the standard fit procedure the parameters c_v, α_v, β_v, c_s and Λ were free. The fit is performed with the help of the MINUIT program.

The resulting Λ and the values of χ^2/DF vs. the weight function parameter α, for different values of N_{max} = 3÷13 are shown in fig. 5b. It is seen that the stable results are obtai-

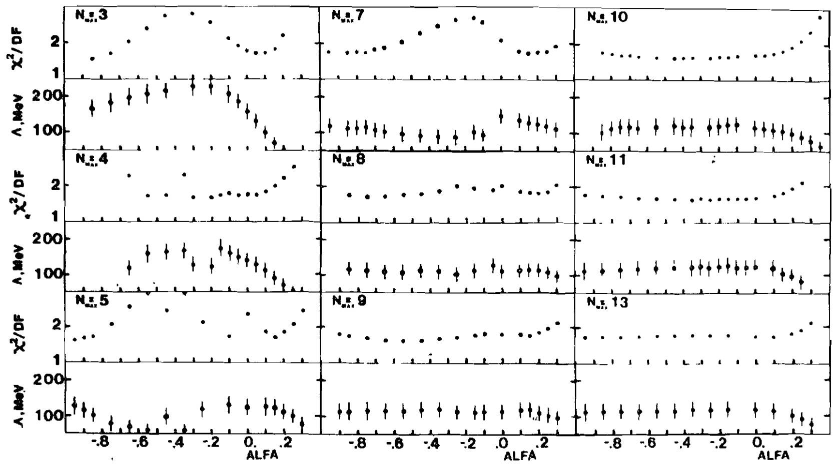


Fig.5b. Results of the "standard fit" of the EMC iron data by our method for singlet structure functions using the leading order QCD formulae. The fit values of χ^2/DF and Λ are shown as a function of the parameter α and number of the expansion (13) terms N_{max} .

ned at $N_{max} \geq 8$. Similar to fig. 5b results are found plotting them as a function of the parameter β . Thus, the final results of the fit have no dependence on the form of the weight function in a wide interval of α and β if the number of expansion terms is large enough , $N_{max} \geq 8$.

The results remained to be stable at $N_{max} \geq 8$ if in addition to the standard fit free parameters we leave the parameter α or β also to be free (see figs. 6 and 7). The average value of Λ for $N_{max} = 7 \div 14$ is Λ_{LO} = 115 MeV with the scattering of points around it by ±1.3 MeV while the statistical error is about ± 20 MeV.

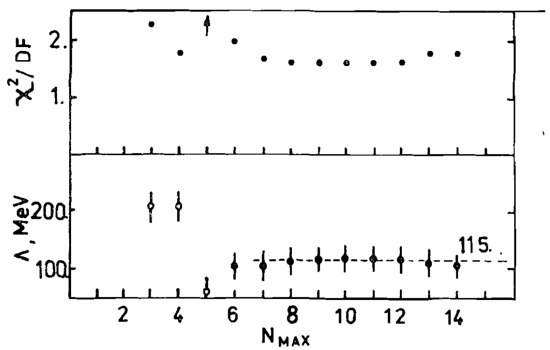


Fig.6. Results of the fit when in addition to the standard fit assumptions the parameter β is assumed to be free (α = -0.5). The fit values of χ^2/DF and Λ are shown as a function of N_{max} .

In Fig. 8 the fit values of Λ are shown as a function of the reference point Q_0^2 in the region $Q_0^2 = 5 \div 180$ GeV. Although within the errors all the results are compatible, the tendency is seen for the fit Λ to be stable at $Q_0^2 \gg 20$ GeV^2.

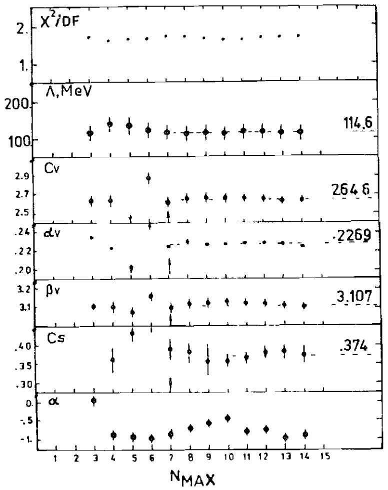
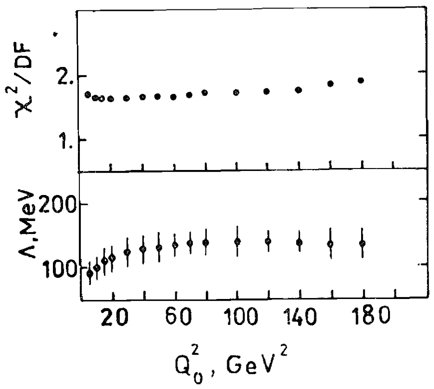


Fig.8. Dependence of the standard fit results on the reference value Q_0^2.

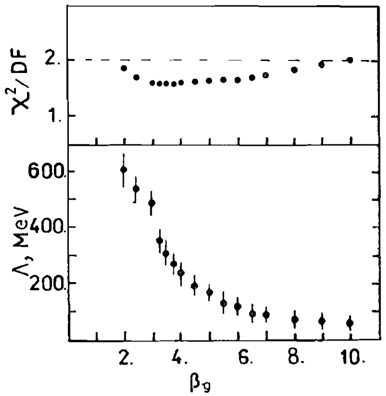
Fig.7. Results of the fit when in addition to the standard fit assumptions the parameter α is assumed to be free (β =3). The fit values of all parameters are shown as a function of N_{max} .



It is well known [14,31] that there exists a strong correlation between the power of the gluon distribution β_g (taken for the above study to be $\beta_g = 5.9$) and the scale parameter Λ . In separate fits with $N_{max} = 11$ we have studied the dependence of Λ on β_g taking β_g to be fixed in steps within the interval $\beta_g = 2 \div 10$. The results are shown in fig. 9. A strong correlation between the fit values of Λ and β_g is clearly seen. In the region of the unpronounced χ^2 minimum corresponding to the $\beta_g = 3 \div 4$ the Λ decreases from 350 to 200 MeV, while it is more stable at $\beta_g \geqslant 5.5$.

We have performed also the singlet analysis taking into account the second order corrections in the $\alpha_s(Q^2)$ constant. The results will be described in a separate paper. Here we mention only that the stable Λ_{SO} (in $\overline{MS}$ - scheme) is by factor ~ 1.4 higher than Λ_{LO} .

Fig.9. Dependence of the fit values χ^2/DF and Λ on the parameter β_g characterising a gluon distribution.



6. Conclusion

A particular realization of the method of the QCD-analysis of structure functions based on its expansion in Jacobi polynomials is proposed. The method allows one to analyse the data on singlet and nonsinglet structure functions both in the leading and next to leading order in the running constant.

An important feature of this method is the following: all the moments of the structure function represented by a finite sum of the N_{max} Jacobi polynomials, i.e. moments $M^{Jacobi}(n, Q^2)$ obeying the condition $n < N_{max}$, exactly coincide with the theoretical moments calculated within the QCD perturbation theory. For this reason the SF expansion with a large enough value of N_{max} seems to be a better parametrization of experimental data as compared with the well-known Buras - Gaemers parametrization.

The accuracy is determined with which the functions can be reconstructed with the help of a series over Jacobi polynomials. The precision of the SF approximation required for a reliable determination of the parameter Λ from the data fit ($\sim 10^{-3}$ and better) can be achieved already at $N_{max} \geqslant 4$.

The role of the weight function $w^{\alpha\beta}(x)$ defining the form of expansion over the Jacobi polynomials is studied. It is shown that the final results of the analysis do not depend on the parameters α and β if the number of the series is large enough ($N_{max} \geqslant 4$ for nonsinglet and $N_{max} \geqslant 8$ for singlet cases).

An application of the method to the analysis of the real data is illustrated using the final $F_2(x,Q^2)$ obtained by the EMC from an iron target. For the nonsinglet analysis that is only performed by the EMC for determination of the parameter Λ' we have obtained results almost identical to EMC. The result of the singlet analysis on Λ is consistent with the nonsinglet one if the power of the gluon distribution β_g is taken to be $\beta_g \geqslant 5$.

An application of the method to other existing data on structure functions will be a subject of our next paper.

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References

1. Politzer H.D. - Phys. Rev. Lett., 1973, v.30, p.1346.
2. Gross D.I., Wilczek F. - Phys. Rev. Lett., 1973, v. 30, p 1343; Phys. Rev., 1973, v.D8, p.3633; ibid. 1974, v.D9, p.980.
3. Floratos E.G., Ross D.A., Sachrajda C.T. - Nucl. Phys. 1977, v.B129, p.66; ibid. 1977, v.B139, p.545 (E); ibid. 1979, v.B152, p.493; Phys.Lett., 1979, v.B80, 269 (Erratum); Bardeen W.A., Buras A.J., Duke D.W., Muta T. - Phys. Rev., 1978, v.D18, p.3998.
4. Furmanski W., Petronzio R. - Phys.Lett., 1980, v.97B, p.437; Curci G., Furmanski W., Petronzio R. - Nucl. Phys., 1980, v.B175, p.27; Floratos E.G., Lacaze R., Kounnas C. - Phys.Lett., 1981, v.98B, p.89.
5. Gribov V.N., Lipatov L.N. - Sov. J. Nucl. Phys., 1972, v.15, p. 438, p.675.
6. Lipatov L.N. - Sov. H. Nucl. Phys., 1975, v.20, p.94. Kogut J., Susskind L. - Phys. Rev., 1974, v.D9, p.697, p. 706, p.3391.
7. Altarelli G., Parisi G. - Nucl. Phys. 1977, v.B126, p.298.
8. Dokshitzer Yu.L. - Preprint of Leningrad Inst. of Nucl. Phys. No. 330, 1977; Phys. Rep., 1980, v.C58, p.269.
9. Dokshitzer Yu.L., Dyakonov D.I., Troyan S.I. SLAC-TRANS-183 1978; Phys.Lett., 1978, v.B79, p.269.
10. Devoto A., Duke D.W., Owens J.F., Roberts R.G. - Phys. Rev., 1983, v. D27, p.508.
11. Yndurain F.J. - Phys. Lett., 1978, v. 74B, p.68.
12. Gonzalez-Arroyo A., Lopez C., Yndurain F.J. - Nucl. Phys., 1979, v. B153, p.161; 1979,v.B159, p.512; Nucl. Phys. 1980, v.B174, p.474.
13. Gonzalez-Arroyo A., Lopez C., Nucl. Phys., 1980, v.B166, p.429.
14. Aubert J.J. et al. Phys. Lett., 1982, v.B114, p.291.
15. Furmansky W., Petronzio R. - Nucl. Phys., 1982, v.B195, p.237.
16. Bergsma F. et al. - Phys. Lett., 1983, v.123B, p.269.
17. Parisi G., Sourlas N. - Nucl. Phys., 1979,v.B151, p.421.
18. Barker I.S., Langensiegen C.S., Shaw G. - Preprint CERN TH-2988, CERN, Geneva, 1980; Nucl. Phys., 1981, v. B186, p.61.
19. Barker I.S., Martin B.R., Shaw G. - Zeit f.Phys., C, 1983, v.19, p.147.
20. Barker I.S., Martin B.R. - Zeit.f.Phys., 1984, v.24, p.255.
21. Petronzio R. - Preprint CERN TH-3085, CERN, Geneva, 1981; Proc. of Conf. on Perturb. QCD, Florida State Univ., Tallahassee, 1981.
22. Pennington M.R. - Rep. Progr. in Phys., 1983, v. 46, N4, p.393.
23. Chyla J., Ramez J. - Zeit f.Phys., 1986,v. C31, p.151 Czech.J.Phys. B, 1986, v.B36, p.534.
24. Buras A.I. - Rev.Mod. Phys., 1980, v.52, p.199.
25. Aurenche P., Faddeev N.G., Lindfors J., Ivanshin Yu.I., Skachkov N.B., Ross D.A. - Nucl.Phys., 1981, v.B177, p.189.
26. Faddeev N.G., Savin I.A., Sanadze V.V., Skachkov N.B. - Phys. Lett. 1982, v.B117, p.349.
27. Aubert J.J. et al. - Phys.Lett., 1982,v.105B, p.322.
28. Buras A.J., Gaemers K.J.F. - Nucl. Phys. 1978, v.B132, p.249.
29. Szego G. - Orthogonal Polynomials, Amer. Math. Colloquium Publ., v. 23, New York, 1959. Courant R., Hilbert D. - Methods of Mathematical Physics, Interscience, New York, 1953.
30. Abramowitz M., Stegun I. - Handbook of Math. Functions, Dover, New York.
31. Abramowicz H. et al. Zeit.f.Phys., 1982, v.C12, p.289.
32. Aubert J.J. et al., CERN - EP/86-05, 1986, Submitted to Nucl. Phys. B.
33. Abbott L.F. et al. Phys. Rev. 1980, v.D22, p.582.

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Метод КХД-анализа синглетных структурных функций с помощью разложения по полиномам Якоби. Основы метода

Развит метод КХД-анализа данных по синглетным структурным функциям, основанный на разложении в ряд по полиномам Якоби. Оценено число членов ряда, обеспечивающих извлечение из экспериментальных данных масштабного параметра Λ с достаточно высокой точностью. В качестве примера работы метода выполнен синглетный анализ данных коллаборации EMC на железной мишени.

Работа выполнена в Лаборатории теоретической физики и в Лаборатории высоких энергий ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1986

Krivokhijin V.G. et al.

E2-86-564

QCD Analysis of Singlet Structure Functions Using Jacobi Polynomials. The Description of the Method

The method of QCD-analysis of data on singlet structure functions is proposed, based on the expansion over orthogonal Jacobi polynonials. The number of polynomials is estimated, necessary to obtain the scale parameter Λ from experimental data with a sufficiently high accuracy. As an example we apply our method to a singlet analysis of EMC-collaboration data on an iron target.

The investigation has been performed at the Laboratory of Theoretical Physics and Laboratory of Hing Energies, JINR.

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