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ON THE PAIR CREATION EFFECT IN RADIATIVE BOTTONIUM TRANSITIONS

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1. GENERAL REMARKS

In the present paper quark pair creation as part of the internal dynamics of meson decay amplitudes is discussed for the case of radiative bottonium transitions.

It is well-known that in the case of the deuteron the contribution from internal nucleon pair creation to the radiative decay width has played an important role in the discussion of the experimental data[1]. The heavy quarkonia are successfully described as bound states of heavy quark-antiquark pairs interacting by a static flavour-independent potential and obeying the Schrödinger equation[2]. In the framework of this nonrelativistic potential model we estimate the contribution from internal b-quark pair creation to the radiative decay width within the Υ family.

Retaining only the time-dependence of the antiquark propagator in the time-ordered pair creation diagrams (Fig.3) and restricting the propagation velocity of the antiquark to a nonrelativistic region, we obtain a quasilocal approximation of the pair creation correction that can be evaluated using time-dependent perturbation theory. The resulting expression contains the two wave functions, the (QQ) potential $V(r)$ and a factor coming from the antiquark propagator that modulates the integrand of the overlap integral. At this point we go beyond the more qualitative investigation of ref.[3].

The presented quasilocal approximation of the pair creation correction has been developed in ref.[4] for radiative charmonium transitions. Since in the Υ system relativistic corrections are expected to be much smaller than in the ψ system, this nonrelativistic approximation of the pair creation correction works more accurately for the radiative bottonium transitions.

As examples we study the electromagnetic transitions $\Upsilon(10023) \to \gamma + \chi_b(9890)$ and $\chi_b(9915) \to \gamma + \Upsilon(9460)$. The pair creation corrections obtained are smaller than 10%.

In section 2 we evaluate the nonpair part of the overlap integrals for the two electromagnetic transitions and in section 3 we obtain the general analytic expression for the pair creation part of the overlap integral. We discuss the applicability of the quasilocal approximation and in Section 4 we rewrite it in a suitable form for numerical evaluation. Discussion of the results concludes section 4.

2. THE NO-PAIR TRANSITION MATRIX ELEMENTS FOR $2^3S_1 \to {}^3P_1$ AND $^3P_2 \to 1^3S_1$

In comparison to lighter $q\bar{q}$ systems, composed of u, d, s or c quarks, in the Υ system relativistic kinematic effects are much smaller. Till now the measurements of electromagnetic transitions among the Υ bound states [5] have been in agreement with the corresponding potential model predictions [2]. Fig.1 shows the energy levels of the Υ(1S) and Υ(2S) as well as those states that can be reached from the latter by radiative transitions.

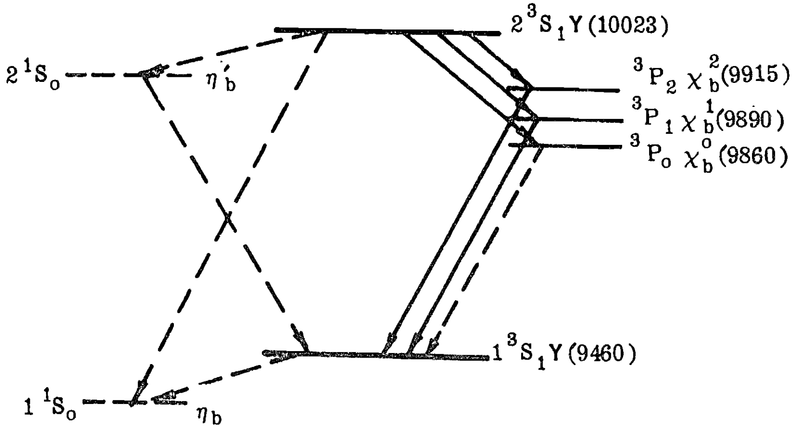


Fig.1. The energy level scheme for $b\bar{b}$ *bound states that can be reached by a radiative transition from the* Υ(2S)*. The solid lines represent the observed transitions.*

In the following we consider the electromagnetic transitions

$$\Upsilon(2S) \to \chi_b^1 + \gamma \tag{1a}$$

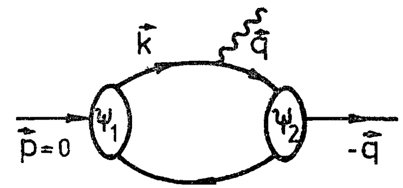
and

$$\chi_b^2 \to \Upsilon(1S) + \gamma . \tag{1b}$$

The partial widths of these electromagnetic E1 transitions are proportional to the square of an overlop integral $F(\vec{q})$ involving the wave functions of the initial and final states:

$$F(\vec{q}) = \int d^3r\, \psi_1^*(\vec{r})\, e^{i\vec{q}\vec{r}}\, \psi_2(\vec{r}) \tag{2}$$

Fig.2. The no-pair diagram of radiative meson decay.



with obvious notation, (see Fig.2), in the rest system of the initial meson. We evaluate the overlap integrals by using for a rough estimate the corresponding wave functions of a three-dimensional harmonic oscillator [6]:

$$\psi^{1S}(\vec{r}) = (\pi R^2)^{-3/4}\, e^{-r^2/2R^2},$$

$$\psi^{2S}(\vec{r}) = \sqrt{\tfrac{3}{2}}\,(\pi R^2)^{-3/4}\left(1 - \tfrac{2}{3}\left(\tfrac{r}{R}\right)^2\right) e^{-r^2/2R^2}, \tag{3}$$

$$\psi^{1P}(\vec{r}) = \sqrt{2}\,(\pi R^2)^{-3/4}\, \frac{r}{R}\cos\theta\, e^{-r^2/2R^2}$$

with R ≈ 0.2 fm in the case of bottonium. We find for the overlap integrals of the decays (1a) and (1b):

$$F_a(q) = i\sqrt{3}(qR)\left[6 - \frac{38}{3}(qR)^2 + \frac{4}{3}(qR)^4\right] e^{-q^2R^2/4} = .78i \tag{4}$$

with q = 0.128 GeV and

$$F_b(q) = i2\sqrt{2}\left(-\frac{1}{qR} + qR\right) e^{-q^2R^2/4} = i(-4,6) \tag{5}$$

with q = 0.445 GeV.

3. THE PAIR CREATION CORRECTION

Quark pair creation in lowest order of perturbation theory is described by the time-ordered diagrams of Fig.3 where the time-ordering of the antiquark propagator prevents inclusion of the soft gluon into the bound states.

We start in time-dependent perturbation theory with

$$\Delta F^1(\vec{q}) = \int_{-\infty}^{+\infty} dt\, W^1_{12}(\vec{q}, t)\, e^{i\omega t}, \tag{6}$$

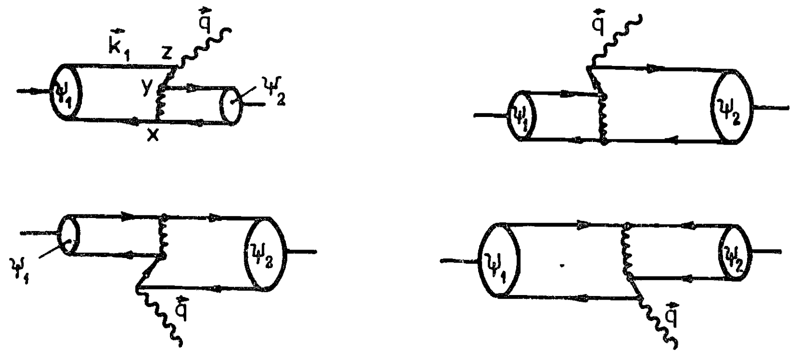


Fig.3. Time-ordered pair creation diagrams to radiative meson decay in lowest order of perturbation theory.

where $\omega = |\vec q|$ and i denotes the corresponding diagram of Fig.3. Each of the diagrams leads to the following general expression for $W^i_{12}(\vec q, t)$:

$$W^i_{12}(\vec q, t) = \int d^3 r_1 d^3 r_2 \psi_1^*(\vec r_1) V(\vec r_2, \vec r_1 - \vec r_2, t, \vec q)\, \psi_2(\vec r_2) \tag{7}$$

with $\vec r_1 = \vec x - \vec z$, $\vec r_2 = \vec x - \vec y$, $\vec r_1 - \vec r_2 = \vec y - \vec z$ and $t = y^0 - z^0$. Depending on $\vec x$, $\vec y$, $\vec z$ the potential part V sandwiched between the wave functions appears now as a nonlocal potential that contains in addition to the radiation operator the antiquark and the soft gluon. The substitution

$$\vec\rho_1 = r_1 + r_2 , \qquad \vec\rho_2 = r_1 - r_2 \tag{8}$$

transforms (7) into

$$W^i_{12}(\vec q, t) = \frac{1}{8}\int d^3\rho_1 d^3\rho_2\, \psi_1^*(\tfrac{1}{2}(\vec\rho_1 + \vec\rho_2)) \cdot V(\vec\rho_1, \vec\rho_2; t, \vec q)\,\psi_2(\tfrac{1}{2}(\vec\rho_1 - \vec\rho_2)) \tag{9}$$

with the nonlocal potential

$$V(\vec\rho_1, \vec\rho_2 ; t, q) = V_{Q\bar Q}(\tfrac{1}{2}(\vec\rho_1 - \vec\rho_2)) \cdot S^c(\vec\rho_2, t)\, e^{i\frac{\vec q}{2}(\vec\rho_1 + \vec\rho_2)} . \tag{10}$$

The flavour-independent quark-antiquark potential $V_{Q\bar Q}(\vec\rho_1 - \vec\rho_2)$ in (10) will be discussed below and for the antiquark propagator we have

$$S^c(\vec\rho_2, t) = (i\gamma_0 \frac{\partial}{\partial t} - i\vec\gamma \cdot \nabla_{\vec\rho_2} + m)\, D^c(\vec\rho_2, t) , \tag{11}$$

(see notation by Bogolubov and Shirkov [7]). Looking for a suitable quasilocal approximation of the nonlocal potential $V(\vec\rho_1, \vec\rho_2; t)$, we require

$$\vec y \approx \vec z, \text{ that means } \vec\rho_2 \ll \vec\rho_1, \text{ and } \rho_1^2 < t^2 \tag{12}$$

introducing a dimensionless parameter a

$$|\vec\rho_2| \le a|\vec\rho_1| , \quad \text{where} \quad 0 < a \ll 1 . \tag{13}$$

In the same manner as in the quark-antiquark potential there remains only the coordinate-dependence of the gluon propagator, under this restriction only the time-dependence survives in the antiquark propagator. In such a way the nonlocal potential factorizes into a local coordinate-dependent part ($Q\bar Q$ potential) and a time-dependent part (resulting from the antiquark propagator), so that we obtain the following quasilocal, time-dependent potential:

$$V(\vec\rho_1; t, \vec q) = \frac{4\pi}{3} a^3 \rho_1^3 V_{Q\bar Q}(\tfrac{1}{2}\vec\rho_1) S^c(0, t)\, e^{i\frac{\vec q}{2}\vec\rho_1} . \tag{14}$$

Considering now the sum of the four diagrams of Fig.3 and, namely, its time dependence, we find

$$\Delta F(\vec q) = \sum_{i=1}^{4} \Delta F^i(\vec q) = \frac{\pi}{3} a^3 \int_0^\infty dt \cdot \cos\omega t (S^{(-)}(0, t) - S^{(+)}(0, -t)) \times \int_0^t d^3\rho_1 \cdot \rho_1^5 \cdot \psi_1(\frac{\vec\rho_1}{2}) V_{Q\bar Q}(\tfrac{1}{2}\vec\rho_1)\, e^{i\frac{\vec q}{2}\vec\rho_1} \cdot \psi_2(\frac{\vec\rho_1}{2}) , \tag{15}$$

where the weak $\vec\rho_2$-dependence in the wave functions ψ_1 and ψ_2 has been neglected and $\omega = |\vec q|$. Further, those terms in the difference $S^{(-)}(0, t) - S^{(+)}(0, -t)$ containing time derivatives cancel each other with the result

$$S^{(-)}(0, t) - S^{(+)}(0, -t) = 2m D^{(-)}(0, t) , \tag{16}$$

where [7]

$$D^{(-)}(0, t) \underset{t>0}{=} \frac{m}{8\pi t}[-I_1(mt) + iN_1(mt)] , \tag{17}$$

($I_1(x)$ and $N_1(x)$ are the Bessel functions in the usual notation). Finally, we end up with the following formula for the quark pair creation correction:

$$\Delta F(\vec q) = \frac{m^2}{12} a^3 \int_0^\infty \frac{dt}{t} \cos\omega t [-I_1(mt) + iN_1(mt)] \cdot g(t, \vec q) . \tag{18}$$

where

$$g(t,\vec q) = \int_0^t d^3\rho_1 \cdot \rho_1^5\, \psi_1\left(\frac{\vec\rho_1}{2}\right) V_{Q\bar Q}\left(\frac{\vec\rho_1}{2}\right) e^{i\frac{\vec q}{2}\vec\rho_1} \cdot \psi_2\left(\frac{\vec\rho_1}{2}\right), \tag{19}$$

and ψ_1, ψ_2 are the initial and final state wave functions of the corresponding electromagnetic transition.

Let us now discuss the conditions under which the quasilocal contribution (14) represents a reasonable approximation of the nonlocal potential (10). Investigation of this point needs physical interpretation of the dimensionless parameter a. Looking at the light-cone picture of Fig.4, we notice that the parameter a determines the angle of the narrow cone which limits the propagation velocity of the antiquark in agreement with (12) and (13). Such a limit corresponds to a cut-off in the internal relative momentum distribution governed by the bound state wave functions. In the oscillator model these are Gaussian distributions and therefore admit a reasonable cut-off. For $R \approx 0.2$ fm in the bottonium system they give an expectation value $\langle|\vec k|\rangle \approx 1$ GeV, so that a cut-off near 2 GeV for $|\vec k_1 - \vec q|$ should be sufficient. Relating the parameter a to the maximum of $|\vec k_1 - \vec q|$ one obtains

$$\frac{\max|\vec k_1 - \vec q|}{m_{\bar q}} = \sigma = \frac{\beta_{max}}{\sqrt{1-\beta^2_{max}}}, \quad \beta_{max} = \frac{\sigma}{\sqrt{1+\sigma^2}} = a. \tag{20}$$

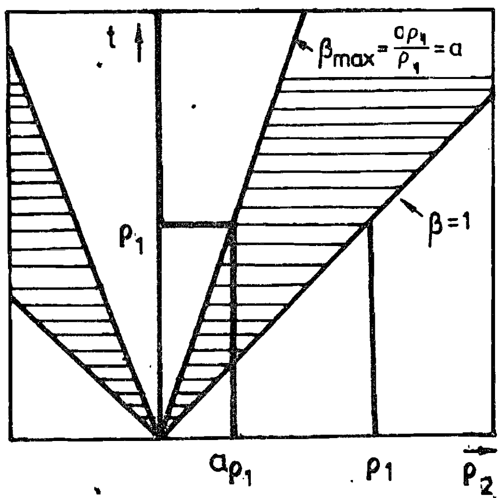


Fig.4. Illustration of the parameter a as maximal propagation velocity of the antiquark in the quasilocal approximation.

It can be seen from Fig.5 how $\beta_{max} = a$ depends on the maximum momentum of the antiquark and on its static mass $m_{\bar q}$. We conclude that the quasilocal approximation should be applicable to the $b\bar b$-system with $a \approx 0.15$–0.25.

To evaluate numerically the pair creation correction $\Delta F(q)$ we have to fix the quark-antiquark potential $V'_{Q\bar Q}(r)$. The ψ and Υ spectroscopies probe distances between 0.1 fm and 1.0 fm. In this range the various

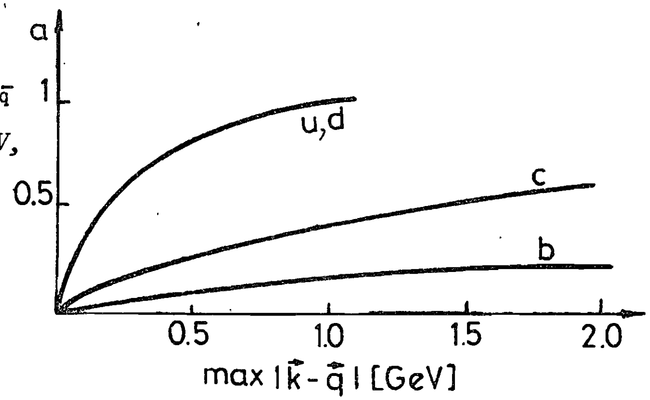


Fig.5. $a = \beta_{max}$ as a function of $\max|\vec k_1 - \vec q|$ with $m_{\bar q}$ as a curve parameter ($m_{u,d} = 0.3$ GeV, $m_c = 1.5$ GeV, $m_b = 4.9$ GeV).

suggested $(Q\bar Q)$ potentials cannot be distinguished within the theoretical uncertainties. We use in the following the particularly simple and successful potential suggested by Richardson /8/

$$V(r) = \frac{8\pi}{33-2n_f}\Lambda\left(\Lambda r - \frac{f(\Lambda r)}{\Lambda r}\right), \tag{21}$$

where

$$f(\kappa) = \frac{4}{\pi}\int_0^\infty dp\, \frac{\sin p\kappa}{p}\left[\frac{1}{\ln(1+p^2)} - \frac{1}{p^2}\right]. \tag{22}$$

The potential $V(r)$ is the Fourier transform of Richardson's interpolating formula

$$\tilde V(\vec p^{\,2}) = -\frac{4}{3}\,\frac{12}{33-2n_f}\cdot\frac{1}{\vec p^{\,2}}\cdot\frac{1}{\ln(1+\vec p^{\,2}/\Lambda^2)} \tag{23}$$

for

$$-p^2 \gg \Lambda^2: \quad \tilde V(p^2) = \frac{4}{3}\alpha_s(p^2)\frac{1}{p^2} = \frac{16\pi}{33-2n_f}\cdot\frac{1}{p^2}\cdot\frac{1}{\ln(-p^2/\Lambda^2)} \tag{24}$$

and

$$-p^2 \ll \Lambda^2: \quad \tilde V(p^2) \longrightarrow \text{const}\,\frac{1}{(p^2)^2}. \tag{25}$$

Equation (21) defines a $(Q\bar Q)$ potential that depends only on a single scale parameter Λ as it is the case for the true QCD potential and describes the ψ and Υ spectroscopies remarkably well ($\Lambda \approx 0.4$ GeV).

4. RESULTS FOR THE ELECTROMAGNETIC TRANSITIONS $\Upsilon(2S) \to \chi_b^1 + \gamma$ AND $\chi_b^2 \to \Upsilon(1S) + \gamma$

Now we are able to estimate the pair creation corrections $\Delta F_a(q)$ and $\Delta F_b(q)$ for the transitions (1a) and (1b), respectively, within our quasilocal approximation. Inserting the wave functions (3) into eq.(19), we obtain

$$\Delta F_{a,b}(q) = \mathrm{Re}(\Delta F_{a,b}) + i\mathrm{Im}(\Delta F_{a,b}) = \tag{26}$$

$$= B_{a,b}(q)\cdot a^3 \int_0^\infty \frac{d\sigma}{\sigma} \cos(\frac{q}{\Lambda}\sigma)[\, N_1(\frac{m}{\Lambda}\sigma) + iJ_1(\frac{m}{\Lambda}\sigma)] \cdot g(\sigma, q) \ ,$$

where in the case of the transition $\Upsilon(2S) \to \gamma + \chi_b^1$ with $q = 0.128$ GeV we have

$$g_a(\sigma, q) = \int_0^\sigma dr \cdot r^5\, V(\frac{r}{2})(\cos\frac{q}{2\Lambda}r - \sin(\frac{q}{2\Lambda}r)\cdot\frac{2\Lambda}{qr})\, e^{-\frac{r^2}{4}(R\Lambda)^{-2}} \times \tag{27}$$

$$\times (1 - \frac{1}{6}r^2(R\Lambda)^{-2}\ ),$$

$$B_a(q) = \sqrt{\frac{\pi}{3}} \cdot \frac{8}{27}(\frac{\Lambda}{q}) \cdot \frac{m^2}{R^4\Lambda^6} \tag{28}$$

and for the transition $\chi_b^2 \to \gamma + \Upsilon(1S)$ with $q = 0.445$ GeV we find

$$g_b(\sigma, q) = \int_0^\sigma dr \cdot r^5\, V(\frac{r}{2})\,(\cos\frac{q}{2\Lambda}r - \frac{2\Lambda}{qr}\sin\frac{q}{2\Lambda}r)\, e^{-\frac{r^2}{4}(\Lambda R)^{-2}}\ , \tag{29}$$

$$B_b(q) = \frac{8\sqrt{2\pi}}{81}(\frac{\Lambda}{q})\, \frac{m^2}{R^4\Lambda^6}. \tag{30}$$

In eqs. (26) and (28) the Richardson potential (21) is denoted by $V(r/2)$ with $r = \rho_1/\Lambda$ and $\Lambda = 0.4$ GeV, $R = 0.2$ fm. Because of their proportionality to the square of the overlap integrals the radiative decay widths get the following relative contribution from quark pair creation;

$$(\frac{F + \Delta F}{F})^2 \approx F^2(1 + \frac{2\,\mathrm{Im}\Delta F}{\mathrm{Im}F}). \tag{31}$$

In the last expression we have taken into account that the overlap integrals F_a and F_b are purely imaginary (see eqs.(4) and (5)) and that the quadratic terms for the decays (1a) and (1b) can be neglected.

According to the relative internal momentum distribution of the b-quarks (see Fig.5) the value of the parameter a should be near 0.2 for both decays: $0.15 \le a \le 0.25$.

The numerical results for the decays $\Upsilon(2S) \to \chi_b^1 + \gamma$ and $\chi_b^2 \to \Upsilon(1S) + \gamma$ are presented in Figs.6a and 6b, respectively.

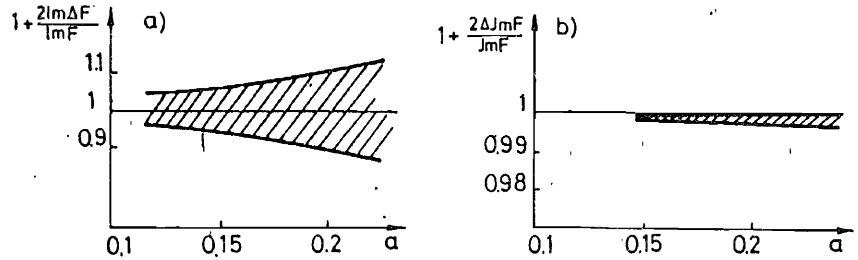


Fig.6. The pair creation correction to the radiative decay widths of $\Upsilon(2S) \to \chi_b^1 + \gamma$ *(a) and* $\chi_b^2 \to \Upsilon(1S) + \gamma$ *(b) as a function of the parameter* a.

We find a pair creation correction which is smaller than 10% for the $2\,^3S_1 \to {}^3P_2$ transition and is negative and smaller than 1% in the case of the $^3P_1 \to 1\,^3S_1$ transition.

The contributions from internal b-quark pair creation to the electromagnetic transition rates of the processes (1a) and (1b) have been estimated in a quasilocal approximation preserving contrary to /3/ the time-dependence of the antiquark propagator and found to be relatively small. But the pair creation correction (18), (19) depends sensitively on quark masses and photon energies and thus cannot be ignored in quantitative investigations of electromagnetic quarkonium transitions.

Within a nonrelativistic treatment of radiative quarkonium transitions the quasilocal approximation of the pair creation correction, preserving the time-dependence of the antiquark propagator, works well for the bottonium system and should give reasonable results for the toponium system, if experimentally observed.

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Левин К., Мотц Г.Б. E2-86-466

Об эффекте рождения пар в радиационных переходах боттониев

Исследуются вклады в ширины радиационных распадов $\Upsilon(10023) \to \gamma + \chi_b(9890)$ и $\chi_b(9915) \to \gamma + \Upsilon(9460)$, связанные с рождением внутренней пары кварков $b\bar{b}$. Эти вклады в рамке представленной квазилокальной аппроксимации, в которой сохраняется временная зависимость антикваркового пропагатора, дают поправку меньше чем 10% по сравнению с расчетом, проведенным без учета рождения пар. Хотя и относительно малы, поправки от рождения пар чувствительно зависят от масс кварков и фотонных энергий и, следовательно, должны учитываться в количественных исследованиях радиационных переходов кваркониев.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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On the Pair Creation Effect in Radiative Bottonium Transitions

The contributions from internal b-quark pair creation to the radiative transition rates of the processes $\Upsilon(2S) \to \chi_b^1 + \gamma$ and $\chi_b^2 \to \Upsilon(1S) + \gamma$ have been estimated in a quasilocal approximation preserving the time-dependence of the antiquark propagator and found to be smaller than 10%. Although relatively small, the pair creation correction depends sensitively on quark masses and photon energies and thus cannot be ignored in quantitative investigations of radiative quarkonium transitions.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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