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IMPACT PARAMETER DEPENDENCE OF THE SPECIFIC ENTROPY AND THE LIGHT PARTICLE YIELD IN RELATIVISTIC HEAVY ION COLLISIONS

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I. Introduction

The study of the entropy production in relativistic heavy ion collisions permits us to investigate the behaviour of matter far from the ground state. This is because the entropy is assumed to grow rapidly when the nucleons make their first collisions and to remain almost constant in the expansion phase when the interaction between the constituents of the nuclear fireball ceases. Thus, measurable characteristics of the hot system as, e.g., the cluster abundances become frozen at this point. The cascade [1,2] and hydrodynamical [3,4] model calculations are capable of describing the complicated collision dynamics and therefore, can provide us with a window at an early stage of the reaction where matter was still hot and dense.

According to the suggestions of Siemens and Kapusta [5] the entropy per baryon produced during the collision process can be extracted in an indirect way via the yield of composite particles. The "experimental" entropy was originally inferred from inclusive measurements that gave significantly larger entropy values than those following from the cascade calculations for central collisions or from estimates based on ordinary equations of state for hot nuclear matter. This discrepancy between theory and experiment made up the so-called entropy puzzle discussed extensively in the literature (see ref.[6] for a recent review). The experimental situation has substantially been improved by measuring the charge multiplicity dependence of the light composite particle yield. In fact, Gutbrod et al.[7] and Doss et al. [8] have shown (cf.fig.1) that the yield of d, t, ^3He, ^4He increases steadily with baryon charge multiplicity and levels off at high multiplicity values. Thus, these experiments demonstrate a strong correlation between the impact parameter and the cluster yield. From these new data it becomes evident that the thermal model assuming chemical equilibrium[9,10] and the original coalescence model [11] both predicting a cluster yield independent of the size of the emitting source may be applied only for large multiplicity events (small impact parameters) when the bulk dynamics limit may asymptotically be reached.

By utilizing the coalescence model of Sato and Yazaki[12] in which the sizes of the source and the fragments are free parameters,

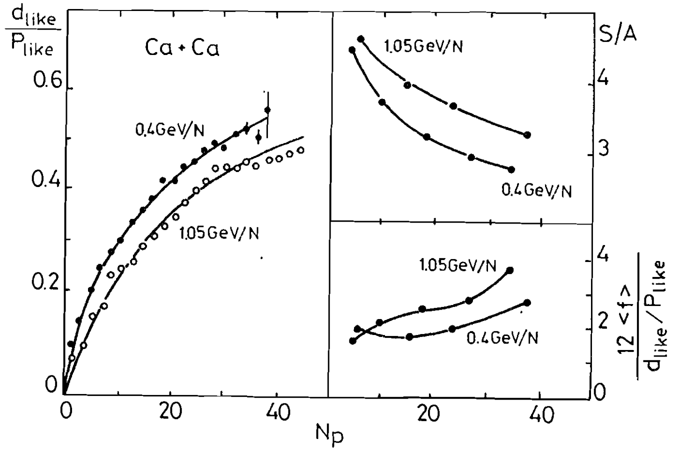

Fig.1.

left: Experimental ratio d_{like}/p_{like} as a function of the charge multiplicity N_p for the reaction Ca + Ca at 0.4 and 1.05 GeV/nucleon (adapted from ref.[8]). The thin lines are the results of a fit according to formula (11) (for details see Ref.[8]).

right: Specific entropy values and the ratio $12\langle f^{(4)}\rangle/(d_{like}/p_{like})$ calculated in the cascade model as a function of the charge multiplicity (for details see text).

the trend of the experimental data for the cluster production as a function of the baryon charge multiplicity N_p has been fitted by Doss et al. [8] introducing a source radius proportional to $N_p^{1/3}$ and a constant temperature inferred from the proton spectrum. These assumptions imply that the break-up density and the associated specific entropy values remain constant as a function of the participant number or the impact parameter. However, such assumptions are unsatisfactory from a physical point of view because the consideration of a fictitious source obscures the complicated dynamical aspects of the freeze-out process. They also contrast with the predictions of the cascade calculations [2,13,14] showing that the specific entropy increases with the impact parameter and that the maximum density reached in the participant region is the lower the more peripheral the collision process is. The variation of the density of the emitting source with the impact parameter is an immediate consequence of the finite size of the nuclei. It is the aim of the present paper

to investigate the evolution of a hot nuclear system by means of the cascade model which seems to be rather suitable for this purpose because the entropy values attained during the collision and the fragment abundances could be calculated separately. This allows us to study their state of being connected in a straight-forward way.

The methods for the calculation of the entropy and the cluster abundances are briefly outlined in the next section. In Section 3 we consider the evolution of these quantities for central and noncentral nucleus-nucleus collisions and compare the results with experimental data of 4π -measurements. Concluding remarks are given in Section 4.

2. Calculation of the Entropy and the Cluster Abundances

The entropy value is calculated by means of the independent particle approximation, i.e. one introduces a probability that a certain single-particle state of a certain phase space volume is occupied. In the Fermi gas approximation one has

$$S = - \int d\Upsilon \; [\, f \ln f + (1-f) \ln (1-f) \,] , \tag{1}$$

where $d\Upsilon$ is the phase space volume element containing the usual spin-isospin degeneracy factors and the distribution function f is normalized to the particle number A of the system. In the classical limit $\langle f \rangle \ll 1$, one finds the expression

$$S/A = 5/2 - \ln [\, 2^{3/2} \langle f \rangle \,] \tag{2}$$

that can be compared with the Siemens-Kapusta formula [5]

$$S/A = 5/2 - \ln [\, \text{"}R_{dp}\text{"} / 3\sqrt{2} \,] , \tag{3}$$

where "R_{dp}" is the ratio of the deuteron-like fragments to the proton-like particles. The cascade model permits us to calculate the phase space occupancy and the "R_{dp}" ratio separately.

The method for the evaluation of the entropy via the phase space distribution function f has been described in a previous paper [2]. There it was assumed that a local thermodynamical equilibrium is established in each subvolume ΔV_i of the whole interacting zone of the two ions. In this case the distribution function for a single cell is

$$f_i(\vec{r},\vec{p},t) = \bar{f}_i \; \exp[-(\vec{p} - m\vec{v}_i(t))^2 / 2m\, T_i(t)\,] \tag{4}$$

with

$$\bar{f}_i = \frac{\bar{N}_i \Lambda_i^3}{4\,\Delta V_i} = \frac{\bar{\rho}_i \Lambda_i^3}{4}, \tag{5}$$

where $\bar{N}_i$ is the mean particle number in ΔV_i, the factor 4 is due to the spin-isospin degeneracy, $\bar{\rho}_i$ denotes the mean particle density in the considered subvolume, $\vec{v}_i(t)$ is the mean velocity of a cell and $\Lambda_i = (2\pi\hbar^2/m\,T_i(t))^{1/2}$ stands for the thermal wave length of a nucleon. The temperatire $T_i(t)$ is determined via energy conservation whereby the cooling effect due to the pion and other particle production is taken into account. In the calculation we have used everywhere relativistic formulae and employed the relativistic Boltzmann distribution function. With the distribution function (4), that reflects the translational motion of the cell, the momentum integration in (I) can be performed and the entropy is finally calculated by replacing the integral by a sum over finite elements. The results for the entropy turned out to be not rather sensitive to the chosen cell subdivision. This profitable feature is mainly due to the fact that the smoothing over the momentum space and the simultaneous account for the translational motion of the cells reflect the enormous deformation of the phase space elements when the free motion sets in; Compared with the results of ref./1/, where a decomposition of the six-dimensional phase space has been performed, our method yields somewhat smaller entropy values.

The ratio "R_{dp}" of quasideuterons d_{like} to the proton-like particles p_{like} is given by /15/

$$"R_{dp}" = \frac{3}{4} \int \frac{d^3R\, d^3P}{(2\pi\hbar)^3}\, f_{np}^{(2)}(\vec{r},\vec{p},\vec{R},\vec{P},t)\, g_d(\vec{r},\vec{p}), \tag{6}$$

where $f_{np}^{(2)}$ is the two-particle distribution function of neutron-proton pairs and g_d stands for the Wigner transform of the deuteron density. The factor 3/4 is due to spin-isospin degeneracy and the integration has to be performed over relative and center-of-mass coordinates. If the deuteron wave function is approximated by a Gaussian one, $\psi_d(r) = (1/\pi r_d^2)^{3/2} \exp(-r^2/2r_d^2)$, the Wigner transform $g_d(r,p)$ is given by

$$g_d(r,p) = \frac{1}{\pi^3} \exp\left(-r^2/r_d^2 - p^2 r_d^2\right). \tag{7}$$

Expression (6), giving the number of deuteron-like pairs in light composites, requires the knowledge of higher order distribution

functions in order to disentangle the asymptotic emission of definite composite particle. In spite of the fact that two-body correlation effects are taken into account in the cascade model and a direct and tedious calculation of R_{dp} according to eq. (6) is in principle possible, we make the replacement

$$f_{np}^{(2)} \simeq f_n^{(1)}(r_1,p_1)\, f_p^{(1)}(r_2,p_2). \tag{8}$$

By assuming an emitting source of radius R_p and a Gaussian spatial nucleon distribution, the integration (6) can be performed and the final result is

$$"R_{dp}" = \frac{12 \langle f^{(1)} \rangle}{\left(1+\frac{r_d^2}{2R_p^2}\right)^{3/2} \left(1+\frac{\hbar^2}{mT r_d^2}\right)^{3/2}}. \tag{9}$$

Here the average phase space occupancy $\langle f^{(1)} \rangle$ is given by

$$\langle f^{(1)} \rangle = \frac{A_{cas}\, \Lambda^3}{4\,(4\pi R_p^2)^{3/2}} \qquad \frac{\bar{\rho}\, \Lambda^3}{4 \cdot 2^{3/2}}. \tag{10}$$

where A_{cas} stands for the number of struck particles calculated in the cascade model and $\bar{\rho}$ is their average density. Inserting (10) into (9) the expression for "R_{dp}" goes over into that derived by Sato and Yazaki/12/ and employed in refs./7,8/ to infer the mean sizes of the source and the deuteron-like pairs from experimental data

$$"R_{dp}" = \frac{6 \cdot N_p}{\left(1+\frac{2R_p^2}{r_d^2}\right)^{3/2} \left(1+r_d^2 \frac{mT}{\hbar^2}\right)^{3/2}}. \tag{11}$$

For the sake of simplicity we have considered symmetric nuclear matter and $A_{cas}/2$ replaced by the number of charges N_p triggered in the experiments of refs./7,8/.

At this place a comment on the fitting procedure of Doss et al. /8/ and on the applicability of expression (11) is in order. The replacement $f_{np}^{(2)} \simeq f_n^{(1)} \cdot f_p^{(1)}$ implies that at the freeze-out stage of the collision process the two-body correlation function can still be expressed through the one-particle function $f^{(1)}$ and that the calculation of the overlap integral with the deuteron wave function in (6) gives a reasonable estimate on how many deuteron-like pairs will be formed during the future evolution of the system. The freeze-out picture is based on a sudden disappearance of the interaction between noncorrelated particles and leads to a rapid transition from an expanding system being in thermal equilibrium to a freely expanding system in which the number of composites does not change anymore. According to this freeze-out scenario, one had to consider at later time steps a two-body correlation function consisting of a genuine bound state distribution function for the formed deuteron-like pairs plus a product of single-particle distribution functions that become completely uncorrelated for large time instants and do not give contribution to the overlap integral (6).

Doss et al.[8] used expression (11) in order to infer directly from the comparison with the data the size of the fictitious source emitting the particles by assuming $R_p = r_0 (N_p A/Z)^{1/3}$ and the parameter r_d characterizing the extension of the deuteron-like pairs. Keeping in mind the freeze-out concept in a cascade model the value $R_p(N_p)$ is a calculable but time-dependent quantity that determines the break-up density. Furthermore, the "source" radius R_p and the temperature T are also not independent quantities. Thus, applying the cascade model, one can play a little with the cluster size parameter r_d and consider different break-up situation in order to fit the experimental cluster abundances.

In the case $R_p \gg 1$, at which the bulk limit dynamics should work, one obtains

$$
"R_{dp}" = \begin{cases} \dfrac{12 \langle f^{(1)} \rangle}{(1 + \hbar^2/mT r_d^2)^{3/2}} , & R_p \gg r_d \qquad (12a) \\[2ex] \dfrac{(3\sqrt{2}/4)(\langle r_d^2 \rangle / r_0^2)^{3/2}}{\left(1 + \frac{2}{3} \frac{mT}{\hbar^2} \langle r_d^2 \rangle \right)^{3/2}} , & R_p = r_0 A^{1/3} \quad A \gg 1 \qquad (12b) \end{cases}
$$

Using the first expression (12a), one calculates within the cascade model the mean phase space occupancy $\langle f^{(1)} \rangle$ that does not change after the free motion sets in and the associated entropy is $S/A = 5/2 - \ln(2^{3/2} \langle f^{(1)} \rangle)$. The quasi-deuteron effective radius r_d has then to be adjusted to fit the experimental R_{dp} values. The second expression (12b) has been used by Doss et al.[8] to predict "asymptotic" R_{dp} values.

In interpreting the radii of the composites, one has to consider that they are embedded in a nuclear medium that affects their effective binding energies in such a way that they become less bound as denser and/or cooler the medium is. Only for a low density medium, one can expect that the effective radii are equal to those of the free composites. In other cases they are larger [16].

In the next section we shall apply the methods described above in order to calculate the entropy evolution and the associated values as a function of the impact parameter or the baryon charge multiplicity.

3. Entropy Evolution of Noncentral Collisions and the Cluster Production

As has been said in the Introduction, the measurements of the charge multiplicity dependence of the ratio d_{like}/p_{like} [7,8] are very useful to gain information on the properties of hot and dense nuclear matter. Since the ratio d_{like}/p_{like} increases with charge multiplicity N_p, the data suggest (see fig.1) that the phase space occupancy $\langle f^{(1)} \rangle$ is significantly larger for central collisions than for peripheral ones. This behaviour has a consequence that the specific entropy should increase with impact parameter because the entropy is the larger the further the nucleons are from each other in phase space, i.e. the smaller the probability of forming composites is. The cascade model calculations reproduce this tendency quite well. From fig.1 it is clearly seen that the calculated entropy decreases steadily as the charge multiplicity N_p increases. At small impact parameters (large multiplicity values) the entropy curve shows a saturation-like tendency. This behaviour is in accord with the results of refs.[13,14].

In the right-hand part of fig.1 we compare these entropy values with those following from formula (3) and show that the ratio $12 \langle f^{(1)} \rangle / (d_{like}/p_{like})$ can be interpreted as a measure of how close the cascade results are to the so-called bulk equilibrium limit. The results point out that this limit is far to be reached. In other words, the specific entropy values exftracted by means of formula (3) are for central collisions between 4 and 5 units whereas the cascade model gives a value which is more than one unit lower. Similar results have been found in ref.[14]. One has, however, to take into consideration under what assumptions formula (3) has originally been derived. In fact, Siemens and Kapusta [5] considered a dilute ideal gas consisting of different species. The experimental data and the cascade calculations show that this limit is not yet reached and that the finite size of the emitting source has to be taken into account.

The situation is not substantially changed when considering a somewhat heavier system. In fact, for the reaction Nb + Nb at 0.4 GeV/nucleon [8], the entropy value following from the cascade calculations for central collisions is $S/A = 3.0$, whereas formula

(3) gives $S/A = 4.3$. This fact and the results for the Ca + Ca system indicate that the finite size effects of the emitting source seem to play a decisive role in explaining the connection between the entropy and fragment production, which is more transparent in fig.2.

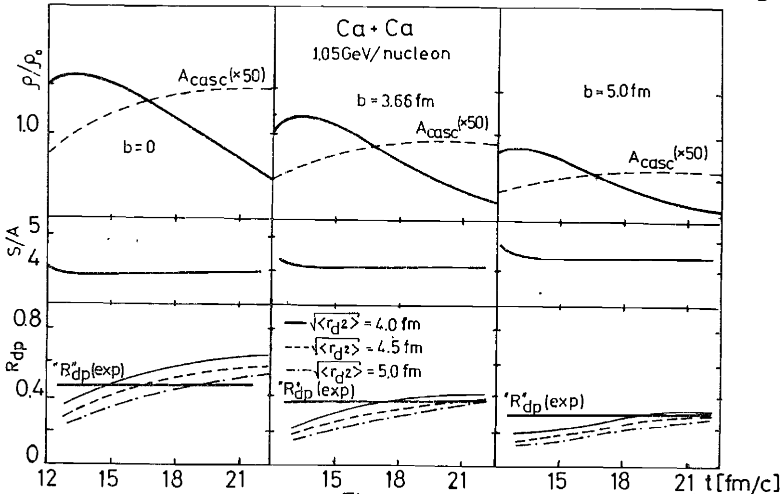

Fig.2

For three impact parameters b = 0, 3.66 fm and 5 fm the cascade results for the time evolution of a number of the struck particles A_{cas}, their density ρ and specific entropy S/A_{cas} are displayed. In the lower part of the figure the "R_{dp}" values calculated according to (9) are compared with the experimental ones of ref./8/ (see also fig.1) using different rms-radii $<r_d^2>$ for the deuteron-like fragments.

There we show for three different impact parameters for the reaction Ca + Ca at 1050 MeV/nucleon the density, the specific entropy and the "R_{dp}" values. The time dependence of these quantities has been rescaled in such a way that t = 0 corresponds to the situation where the two nuclei having initially a Woods – Saxon density profile touch and the value of the overlapping density at this point amounts to $\rho_0 / 10$ ($\rho_0 = 0.15$ fm^{-3}).

The curves for the entropy evolution illustrate that the system formed by the participants gains entropy mostly during a short time interval at the compression stage (see also refs./1,2,13,14/) and that the entropy remains fairly constant when the expansion phase sets in.

In the lower part of fig.2 the theoretical "R_{dp}" values are compared with the experimental ones. It is seen that "R_{dp}" is a slightly increasing function of time, although the entire system expands adiabatically. This behaviour is an artifact that is mainly due to

a too simple picture behind the sudden freeze-out concept. This can be seen in the upper part of fig.2 where the number of struck particles considered in calculating the density ρ is shown. For the central collision, the density ρ reaches its maximum value when only 60% of the nucleons has suffered a collision. Thus, such a behaviour of the entire collision process would require a dynamical treatment of the freeze-our mechanism itself (cf. ref./17/), which is beyind the present consideration.

Interesting enough, the results obtained for "R_{dp}" according to (9) suggest that with a reasonable choice of the radii of the clusters and by fixing the break-up moment by choosing the break-up density of say $\rho_0/3$ the experimental data can be reproduced. As concerns the rms – radii of the fragments there is a tendency that smaller $<r_d^2>$ values are required to fit the data the higher the specific entropy is. This agrees with the predictions of ref. /16/ in which the changes of the rms – radii of clusters embedded in a hot nuclear medium have been calculated.

The results shown in figs. 1 and 2 demonstrate that the cascade model predicts entropy values that are smaller than those following from the bulk equilibrium limit, and that at the same time the model is able to reproduce the cluster abundances reasonably well although the consideration of the freeze-out process itself is lacking desirable qualities. Our analysis of central collisions of Nb + Nb at 400 MeV/nucleon gives also strong support of the fact that the bulk equilibium limit is not yet reached in this heavy system as well. From our calculation it follows that the colliding systems have to be larger than Nb + Nb to diminish the influence of the finite size effects in the fragment formation process. Furthermore, to retain the validity of a simple relationship like (3) between theory and experiment (i.e. between the specific entropy and the cluster abundances) a low occupancy $<f^{(1)}>$ of the available phase space is necessary. It could be achieved by passing to higher beam energies because the available phase space is determined by that portion of the beam energy that is converted into thermal motion. From the experimental data of ref./8/ such a tendency can be seen when comparing, e.g., the cluster yield of Ca + Ca at 0.4 with 1.05 GeV/nucleon. But it is still an open question which degree of global thermalization at higher beam energies say (2 – 3) GeV/nucleon is established.

4. Summary and Concluding Remarks

Analyzing the entropy evolution and the light cluster formation within the cascade model, we have found that the confirmation of a simple relationship between entropy and cluster yield as following from the bulk equilibrium limit is still obscured by the effects of finiteness of the system and the dynamics of the freeze-out process. We have obtained the result that the specific entropy of the participants increases with impact parameter and shows a saturation-like tendency in central collision events. However, for central collisions the specific entropy is more than one unit smaller than that following from the Siemens - Kapusta relation (3). Even for a large system such as Nb + Nb at 400 MeV/nucleon the bulk equilibrium limit is not reached. Furthermore, it has been found that the break-up density of the system becomes the smaller the more peripheral the collision is whereby the number of participants reaches its final value when the density dropps to about 1/3 of its maximum value. This behaviour of the hot system requires a dynamical treatment of the freezd-out mechanism that goes beyond the sudden approximation applied in the present work.

As concerns more peripheral collision processes, the influence of the decay of the spectator matter and its contribution to the entropy and the fragment yield should also be investigated. The cascade approach permits us to distinguish between the spectators and participants and allows us to calculate their disintegration process separately. It could well be that the cluster yield as measured by the 4π detector could turn out to be somewhat lower for low multiplicity events when experimentally a clear distinction between spectator and participant matter would be possible.

In summary, we would like to say that in spite of the fact that there exist still open problems mentioned above, the theoretical investigation of the charge multiplicity and impact parameter dependence of the light particle yield has substantially reduced the original puzzling of the entropy problem.

References

1. Bertsch, Cugnon J. Phys.Rev., 1981, C24, p.2514.
2. Gudima K.K. et al. Phys.Rev., 1985, C32, p.1605.
3. Stöcker H. et al. Prog.Part.Nucl.Phys., 1981, 4, p.133.
4. Kapusta J., Strottman D. Phys.Rev., 1981, C23, p.1282.
5. Siemens P., Kapusta J. Phys.Rev.Lett., 1979, 43, p.1486.
6. Csernai L., Kapusta J. Phys.Rep., 1986, 131, p.223.
7. Gutbrod H.H. et al. Phys.Lett., 1983, 127B, p.317.
8. Doss K.G.R. et al. Phys.Rev., 1985, C32, p.116.
9. Mekjian A.Z. Phys.Rev., 1978, C17, p.1051.
10. Kapusta J. Phys.Rev., 1980, C21, p.1301.

11. Gutbrod H.H. et al. Phys.Rev.Lett., 1976, 37, p.667.
12. Sato H., Yazaki K. Phys.Lett., 1981, 98B, p.153.
13. Cugnon J., L'Hote D. Proc. 7th High Energy Heavy Ion Study. GSI-Report 85-10 (1985).
14. Cugnon J. Lectures at Cargese Sommer School, september 1984.
15. Remler E.A. Phys.Rev., 1982, C25, p.2974.
16. Schulz H. et al. Phys.Lett., 1983, 124B, p.458; Phys.Rev., 1986, C32, p.1095.
17. Barz H.W. et al. Z.Phys., 1982, A308, p.187.

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Гудима К.К. и др. E2-86-449

Зависимость от параметра удара удельной энтропии и выход легких фрагментов в релятивистских столкновениях тяжелых ионов

В рамках каскадного подхода исследована связь между выходом ядерных фрагментов и ассоциативной удельной энтропией частиц, образованных во взаимодействии тяжелых ионов при высокой энергии. Обнаруженная существенная зависимость выхода фрагментов от параметров удара проявляется в том, что удельная энтропия возрастает с увеличением параметра удара, а критическая плотность распада системы оказывается выше для более центральных столкновений. Показано, что при образовании энтропии предел полного термодинамического равновесия не достигается даже для таких тяжелых систем как Nb + + Nb при энергии 400 МэВ/нуклон и что эффекты конечного размера системы и динамика процесса "замораживания" являются доминирующими факторами при определении относительного выхода кластеров.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Gudima K.K. et al. E2-86-449

Impact Parameter Dependence of the Specific Entropy and the Light Particle Yield in Relativistic Heavy Ion Collisions

The connection between the fragment yield and the associated specific entropy of participant matter produced in the course of a relativistic heavy ion collision is studied within the cascade approach. The essential impact parameter dependence of the fragment yield indicates that the specific entropy increases with impact parameter and that the break-up density is the larger the more central the collision process is. The results show that the bulk equilibrium limit for the entropy production is not reached for such heavy systems as Nb + Nb at 400 MeV/nucleon and that the finite size effects and the dynamical freeze-out process are dominant factors in determining the cluster abundances.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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