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POSSIBLE TESTS
FOR SUPERSYMMETRY IN $\mathbf{e}^{+}$. $e^{-}$COLLISIONS WITH POLARIZED BEAMS

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1. It seems most probable that the supersymmetry will have an important role in a future unified theory. A direct generalization of the standard model is likely to be the $\mathrm{N}=1$ supersymmetry/ ${ }^{1 /}$. It is known as well that the masses of supersymmetric partners of the ordinary particles may be of an order of (or less than) 1 TeV .

As a reflection of the chiral structure of the standard theory, definite chiral properties are characteristic of the $N=1$ supersymmetric Lagrangian. They lead to specific dependences of the amplitudes for producing supersymmetric particles on the helicities of the initial particles. It is clear that these dependences would manifest themselves most brightly in experiments with polarized initial particles.

In recent papers ${ }^{\prime 2 \cdots 4 /}$ attention has been drawn to the importance of using polarized beams in the search of supersymmetric particles. In thesc papers the cross-sections and spin asymmetries for pair productions of spin 0 or spin $1 / 2$ supersymmetric particles in $\mathrm{e}^{+}-\mathrm{e}^{-}$and $\mathrm{p}-\overline{\mathrm{p}}-\mathrm{collisions}$ with polarized beams have been calculated in the framework of $N=1$ supersymmetry. The polarization effects in these processes have been shown to be large.

The values of the asymmerties obtained in ${ }^{/ 2-4 /}$ depend on the masses of supersymmetric particles and on the other parameters of the theory. It would be interesting to find general relations between polarization characteristics of the processes of supersymmetric-particle production which would reflect only the chiral properties of the Lagrangian and would not depend on the parameters determined by the mechanism of supersymmetry breaking. To obtain such realtions for polarized $\mathrm{e}^{+}-\mathrm{e}^{-}$ collisions is the aim of the present paper.
2. Let us consider the production of a pair of supersymmetric particles with spin 0 or $1 / 2$ in polarized $e^{+}-e^{--}$collision $\mathrm{e}^{+}+\mathrm{e}^{-} \rightarrow \overline{\tilde{s}}+\ddot{\mathrm{s}}$.
The supersymmetric particles $\ddot{\mathrm{s}}$ and $\overline{\overline{\mathrm{s}}}$ can either possess some charges or be truly neutral. We suppose the incident $e^{+}$and $e^{-}$ to be polarized. We are not interested in the spin states of the final particles. Correspondingly, we write down the matrix element of process (1) in the form
$\langle f|(S-1)|i\rangle=N_{p} N_{p}, N_{f} \ddot{u}^{\prime} r^{\prime}\left(-p^{\prime}\right) M u^{r}(p)(2 \pi)^{4} \delta\left(P^{\prime} \ldots P\right)$.

Here $p, r$ and $p{ }^{\circ}$, r" are the momenta and helicities of initial $e^{--}$ and $e^{+}, N_{p}, N_{p}, \ldots$ are standard normalizing factors
$\left(N_{p}=\frac{1}{(2 \pi)^{3 / 2}} \frac{1}{\sqrt{2 p_{o}}}\right)$,
$P$ and $P^{\prime}$ are total 4 -momenta of the initial and final particles, $M$ is a matrix acting on spin variables of the electron and positron.

We shall show in Sect. 3 that for $N=1$ supersymmetry to the lowest order of perturbation theory the matrix $M$ satisfies the chirality conservation relation
$\gamma_{5} \mathrm{M} \gamma_{5}=-\mathrm{M}$.
In the ultrahigh energy range we are interested in we have $\gamma_{5} u^{r}(\mathrm{p})=-\mathrm{r}^{\mathrm{r}}(\mathrm{p})$ and $\gamma_{5} \mathrm{u}^{\mathrm{r}^{\prime}}\left(-\mathrm{p}^{\prime}\right)=\mathrm{r}^{\prime} \mathrm{u}^{\mathrm{r}^{\prime}}\left(-\mathrm{p}^{\prime}\right)$, and from (2) and (3) it follows

$$
r^{\prime}=-r
$$

So, if the relation (3) holds, process (1) is possible only in the case of opposite electron and positron helicities.

Let us look now for relations between measurable quantities following from (3)*. In the general case of polarized initial particles the differential cross section for process (1) in the c.m.s. has the form
$\sigma_{\xi^{\prime} ; \xi}=\mathrm{B}(\mathrm{s}) \operatorname{SpM} \rho(\mathrm{p}, \xi) \overline{\mathrm{M}} \rho\left(-\mathrm{p}^{\prime}, \xi^{\prime}\right)$.
Here
Here
$B(s)=\frac{1}{16 \sqrt{s}} \frac{1}{(2 \pi)^{2}} \sqrt{1-4 \frac{M_{s}^{2}}{s}}$
$s=-\left(p+p^{\prime}\right)^{2}, M_{\widetilde{s}}$ is the mass of the supersymmetric particles, $\rho(\mathrm{p}, \xi)$ is the spin density matrix of the electron and the matrix $\rho\left(-\mathrm{p}^{\prime}, \xi^{\prime}\right)$ is connected with the positron density matrix $\rho\left(\mathrm{p}^{\prime}, \xi^{\prime}\right)$ as follows
$\rho\left(-\mathrm{p}^{\prime}, \xi^{\prime}\right)=-\mathrm{C} \rho^{\mathrm{T}}\left(\mathrm{p}^{\prime}, \xi^{\prime}\right) \mathrm{C}^{-1}{ }^{\prime}$
where $C$ is the charge conjugation matrix ( $C^{T}=-C, C \gamma_{a}^{\mathrm{T}} \mathrm{C}^{-1}=-\gamma_{a}$ ), $\xi$ and $\xi^{\prime}$ are 4-vectors of the electron and positron polarization ( $\xi \cdot \mathrm{p}=0 ; \xi^{*} \cdot \mathrm{p}^{\prime}=0$ ). In the c.m.s. $\mathrm{p}_{\mathrm{o}} \gg \mathrm{m}(\mathrm{m}$ is the electron mass) and up to terms linear in $m / p_{o}$ for the electron density matrix we have ${ }^{/ 6 /}$.

[^0]$\rho(\mathrm{p}, \xi)=\frac{1}{2} .\left(1-\mathrm{P}_{\mathrm{L}} \gamma_{5}+\mathrm{i} \gamma_{5} \vec{y} \overrightarrow{\mathrm{P}}_{\mathrm{T}}\right) \Lambda(\mathrm{p})$.
Here $\Lambda(p)=\frac{y p}{i}, P_{L}$ and $P_{T}$ are longitudinal and transverse polarization of the electron in its rest frame (in an arbitrary frame $\vec{\xi}_{\mathrm{L}}=\mathrm{P}_{\mathrm{L}} \frac{\overrightarrow{\mathrm{p}}}{\mathrm{m}} ; \vec{\xi}_{\mathrm{T}}=\overrightarrow{\mathrm{P}}_{\mathrm{T}}$ ). For the density matrix $\rho\left(-\mathrm{p}^{\prime}, \xi^{\prime}\right)$ by using Eqs.(6) and (7) we get.
$\rho\left(-\mathrm{p}^{\prime}, \xi^{\prime}\right)=-\frac{1}{2}\left(1+\mathrm{P}_{\mathrm{L}}^{\prime} \gamma_{5}+\mathrm{i} \gamma_{5} \vec{\gamma} \overrightarrow{\mathrm{P}}_{\mathrm{T}}^{\prime}\right) \Lambda\left(-\mathrm{p}^{\prime}\right)$,
where $P_{L}^{\prime}$ and $\vec{P}_{T}^{\prime}$ are longitudinal and transverse polarizations of the positron in its rest frame.

We suppose first that the electrons and positrons are longitudinally polarized. Obviously, the differential cross-section has in this case the following general form
$\sigma_{\mathrm{P}_{\mathrm{L}}^{\prime} \mathrm{P}_{\mathrm{L}}}(\theta)=\sigma_{0}(\theta)+\mathrm{P}_{\mathrm{L}} \sigma(\theta)+\mathrm{P}_{\mathrm{L}}^{\prime} \sigma_{+}(\theta)+\mathrm{P}_{\mathrm{L}} \mathrm{P}_{\mathrm{L}}^{\prime} \sigma_{+-}(\theta)$.
Here $\theta$, is the angle between the momenta of $\mathrm{e}^{+}$and $\overrightarrow{\mathrm{s}}, \sigma_{0}(0)$ is the cross section for the process with unpolarized $e^{+}$and $e^{-}$, $\sigma_{-}(\theta), \sigma_{+}(\theta)$ and $\sigma_{+-}(\theta)$ characterize the contributions of the incident-particle polarizations to the cross section. If the matrix M obeys Eq. (3), the following relations take place:
$\sigma_{-}(\theta)=-\sigma_{+}(\theta)$,
$\sigma_{+}-(\theta)=-\sigma_{o}(\theta)$.
Really, by using Eqs. (3), (5), (7), (9), we have*

* It is not difficult to see that relations (10) and (11) are direct consequences of equality (4). Indeed, in the helicity basis the cross section for process (1) with longitudinally polarized $e^{+}$and $e^{-}$has the form
$\sigma=\sum_{r, r^{\prime}}\left|f_{r^{\prime} r^{\prime}}\right|^{2} \frac{1}{2}\left(1+\mathrm{P}_{\mathrm{L}} \mathrm{r}\right) \frac{1}{2}\left(1+\mathrm{P}_{\mathrm{L}}^{\prime} \mathrm{r}^{\prime}\right)$.
Therefore, if $r^{\prime}=-r$, then
$\sigma_{-}=\frac{1}{4} \sum_{\mathrm{r}}\left|\mathrm{f}_{-\mathrm{rr}}\right|^{2} \mathrm{r}=-\sigma_{+}, \sigma_{+-}=-\frac{1}{4} \sum_{\mathrm{r}}\left|\mathrm{f}_{-\mathrm{rr}}\right|^{2}=-\sigma_{\mathrm{o}}$.
$\sigma_{-}(\theta)=\frac{1}{4} \mathrm{~B}(\mathrm{~s}) \operatorname{SpM} \gamma_{5} \Lambda(\mathrm{p}) \widehat{\mathrm{M}} \Lambda\left(-\mathrm{p}^{0}\right)=\frac{1}{4} \mathrm{~B}(\mathrm{~s}) \operatorname{SpM} \Lambda(\mathrm{p}) \widehat{\mathrm{M}} \gamma_{5} \Lambda\left(-\mathrm{p}{ }^{0}\right)=-\sigma_{4}(\theta)$.
$\left.\sigma_{+-}(\theta)=\frac{1}{4} \mathrm{~B}(\mathrm{~s}) \operatorname{SpM} \gamma_{5} \Lambda(\mathrm{p}) \overline{\mathrm{M}} \gamma_{5} \Lambda(-\mathrm{p})^{\prime}\right)=\frac{1}{4} \mathrm{~B}(\mathrm{~s}) \operatorname{Sp} \mathrm{M} \Lambda(\mathrm{p}) \overline{\mathrm{M}} \Lambda\left(\ldots \mathrm{p}{ }^{\mathrm{o}}\right)=-\sigma_{o}(\theta)$.
To obtain from Eqs.(10) and (11) relations between measurable quantities, let us define the asymmetries
$\mathrm{A}_{-}(\theta)=\frac{1}{\mathrm{P}_{\mathrm{L}}}\left[\frac{\sigma_{0 ; \mathrm{P}_{\mathrm{L}}}(\theta)-\sigma_{0 ;-\mathrm{P}_{\mathrm{L}}}(\theta)}{\sigma_{0 ; \mathrm{P}_{\mathrm{L}}}(\theta)+\sigma_{0 ;-\mathrm{P}_{\mathrm{L}}}(\theta)}\right]$.
$A_{+}(\theta)=\frac{1}{\mathrm{P}_{\mathrm{L}}^{\prime}}\left[\frac{\sigma_{\mathrm{P}_{\mathrm{L}}{ }^{\prime} ; 0}(\theta)-\sigma_{-\mathrm{P}_{\mathrm{L}}^{\prime} ; 0}(\theta)}{\sigma_{\mathrm{P}_{\mathrm{L}}^{\prime} ; 0}(\theta)+\sigma_{-\mathrm{P}_{\mathrm{L}}{ }^{\prime} ; 0}(\theta)}\right]$.
From Eq. (10) it follows that the asymmetries $A_{-}$and $A_{+}$are connected by the relation

$$
\begin{equation*}
A_{-}(\theta)=-A_{+}(\theta) \tag{14}
\end{equation*}
$$

The quantity $\sigma_{+--}(\theta)$ can be determined in experiments with both electron and positron beams polarized. As a consequence of Eq.(11) we have
$\mathrm{A}_{+-}(\theta)=-1$,
where the asymmetry $A_{+-}$is defined as follows
$\mathrm{A}_{+-}(\theta)=\frac{1}{\mathrm{P}_{\mathrm{L}}^{\prime} \mathrm{P}_{\mathrm{L}}}\left[\frac{\sigma_{\mathrm{P}_{\mathrm{L}}^{\prime} ; \mathrm{P}_{\mathrm{L}}}(\theta)-\sigma_{-\mathrm{P}_{\mathrm{L}}^{\prime} ; \mathrm{P}_{\mathrm{L}}}(\theta)-\sigma_{\mathrm{P}_{\mathrm{L}}^{\prime} ;-\mathrm{P}}(\theta)+\sigma_{-\mathrm{P}_{\mathrm{L}}^{\prime} ;-\mathrm{P}_{\mathrm{L}}(\theta)}(\theta)}{\sigma_{\mathrm{L}}(\theta)+\sigma_{-\mathrm{P}_{\mathrm{L}}^{\prime} ; \mathrm{P}_{\mathrm{L}}}(\theta)+\sigma_{\mathrm{P}_{\mathrm{L}}^{\prime} ;-\mathrm{P}_{\mathrm{L}}}(\theta)+\sigma_{-\mathrm{P}_{\mathrm{L}}^{\prime} ;-\mathrm{P}_{\mathrm{L}}}(\theta)}\right]$.
So, if the matrix M satisfies the helicity conservation relation (3) the asymmetry $A_{-}(\theta)$ in process (1) with polarized $\mathrm{e}^{-}$and unpolarized $\mathrm{e}^{+}$is equal in magnitude and opposite in sign to the asymmetry $\mathrm{A}_{+}(\theta)$ in the same process with polarized $\mathrm{e}^{+}$and unpolarized $\mathrm{e}^{-*}$; the acymmetry $\mathrm{A}_{+-}(\theta)$ in process (1) when both $\mathrm{e}^{-\infty}$ and $\mathrm{e}^{+}$are polarized is equal to -1 .

It is easy also to obtain relations between integral asymmetries following from the helicity conservation. From Eqs. (10) and (11) we find
$\sigma_{-}^{\mathrm{F}, \mathrm{B}}=-\sigma_{+}^{\mathrm{F}, \mathrm{B}}, \sigma_{+-}^{\mathrm{F}, \mathrm{B}}=-\sigma_{\mathrm{o}}^{\mathrm{F}, \mathrm{B}}$.
where ,

$$
\begin{equation*}
\sigma_{ \pm}^{\mathrm{F}}=2 \pi \int_{\theta_{0}}^{\pi / 2} \sigma_{ \pm}(\theta) \sin \theta \mathrm{d} \theta ; \quad \sigma_{ \pm}^{\mathrm{B}} \div 2 \pi \int_{\pi / 2}^{\pi-\theta_{\mathrm{o}}} \sigma_{ \pm}(\theta) \sin \theta \mathrm{d} \theta . \tag{18}
\end{equation*}
$$

(the angle $\theta_{0}$ is determined by the experimental set up). From Eqs.(17) the following relations between asymmetries can be easily obtained
$A_{-}^{F, B}=-A_{+}^{F, B}, \quad A_{+-}^{F, B}=-1$,
where the quantities $A_{ \pm}^{F, B}$ and $^{\prime} A_{+}^{F, B}$ are defined analogously as (12), (13) and (16). Finally, by integrating Eqs. (10) and (11) over the total solid angle we find for the integral asymmetries
$A_{-}=-A_{+}, A_{+-}=1$.
We shall now consider process (1) in the general case of. arbitrarily polarized electrons and positrons. If the matrix $M$ satisfies the helicity conservation relation (3), it is not hard to see that in the cross section all terms linear in transverse polarization vanish. In fact, by using Eq. (3) we have
$\left.\operatorname{Spm} \gamma_{5} \vec{\gamma} \Lambda(\mathrm{p}) \overline{\mathrm{M}} \Lambda(-\mathrm{p})^{\prime}\right)=\operatorname{Sp} \mathrm{M} \Lambda(\mathrm{p}) \dot{\gamma}_{5} \vec{\gamma} \Lambda\left(-\mathrm{p}{ }^{\prime}\right)=0$.
Similarly, we have
$\operatorname{Sp} \mathrm{M} \gamma_{5} \vec{\gamma} \Lambda(\mathrm{p}) \overline{\mathrm{M}} \gamma_{5} \Lambda\left(-\mathrm{p}^{\prime}\right)=\operatorname{Sp} \mathrm{M} \gamma_{5} \Lambda(\mathrm{p}) \overline{\mathrm{M}} \gamma_{5} \vec{\gamma} \Lambda\left(-\mathrm{p}^{\prime}\right)=0$.
It is easy to convince oneself that the helicity conservation does not impose any restrictions on the terms bilinear in electron and positron transverse polarizations. Eqs.(21) mean. that
$\sigma_{\vec{P}_{\mathbf{T}}^{\prime} ; 0}=\sigma_{0 ; \vec{P}_{T}}=\sigma_{0}$.
$\sigma_{\mathrm{P}_{\mathrm{L}} ;} ; \overrightarrow{\mathrm{P}}_{\mathrm{T}}=\sigma_{\mathrm{P}_{\mathrm{L}}^{\prime} ; 0}: \quad \sigma \overrightarrow{\mathrm{P}}_{\mathrm{T}}^{\prime} ; \mathrm{P}_{\mathrm{L}}=\sigma_{0 ; \mathrm{P}_{\mathrm{L}}}$.
Thus, if matrix $M$ conserves helicity, the cross section for process (1) with one of the initial beams transversely polarized and the other unpolarized is equal to the cross section for the process with unpolarized $e^{+}$and $e^{-}$. If one of the beams is longitudinally polarized, the cross section for process (1) is the same in both the cases of the second beam being either transversely polarized or unpolarized.

In conclusion, we make the following remark. As is evident from Eq. (23), the asymmetries in the case, when one of the beams is longitudinally polarized and the other is transversely polarized, are identical with the corresponding asymmetries defined by Eqs.(12) and (13). Therefore, relation (14) remains valid in this case too.
3. We shall now demonstrate that for $N=1$ supersymmetry the matrix M determined by (2) satisfies to the lowest order of perturbation theory the helicity conservation relation (3). Let us first consider the production of a pair of scalar leptons $\vec{\mu}^{+}-\vec{\mu}^{-}$or $\vec{\tau}^{+}-\vec{\tau}^{-}$.
$e^{+}+\mathrm{e}^{-} \rightarrow \vec{\mu}_{\mathrm{L}}^{+}+\tilde{\mu}_{\mathrm{L}}^{-}\left(\vec{\mu}_{R}^{+}, \tilde{\mu}_{\mathrm{R}}^{-}\right)$.
$\mathrm{e}^{+}+\mathrm{e}^{-} \rightarrow \vec{\tau}_{\mathrm{L}}^{+}+\vec{\tau}_{\mathrm{L}}^{-}\left(\vec{r}_{\mathrm{R}}^{\gamma}, \vec{\tau}_{\mathrm{R}}^{-}\right)$.
Only diagrams with $\gamma$ and $Z$ exchange in the $s-c h a n n e l$ shown in Fig. 1 contribute to the amplitude of these processes


Fig. 1. Diagrams of the process $e^{+}+e^{-} \rightarrow$ $\rightarrow \tilde{\ell}_{\mathrm{L}, \mathrm{R}^{+}}+\widetilde{\mathrm{L}}_{\mathrm{L}, \mathrm{R}}$.

The couplings of photon and Z boson to the pair $e^{+}-e^{-}$are given by the standard theory and are of a vector and an axial vector type. So,
the matrix $M$ for processes (24) and (25) anticommutes with $\gamma_{5}{ }^{*}$. Next, we consider the processes
$e^{+}+e^{-} \rightarrow \tilde{e}_{L}^{+}+\tilde{e}_{L}^{-}$,
$\mathrm{e}^{+}+\mathrm{e}^{-} \rightarrow \overrightarrow{\mathrm{e}}_{\mathrm{R}}^{+}+\overrightarrow{\mathrm{e}}_{\mathrm{R}}^{-}$.
In addition to the diagrams of Fig. 1, diagrams with exchange of Majorana supersymmetric fermions $\chi_{i}$ in the t-channel (see Fig. 2). .contribute to the amplitude of these processes


Fig. 2. Diagrams of the process $\mathrm{e}^{+}+\mathrm{e}^{-} \rightarrow \overrightarrow{\mathrm{e}}_{\mathrm{L}}^{+}+\overline{\mathrm{e}}_{\mathrm{L}}^{-}\left(\tilde{\mathrm{e}}_{\mathrm{R}^{+}}^{+} \tilde{\mathrm{e}}_{\mathrm{R}}^{-}\right)$with exchange of Majorana fermions $X_{1}$ in the t -channel.

* Obviously,this also refers to the production of squark-antisquark pair ịn $\mathrm{e}^{+}-\mathrm{e}^{-}$collisions. Note that here and further on the diagrams with Higgs-boson exchange are not taken into. accounț. Their contributions to the maṭrị element of the processes are small compared to the contribution of diagrams Fig. 1.

Relevant terms in the interaction Lagrangian have the general form

where $C_{i}^{L}$ and $C_{i}^{R}$ are constants determined by the supersymmetry breaking mechanism. It is clear from Eq. (28) that only the operators ( $e_{L} e_{I}$ ) and ( $\vec{e}_{R} e_{R}$ ) contribute to the matrix elements of processes (26) and (27), respectively*. Therefore, the whole matrix $M$ which includes contributions both from diagrams of
Fig. 1 and Fig. 2 satisfies relation (3) for processes (26) and (27) as well.

We proceed further with considering the production of sneu-trino-antisneutrino pair
$\mathrm{e}^{r}+\mathrm{e}^{-} \ldots \dot{\bar{v}}_{\mathrm{p}}+\overline{\mathrm{i}} p$.
If $\ell=\mu, T \ldots$ only the diagrams with $Z$-exchange in the s-channel contribute to the amplitudes of these processes. In the case of electron sneutrino production
$\mathrm{e}^{+}+\mathrm{e}^{-} \rightarrow \overrightarrow{\vec{v}}_{\mathrm{e}}+\tilde{v}_{\mathrm{e}}$
there is an additional diagram with $\tilde{W}$ exchange in the t-channel. Since in the interaction Lagrangian of $\mathrm{N}=1$ supersymmetry the field $\bar{\nu}_{e}$ is coupled to the field $e_{L}$, only the operator ( $\bar{e}_{L} e_{L}$ ) contributes to the corresponding. amplitude. As a result, the helicity is again conserved by matrices $M$ of processes (29).

Let us finally turn to the production of a pair of winos $\mathrm{e}^{+}+\mathrm{e}^{-} \rightarrow \overrightarrow{\mathrm{W}}^{+}+\vec{W}^{-}$.

The corresponding diagrams include $\gamma$ and $Z$ exchange in the s-channel and $\vec{V}_{e}$ exchange in the t-channel. Obviously, the matrix $M$ in this case satisfies relation (3) for the same reasons as aboye. Note that this remains valid also for the production of charged higgsinos (as well as in the general case of higgsino-wino mixing).
4. Up to now we have considered the production of a pair of supersymmetric particles with equal masses. Here we turn to the processes of production of two different selectrons $e^{+}+e^{-} \rightarrow \tilde{e}_{L}^{+}+\tilde{e}_{R}^{-}$,

[^1]$$
\mathrm{e}^{+}+\mathrm{e}^{-} \Rightarrow \overrightarrow{\mathrm{e}}_{\mathrm{R}}^{+}+\overrightarrow{\mathrm{e}}_{\mathrm{L}}^{-}
$$

These reactions proceed via the t-channel exchange of neutral gauginos $\chi_{i}$ (see Fig.2). As it is easy to see from Eq. (28), the operators contributing to the amplitudes of processes (31) and (32) are, respectively, ( $\left.\bar{e}_{R} e_{L}\right)$ and ( $\left.\bar{e}_{L} e_{R}\right)$. So, the matrix $M$ of process (31), ((32)) satisfies the relation.
$\gamma_{5} \mathrm{M} \gamma_{5}=\mathrm{M}$,
It follows from Eq. (33) that (unlike the previously considered cases) the amp1itudes of reactions (31) and (32) are nonvanishing if $e^{-}$and $e^{+}$have equal helicities $\left(r=r^{\prime}=-1\right.$ for process (31) and $r=r^{\prime}=1$ for process (32)).

For the collision of longitudinally polarized $\mathrm{e}^{+}$and $\mathrm{e}^{-}$the cross-section for process (31) ((32)) has the general form, Eq. (9). From Eq. (33) analogously to Eqs.(10) and (11) we have $\sigma_{-}(\theta)=\sigma_{+}(\theta), \sigma_{+-}(\theta)=\sigma_{0}(\theta)$.
For the asymmetries $\mathrm{A}_{\mp}(\theta)$ and $\mathrm{A}_{+-}(\theta)$ defined by Eqs.(12), (13) and (16) we find
$A_{-}(\theta)=A_{+}(\theta), \quad A_{+-}(\theta)=1$.
Note that relations (22) and (23) remain valid for processes (31) and (32) too.
5. Thie production of two Majorana supersymmetric fermions $\mathrm{e}^{+}+e^{-} \rightarrow \chi_{i}+\chi_{k}$
needs a separate consideration. Processes (36) occur via $Z^{-e x-}$ change in the s-channel and $\bar{e}_{L}$ and $\tilde{E}_{R}$ exchange in the $t$-channel. If there is no mixing between the fields $\overrightarrow{\mathrm{e}}_{\mathrm{L}}$ and $\overrightarrow{\mathrm{e}}_{\mathrm{R}}$, it is easy to see with the help of Eq. (28) that the operators contributing to the matrix elements of these processes are ( $\bar{e}_{\mathrm{L}} \mathrm{e}_{\mathrm{L}}$ ) and $\left({ }^{-}{ }_{R} e_{R}\right)$. So, the matrices of processes (36) satisfy the helicity conservation relation (3) and therefore in this case the general relations (10), (11), (22) and (23) hold. The cross section for producing a pair of Majorana particles is bound to fulfil the following additional relation ${ }^{1 / 7 /}$
$\sigma_{\mathrm{P}_{\mathrm{L}}^{\prime} ; \mathrm{P}_{\mathrm{L}}}(\theta)=\sigma_{-\mathrm{P}_{\mathrm{L}} ;-\mathrm{P}_{\mathrm{L}}^{\prime}}(\pi-\theta)$

[^2]which is based on the CPT-invariance and unitary of the $S$ matrix. From Eqs.(9) and (37) we find $/ 7,8$ /
$\sigma_{o}(\theta)=\sigma_{o}(\pi-\theta)$,
$\sigma_{+}(\theta)=-\sigma_{-}(\pi-\theta)$.
The combination of Eqs. (39) and (11) gives
$\sigma_{ \pm}(\theta)=\sigma_{ \pm}(\pi-\theta)$.
With the help of Eqs. (38) and (40) we find that the asymmetries $A_{\mp}$ satisfy the relation
$\mathrm{A}_{F}(\theta)=\mathrm{A}_{\mp}(\pi-\theta)$.
Further, by integrating Eq. (40) over $\theta$ from $\theta_{0}$ to $\pi / 2$ we have $\sigma_{ \pm}{ }^{F}=\sigma{ }_{ \pm}{ }_{ \pm}$,
where the quantities $\underset{ \pm}{\sigma_{ \pm}, B}$ are given by Eq. (18). Relation (42) can be checked by measuring the asymmetries
\[

$$
\begin{equation*}
\mathrm{A}_{\mathrm{P}_{\mathrm{L}}^{\prime} ; 0}^{\mathrm{FB}}=\frac{\sigma_{\mathrm{P}_{\mathrm{L}}^{\prime} ; 0}^{\mathrm{F}}-\sigma \stackrel{\mathrm{P}}{\mathrm{~L}}_{\mathrm{B}}^{\mathrm{B}}}{\sigma_{\mathrm{P}_{\mathrm{L}}^{\prime} ; 0}^{\mathrm{F}}+\sigma_{\mathrm{P}_{\mathrm{L}}^{\prime} ; 0}^{\mathrm{B}}}, \quad \mathrm{~A}_{0 ; \mathrm{P}_{\mathrm{L}}}^{\mathrm{FB}}=\frac{\sigma_{0 ; \mathrm{P}_{\mathrm{L}}}^{\mathrm{F}}-\sigma_{0 ; \mathrm{P}_{\mathrm{L}}}^{\mathrm{B}_{0}^{\mathrm{B}}}}{\sigma_{0 ; \mathrm{P}_{\mathrm{L}}}^{\mathrm{F}}+\sigma_{0 ; \mathrm{P}_{\mathrm{L}}}^{\mathrm{B}}} \tag{43}
\end{equation*}
$$

\]

Obviously, the asymmetries $A_{P_{L} ; 0}^{F B} \quad$ and $A A_{0 ; P_{L}}^{F B}$ can be nonvanishing only if the helicity conservation relation, Eq. (3); is violated. As a consequence of Eq. (38), the asymmetry $A_{0 ; 0}^{\mathrm{FB}}$ corresponding to unpolarized $e^{+}$and $e^{-}$is always zero.

Let us note that the productions of Majorana particles with equal and different masses must be considered separately. In the case of equal final particle masses, as a consequence of the Pauli principle we have
$\sigma_{\mathrm{P}_{\mathrm{L}}^{\prime} ; \mathrm{P}_{\mathrm{L}}}(\theta)=\sigma_{\mathrm{P}_{\mathrm{L}}^{\prime}} ; \mathrm{P}_{\mathrm{L}}(\pi-\theta)$.
From Eqs.(9) and (44) we find
$\sigma_{ \pm}(\theta)=\sigma_{ \pm}(\pi-\theta)$.
By combining Eqs. (39) and (45) we obtain relation (10). Thus, for identical final Majorana particles, the relation (10) always holds. In this case it is possible to test the helicity conservation by checking relations (15) or (22) and (23).
6. So far we have been neglecting the mixing of slepton fields $\tilde{\ell}_{L}$ and $\widetilde{\ell}_{R}$. In all standard supersymmetric models the angle $\theta$ of mixing of $\tilde{\ell}_{L}$ and $\bar{\ell}_{R}$ is small $1^{\prime 9}\left(\operatorname{tg} 20-\frac{m_{q}}{\tilde{m}} \ll 1\right.$, where $\mathrm{m}_{\ell}$ is the lepton mass and $\vec{m}$ is a characteristic scalar lepton mass).

In the presence of a considerable mixing of $\tilde{\mu}_{\mathrm{L}}$ and $\vec{\mu}_{\mathrm{R}}$ or $\tilde{r}_{\mathrm{L}}$ and $\tilde{\tau}_{\mathrm{R}}$ fields not only sleptons with equal masses but also sleptons with different masses will be produced in $\mathrm{e}^{+}-\mathrm{e}^{-}$collisions. As only the diagrams with $\gamma$ and $Z$ exchange in the schannel contribute to the amplitude of these processes, it is obvious that Eq. (3) and all the relations between observable quantities following from Eq.(3) remain valid in this case.

If the mixing of $\vec{e}_{\mathrm{L}}$ and $\overrightarrow{\mathrm{e}}_{\mathrm{R}}$ takes place, the fields $\mathrm{e}_{\mathrm{L}}$ and $e_{R}$ couple to the same massive selectron field in the $N=1$ supersymmetry Lagrangian. This leads to the violation of relation (3) (as well as (33.)) for processes $\mathrm{e}^{+}+\mathrm{e}^{+} \rightarrow \tilde{\mathrm{e}}^{+}+\tilde{\mathrm{e}}^{-}$and $e^{+}+e^{-} \rightarrow \chi_{i}+\chi_{k}$. Thus, the check of relations (14), (15), etc., would allow one to gain information about the mixing of the fields $\vec{e}_{L}$, and $\tilde{e}_{R}$.

Let us note in conclusion that the questions of detecting the supersymmetric particles have not been discussed here. A detailed consideration of this subject can be found, for example, in a recent CERN preprint ${ }^{\prime 10 /}$.

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ипенький С...., Неделчееа Н.П.
Bи зможнне тесты суперсимметрии:
в олытах с поляризованными © ${ }^{+}$и а
В рамках $N=1$ суперсимметрии рассматриааптся различные процессы рождения суперсимметричных частиц я столкновении поляризсванных е ${ }^{+}$e . Полу сены обцие соотношения мєжду поляризационными характеристиками процессов отрэжаэщие киральные свойства лагранжиана взаимодействия и не зэвисящие от параметров, значения которых определяптся механизмом нарушения суперсимметрии. В частности, показано, что асимметрия, зознккаэщая при взаимодействии продольно поляризованных $\mathrm{e}^{+}$с неполяризовапными $\mathrm{e}^{-}$, раана по вели чиме и протияиположна по знаку асимметрия, возникающей при взаимодействии поляризованных $e^{-}$- неполяризованными $e^{+}$.

Работа внлолнена в Лабораторим теоретической ризики оияи.

Препринт Объединенного института пдерных исследований. Дубна 1986

Bilenky S.M. Nedelcheva N.F.
Possible Tests for Supersymmetry
in $\mathrm{e}^{+}$- $\mathrm{e}^{-}$Collisions with Polarized Beams
Pair production of various supersyme: ric particles in $e^{+}-\mathrm{e}^{-}$coll. sions with polarized beams is considered in the framework of $\mathrm{N}=1$ supersymmetry. General relations between the polarization characteristics are and don't depend on the parameters determined by the supersvmetry breaking It is shown, in particular, that for a wide class of processes the spin asymmerty arising in any process with longitudinally polarized $e^{+}$and unpolarized $\mathrm{e}^{-}$is equal in magnitude and opposite in sigr: to the asymmetry arising in the same process with $\operatorname{Dolarized~} \mathrm{e}^{-}$and unpolarized $\mathrm{e}^{+}$

The investigation has been performed at the Laboratory of Theoretical Physics, JiNR


[^0]:    * Let us note that polarization characteristics of strong interaction processes in the framework of approximate $\gamma_{5}$-invariance have been considered in ref. ${ }^{5 /}$.

[^1]:    * That means, the diagrams of Fig. 2 give nonvanishing contributions to the amplitude of process (26) with $r=-1, r^{\prime}=1$, and to the amplitude of process (2,7) with $r=1, r{ }^{\prime \prime}=-1$ ( $r$ and $r$ ' are the electron and positron helicities).

[^2]:    * Really, by using the Fierz transformation we have
    $\left(\bar{e}_{\mathrm{L}} \chi_{\mathrm{iR}}\right)\left(\overline{\dot{x}}_{\mathrm{kR},}{ }_{\mathrm{L}}\right)=\frac{1}{2}\left(\overline{\mathrm{e}}_{\mathrm{L}} \gamma_{a} \mathrm{e}_{\mathrm{L}}\right)\left(\bar{\chi}_{\mathrm{kR}} \gamma_{a} \chi_{{ }_{\mathrm{iR}}}\right)$,
    $\left(\bar{e}_{R} \chi_{i L}\right)\left(\bar{\chi}_{k L} e_{R}\right)=-\frac{1}{2}\left(\dot{\bar{e}}_{R} \gamma_{\alpha} e_{R}\right)\left(\bar{\chi}_{k L} \gamma_{a} \chi_{i L}\right)$.

