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THE CHIRAL QUARK-LOOP MODEL AND THE $\Delta I = 1/2$ RULE IN KAON DECAYS

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1. Introduction

In the note, electroweak decays of kaons: $K_{L,S} \to 2\gamma$ and $K \to 2\pi$ are considered[1]. The decays $K_{L,S} \to 2\gamma$ and $K \to 2\pi$ are due to both electroweak and strong low-energy interactions. For describing weak interactions we use an effective Lagrangian obtained in [2]. In terms of this Lagrangian weak vertices take the form of four-quark operators, the structure of which is defined by the Standard Kobayashi-Maskawa Model (KM) [3] with the account of the QCD-interaction. Matrix elements of four-quark operators are due to low-energy strong interactions. Feynman diagrams of matrix elements contain strong quark-meson vertices and divergent quark loops. For describing strong quark-meson vertices and quark loops it is convenient to employ the Chiral Quark-Loop Model (the CQL-model) [4,5]. Matrix elements of four-quark operators can be expressed in terms of square- and logarithmic-divergent integrals

$$I_1(m_i) = \frac{-3i}{(2\pi)^4}\int \frac{d^4k}{m_i^2-k^2} = \frac{3}{16\pi^2}\left[\Lambda^2 - m_i^2 \ln\left(1+\Lambda^2/m_i^2\right)\right],$$

$$I_2(m_i,m_j) = \frac{-3i}{(2\pi)^4}\int \frac{d^4k}{(m_i^2-k^2)(m_j^2-k^2)} = \frac{3}{16\pi^2}\frac{1}{m_i^2-m_j^2}\cdot$$

$$\cdot\left[m_i^2 \ln\left(1+\Lambda^2/m_i^2\right) - m_j^2 \ln\left(1+\Lambda^2/m_j^2\right)\right], \tag{1}$$

where Λ is a cut-off parameter, and $m_i\,(i=u,d,s)$ is the mass of a constituent quark i. In the CQL-model Λ =1.25 GeV, $m_d \simeq m_u$ = =0.28 GeV and m_s =0.46 GeV. With the help of these parameters one can calculate both all strong low-energy coupling constants of four meson nonets (scalar, pseudoscalar, vector and axial-vector) and such important characteristics of strong low-energy interactions of mesons as scattering lengths, slope parameters, electric radii and so on. The employment of the CQL-model for describing low energy strong interactions in decays $K_{L,S} \to 2\gamma$ and $K \to 2\pi$ does not result in new low-energy parameters. Emphasize that in the calculation, the relationship $I_1(m_u)/m_u^2 I_2(m_u,m_u)=8$ is employed.

[1] The wave-functions of K_L and K_S mesons are connected with the wave-functions of $\bar{K}^0$ and K^0 mesons in a standard manner [1].

The effective Lagrangian of weak interactions takes the form [2]

$$\mathcal{L}_{eff} = \frac{G_F}{\sqrt{2}} s_1 c_1 c_3 (-1.000\, Q_1 + 1.600\, Q_2 - 0.033\, Q_3 + 0.018\, Q_5 - 0.100\, Q_6) \equiv \frac{G_F}{\sqrt{2}} s_1 c_1 c_3 \sum_{i=1}^{6} C_i \cdot Q_i , \qquad (2)$$

where $(G_F/\sqrt{2})\, s_1 c_1 c_3 = 1.77 \times 10^{-6}\ GeV^{-2}$, $s_i = \sin\theta_i$, $c_i = \cos\theta_i$ $(i=1,3)$ are KM-matrix elements [3], and Q_i $(i=1,2,3,5,6)$ are four-quark operators:

$$Q_1 = [\bar{s}_a \gamma^\alpha (1-\gamma^5) d_a][\bar{u}_b \gamma_\alpha (1-\gamma^5) u_b],$$

$$Q_2 = [\bar{s}_a \gamma^\alpha (1-\gamma^5) d_b][\bar{u}_b \gamma_\alpha (1-\gamma^5) u_a],$$

$$Q_3 = [\bar{s}_a \gamma^\alpha (1-\gamma^5) d_a] \sum_{q=u,d,s} [\bar{q}_b \gamma_\alpha (1-\gamma^5) q_b],$$

$$Q_5 = [\bar{s}_a \gamma^\alpha (1-\gamma^5) d_a] \sum_{q=u,d,s} [\bar{q}_b \gamma_\alpha (1+\gamma^5) q_b],$$

$$Q_6 = [\bar{s}_a \gamma^\alpha (1-\gamma^5) d_b] \sum_{q=u,d,s} [\bar{q}_b \gamma_\alpha (1+\gamma^5) q_a]; \qquad (3)$$

a,b=1,2,3 are colour indices. The effective Lagrangian (2) satisfies the selection rules: $|\Delta S| = 1$, $|\Delta I| = 1/2$ and $|\Delta I| = 3/2$, where S and I are strangeness and isospin. The $|\Delta I| = 3/2$ transitions are carried out by the four-quark operator

$$O_{|\Delta I|=3/2} = [\bar{s}_a \gamma^\alpha (1-\gamma^5) d_a][\bar{u}_b \gamma_\alpha (1-\gamma^5) u_b] - [\bar{s}_a \gamma^\alpha (1-\gamma^5) d_a] \cdot [\bar{d}_b \gamma_\alpha (1-\gamma^5) d_b] + [\bar{s}_a \gamma^\alpha (1-\gamma^5) u_a][\bar{u}_b \gamma_\alpha (1-\gamma^5) d_b], \qquad (4)$$

that is contained in the Q_1 and Q_2 operators with factor 1/3 [2]. The remaining part of Lagrangian (2) leads to the $|\Delta I| = 1/2$ transitions.

It should be noted that the coefficients for the Q_i-operators in the effective Lagrangian describing weak interactions in kaon decays depend on $\alpha_s(\mu)$, where $\alpha_s(\mu)$ is the QCD- coupling constant for three quark flavours [2], and μ is the normalization point. In the effective Lagrangian (2) the coefficients are calculated for $\alpha_s(\mu) = 1$ that corresponds to μ =0.24 GeV. In general, $\alpha_s(\mu)$ is a free parameter in the CQL-model. The choice

$\alpha_s(\mu) = 1$ is explained by the fact that only for $\alpha_s(\mu) = 1$ one can agree the theoretical values of decay $K_{L,S} \to 2\gamma$ and $K \to 2\pi$ amplitudes with experimental ones.

The paper is organized as follows. In Sec.2 decay $K_{L,S} \to 2\gamma$ amplitudes are calculated. Section 3 is devoted to the calculation of decay $K \to 2\pi$ amplitudes that are due to $|\Delta I| = 3/2$ transitions. In Sec. 4 decay $K \to 2\pi$ amplitudes that are due to $|\Delta I| = 1/2$ transitions are calculated, In Sec. 5 we discuss the results obtained.

2. Decays $K_{L,S} \to 2\gamma$.

In the CQL-model decay $K_{L,S} \to 2\gamma$ amplitudes are defined by contact and pole diagrams in Fig. 1. Pole diagrams are due to the exchange of pseudoscalar mesons π^0, η, η' and scalar meson ε (700).

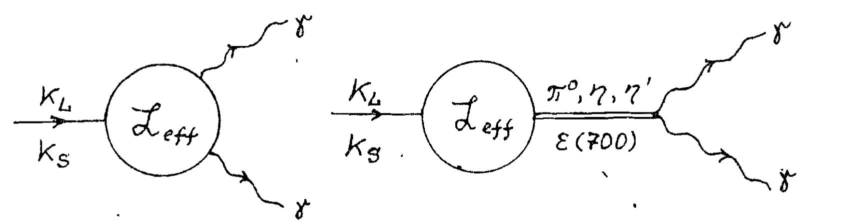
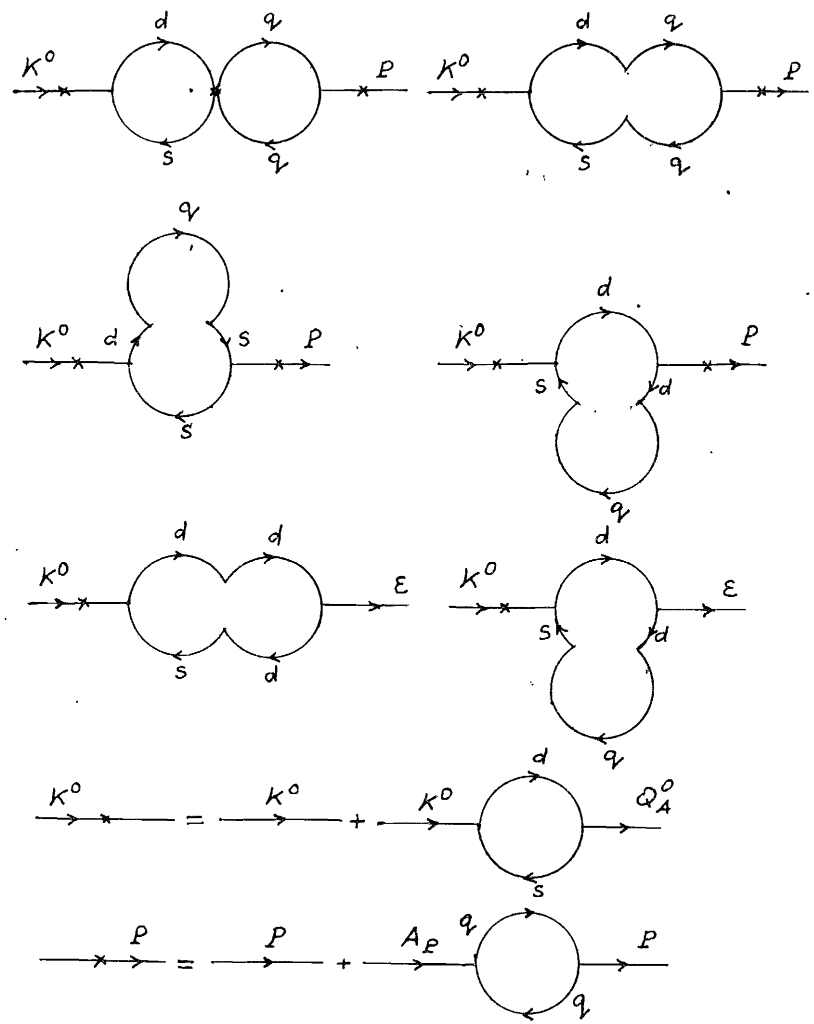


Fig.1. Contact and pole diagrams, defining decay $K_{L,S} \to 2\gamma$ amplitudes

Within the CQL-model accuracy (15÷20%) the contact-diagram contribution can be neglected with respect to the pole diagram one [7].

To calculate the pole diagram contribution , consider quark diagrams determining matrix elements of four-quark operators between states K^0 and X (where X= π^0, η, η' or ε(700)), (Fig.2).



<u>Fig.2.</u> Quark-diagrams, defining matrix elements of $K^0 \to P$ and $K^0 \to \varepsilon$ transitions, where $P = \pi^0$, η and η'.

Write down the result of the calculation:

$$\langle\pi^0|Q_2|K^0\rangle = -\langle\pi^0|Q_3|K^0\rangle = 1/3\langle\pi^0|Q_1|K^0\rangle = \sqrt{2}/3\, F_\pi F_K m_K^2 \equiv 1/3\, X,$$
$$\langle\pi^0|Q_5|K^0\rangle = 1/3\langle\pi^0|Q_6|K^0\rangle = 1/3\,\rho X;$$
$$\langle\eta|Q_2|K^0\rangle = 1/3\langle\eta|Q_1|K^0\rangle = 1/3\sin(\theta_0-\theta_P)X,$$

$$\langle\eta|Q_3|K^0\rangle = [(2+1/3)\sin(\theta_0-\theta_P) - \sqrt{2}(F_s/F_\pi)(1+1/3)\cos(\theta_0-\theta_P)]X,$$
$$\langle\eta|Q_5|K^0\rangle = [-(2+\rho/3)\sin(\theta_0-\theta_P) + \sqrt{2}(F_s/F_\pi)(1+\rho'/3)\cos(\theta_0-\theta_P)]X,$$
$$\langle\eta|Q_6|K^0\rangle = [-(2/3+\rho)\sin(\theta_0-\theta_P) + \sqrt{2}(F_s/F_\pi)(1/3+\rho')\cos(\theta_0-\theta_P)]X;$$
$$\langle\eta'|Q_2|K^0\rangle = 1/3\langle\eta'|Q_1|K^0\rangle = 1/3\cos(\theta_0-\theta_P)X,$$
$$\langle\eta'|Q_3|K^0\rangle = [(2+1/3)\cos(\theta_0-\theta_P) + \sqrt{2}(F_s/F_\pi)(1+1/3)\sin(\theta_0-\theta_P)]X,$$
$$\langle\eta'|Q_5|K^0\rangle = [-(2+\rho/3)\cos(\theta_0-\theta_P) - \sqrt{2}(F_s/F_\pi)(1+\rho'/3)\sin(\theta_0-\theta_P)]X,$$
$$\langle\eta'|Q_6|K^0\rangle = [-(2/3+\rho)\cos(\theta_0-\theta_P) - \sqrt{2}(F_s/F_\pi)(1/3+\rho')\sin(\theta_0-\theta_P)]X;$$
$$\langle\varepsilon|Q_1|K^0\rangle = \langle\varepsilon|Q_2|K^0\rangle = \langle\varepsilon|Q_3|K^0\rangle = 0,$$
$$\langle\varepsilon|Q_5|K^0\rangle = 1/3\langle\varepsilon|Q_6|K^0\rangle = 1/3\, i\rho'' X. \tag{5}$$

Here θ_P is a singlet-octet mixing angle of pseudoscalar mesons, $tg\theta_0 = 1/\sqrt{2}$, $F_\pi = 0.093\,GeV$, $F_K = 1.15F_\pi$ and $F_s = 1.27F_\pi$ are PCAC (Partial Conservation Axial Current) constants of π, K-mesons and a pseudoscalar state containing only strange quarks. In the QCL-model constants F_π, F_K and F_s are defined by the formulas [4]:

$$F_\pi = 2m_u[I_2(m_u,m_u)/Z]^{1/2} = 0.093\,GeV,$$
$$F_K = (m_u+m_s)[I_2(m_u,m_s)/Z]^{1/2} = \left(\frac{1+\lambda}{2}\right)\left[\frac{I_2(m_u,m_s)}{I_2(m_u,m_u)}\right]^{1/2} F_\pi = 1.15F_\pi,$$
$$F_s = 2m_s[I_2(m_s,m_s)/Z]^{1/2} = \lambda\left[\frac{I_2(m_s,m_s)}{I_2(m_u,m_u)}\right]^{1/2} F_\pi = 1.27F_\pi, \tag{6}$$

where $Z^{-1} = 0.71$ is the renormalization constant of 0^--meson wave-functions, that is due to nondiagonal $0^- \leftrightarrow 1^+$ transitions (1^+ is an axial meson). The constant Z is identical for all components of 0^--meson nonet [8] . The theoretical values of PCAC constants are in good agreement with experimental data: $F_\pi = (0.09324 \pm 0.0013)$ GeV and $F_K/F_\pi = 1.17 \pm 0.01$ [1,9] .

The parameters ρ, ρ' and ρ'' are defined by the expressions:

$$\rho = Z^2\frac{64(1+\lambda)m_u^2}{m_K^2}\cdot\frac{F_\pi^2}{F_K^2}\left\{1-\frac{\lambda}{2(1+\lambda)^2}\frac{F_K^2}{F_\pi^2}\left[1+\lambda\frac{I_1(m_s)}{I_1(m_u)}\right]\right\} = 51,$$
$$\rho' = Z^2\frac{64\lambda(1+\lambda)m_u^2}{m_K^2}\cdot\frac{F_\pi^4}{F_s^2F_K^2}\cdot\frac{I_1(m_s)}{I_1(m_u)}\left\{1-\frac{\lambda}{2(1+\lambda)^2}\frac{F_K^2}{F_\pi^2}\left[1+\frac{1}{\lambda}\frac{I_1(m_u)}{I_1(m_s)}\right]\right\} = 50,$$
$$\rho'' = Z^{3/2}\cdot\frac{64(1+\lambda)m_u^2}{m_K^2}\cdot\frac{F_\pi^2}{F_K^2} = 68. \tag{7}$$

Table 1.

Numerical values of matrix elements of Q_i-operators

Q_i	$K^0 \to \pi^0$	$K^0 \to \eta$	$K^0 \to \eta'$	$K^0 \to \varepsilon$
	in units by $\langle \pi^0 \vert Q_1 \vert K^0 \rangle = 3.5\times10^{-3}$ GeV^4			
Q_1	1.0	0.8	0.6	0
Q_2	0.3	0.3	0.2	0
Q_3	- 0.3	0.4	3.3	0
Q_5	17.0	3.8	-37.0	123
Q_6	51.0	13.0	-104.0	168

Numerical values of Q_i-operator matrix elements are presented in Table 1. Matrix elements of $K^0 \to \eta, \eta'$ transitions are calculated for $\theta_P = -21^0$ ((θ_P)$_{exp}$ $= -17.3 \pm 3.6^0$ [10]). It should be noted that matrix element of the $K^0 \to \eta$ transition strongly depends on the quark-mass difference and θ_P mixing angle. For example, for a fixed quark-mass difference the variation of θ_P by three degrees ($\theta_P = -18^0$) almost twice changes the matrix element of $K^0 \to \eta$ transition. The same strong variation takes place for a fixed mixing angle and a variation of quark-mass difference. Such a strong dependence can be explained by the fact that the contributions of quark-components $(\bar{u}u+\bar{d}d)/\sqrt{2}$ and $\bar{s}s$ are subtracted in the $K^0 \to \eta$ transition matrix element. The contributions of these quark components are of the same order, that is why, a slight variation of one of them leads to a significant variation of their difference. In the $K^0 \to \eta'$ transition matrix element the contributions of quark-components $(\bar{u}u+\bar{d}d)/\sqrt{2}$ and $\bar{s}s$ are added so, the influence of their variation is not so essential.

The choice $\theta_P = -21^0$ is agreed with experimental data on two-photon decays of η, η'-mesons. For $\theta_P = -21^0$ partial widths of decays $\eta \to 2\gamma$ and $\eta' \to 2\gamma$, calculated in the CQL-model, are in good agreement with experimental data:

$$\Gamma(\eta\to 2\gamma) = \left(\frac{\alpha}{24F_\pi}\right)^2\left[5\sin(\theta_0-\theta_P)-\sqrt{2}\frac{F_\pi}{F_s}\cos(\theta_0-\theta_P)\right]^2\left(\frac{m_\eta}{\pi}\right)^3 = 0.71\ keV,$$

$$\Gamma(\eta\to 2\gamma)_{exp} = (0.56 \pm 0.12 \pm 0.10)\ keV\ [10],$$

$$\Gamma(\eta'\to 2\gamma) = \left(\frac{\alpha}{24F_\pi}\right)^2\left[5\cos(\theta_0-\theta_P)+\sqrt{2}\frac{F_\pi}{F_s}\sin(\theta_0-\theta_P)\right]^2\left(\frac{m_{\eta'}}{\pi}\right)^3 = 4.2\ keV,$$

$$\Gamma(\eta'\to 2\gamma)_{exp} = (4.16\pm0.43)\ keV\ [11],\ (4.42 \pm 0.34)\ keV\ [12]. \tag{8}$$

Let us write down analytical expressions for decay $K_{L,S}\to 2\gamma$ amplitudes:

$$A(K_L\to 2\gamma) = -\frac{\alpha}{\pi F_\pi}(G_F s_1 c_1 c_3)\left\{\frac{1}{m_K^2-m_{\pi^0}^2}\langle\pi^0|Q_{|\Delta S|=1}|K^0\rangle + \right.$$
$$+\frac{1}{3}\left[5\sin(\theta_0-\theta_P)-\sqrt{2}\frac{F_\pi}{F_s}\cos(\theta_0-\theta_P)\right]\frac{1}{m_K^2-m_\eta^2}\langle\eta|Q_{|\Delta S|=1}|K^0\rangle +$$
$$\left.+\frac{1}{3}\left[5\cos(\theta_0-\theta_P)+\sqrt{2}\frac{F_\pi}{F_s}\sin(\theta_0-\theta_P)\right]\frac{1}{m_K^2-m_{\eta'}^2}\langle\eta'|Q_{|\Delta S|=1}|K^0\rangle\right\} =$$
$$= 3.3\times10^{-9}\ GeV^{-1}, \tag{9}$$

$$A(K_S\to 2\gamma) = \frac{10}{9}\frac{\alpha}{\pi F_\pi Z^{1/2}}(G_F s_1 c_1 c_3)\frac{i\langle\varepsilon|Q_{|\Delta S|=1}|K^0\rangle}{m_\varepsilon^2-m_K^2}\cdot \exp i\delta_\varepsilon(m_K)\cdot$$
$$\cdot\cos\delta_\varepsilon(m_K) = 4.0\times10^{-9}\times\cos\delta_\varepsilon(m_K)\times\exp i\delta_\varepsilon(m_K)\ GeV^{-1}. \tag{10}$$

Here $Q_{|\Delta S|=1} = \sum C_i Q_i$, $m_\varepsilon = 0.73\ GeV$ is the mass of ε-meson, $\delta_\varepsilon(m_K) = arctg[m_K\Gamma_\varepsilon(m_K)/(m_\varepsilon^2-m_K^2)] = 61.4^0$ and $\Gamma_\varepsilon(m_K) = 1\ GeV$ is the partial width of decay $\varepsilon \to 2\pi$ of the virtual ε-meson with energy m_K [4,13] 2). The theoretical value of $A(K_L\to 2\gamma)$ is in good agreement with the experimental one:

$$|A(K_L\to 2\gamma)|_{exp} = (3.18\pm0.14)\times10^{-9}\ GeV; \tag{11}$$

the value of $A(K_S\to 2\gamma)_{th}$ satisfies the experimental constraint: $|A(K_S\to 2\gamma)|_{exp} < 7\times10^{-8}\ GeV\ [1]$.

2) $\Gamma_\varepsilon(m_K) = (3m_u^2 g/2\pi m_K)F_{\varepsilon\pi\pi}^2(m_K)(1-4m_\pi^2/m_K^2)^{1/2} = 1\ GeV$, where $4m_u g = 4m_u^2 Z^{1/2}/F_\pi = 4\ GeV$ and $F_{\varepsilon\pi\pi}(m_K) = 1+(m_\varepsilon^2-m_K^2)/(4\pi F_\pi)^2 Z = 1.15$ are respectively the coupling constant and form factor of decay $\varepsilon \to 2\pi$ of the virtual ε-meson with energy m_K [4, 13].

3. $|\Delta I| = 3/2$ transitions in decays $K \to 2\pi$.

The effective Lagrangian, satisfying the selection rules $|\Delta S| = 1$ and $|\Delta I| = 3/2$, takes the form [13] :

$$\mathcal{L}_{eff}^{|\Delta I|=3/2} = \frac{G_F}{\sqrt{2}} s_1 c_1 c_3 \times 0.2 \times Q_{|\Delta I|=3/2} \quad (12)$$

An amplitude $A^{3/2}(K \to 2\pi)$ is proportional to a matrix element $\langle 2\pi | Q_{|\Delta I|=3/2} | K \rangle$:

$$A^{3/2}(K \to 2\pi) = 3.54 \times 10^{-7} \langle 2\pi | Q_{|\Delta I|=3/2} | K \rangle \, (GeV)^{-2}. \quad (13)$$

In the CQL-model matrix elements $\langle 2\pi | Q_{|\Delta I|=3/2} | K \rangle$ are defined by contact quark diagrams in fig.3.

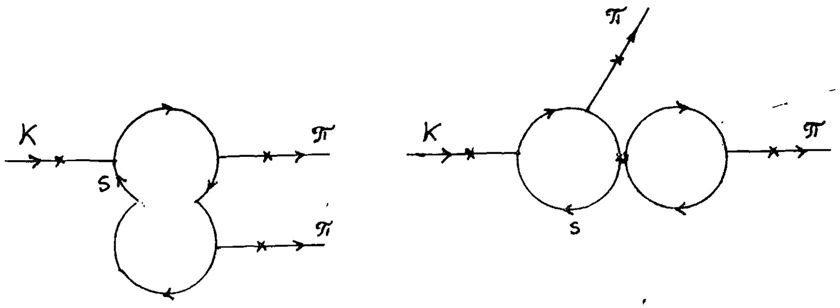


Fig.3. Contact quark diagrams, defining matrix elements $\langle 2\pi | Q_{|\Delta I|=3/2} | K \rangle$.

As a result of the calculation we get

$$\langle \pi^+\pi^- | Q_{|\Delta I|=3/2} | K^0 \rangle = -1/2 \langle \pi^0\pi^0 | Q_{|\Delta I|=3/2} | K^0 \rangle =$$
$$= \sqrt{2}/3 \langle \pi^+\pi^0 | Q_{|\Delta I|=3/2} | K^+ \rangle, \quad (14)$$

where

$$\langle \pi^+\pi^0 | Q_{|\Delta I|=3/2} | K^+ \rangle = -i \frac{8}{1+\lambda} F_K m_K^2 \left[1 - \left(\frac{1+\lambda}{2}\right)^2 (F_\pi^2 m_\pi^2 / F_K^2 m_K^2) - (Z-1)\left(\frac{1+\lambda}{4}\right)(1 - F_\pi^2/F_K^2)\right] = -i \times 6.55 \times 10^{-2} (GeV)^3. \quad (15)$$

Numerical values of amplitudes and partial widths of decays $K \to 2\pi$ that are due to $|\Delta I| = 3/2$ transitions are presented in Table 2.

Table 2.

Numerical values of amplitudes and partial widths of decays $K \to 2\pi$.

Decay	THEORY				EXPERIMENT	
	Δ I=3/2 transition		Δ I=1/2 transition			
	A	Γ	A	Γ	A	Γ
$K^+ \to \pi^+ \pi^0$	2.33	1.80	0	0	1.84	1.13
$K^0 \to \pi^+ \pi^-$	1.10	0.40	30.0	297	27.7	253
$K^0 \to \pi^0 \pi^0$	2.20	0.80	30.0	149	26.3	116

Here A is the absolute value of decay $K \to 2\pi$ amplitude in units 10^{-8} GeV, and Γ is a partial decay $K \to 2\pi$ width in units 10^{-17} GeV.

4. $|\Delta I| = 1/2$ transitions in decays $K \to 2\pi$.

$|\Delta I| = 1/2$ transitions take place in decays $K^0 \to \pi^+ \pi^-$ and $K^0 \to \pi^0 \pi^0$. The effective Lagrangian of weak interactions describing $|\Delta I| = 1/2$ transitions, can be obtained from (2) by subtracting (12):

$$\mathcal{L}_{eff}^{|\Delta I|=1/2} = \mathcal{L}_{eff} - \mathcal{L}_{eff}^{|\Delta I|=3/2} = \frac{G_F}{\sqrt{2}} s_1 c_1 c_3 \times Q_{|\Delta I|=1/2} . \quad (16)$$

The decay $K^0 \to 2\pi$ amplitudes are defined by contact and pole diagrams. The main contribution comes from the pole diagram with the scalar $\varepsilon(700)$-meson exchange. Within the accuracy of CQL model the contribution of contact diagrams and pole diagrams with other resonance exchange can be neglected as compared to the ε-meson one.

The matrix element $\langle \varepsilon | Q_{|\Delta I|=1/2} | K^0 \rangle$ is connected with the matrix element $\langle \varepsilon | Q_6 | K^0 \rangle$ by the equality:

$$\langle \varepsilon | Q_{|\Delta I|=1/2} | K^0 \rangle = -0.09 \langle \varepsilon | Q_6 | K^0 \rangle = -i \times 2.2 \times 10^{-2} (GeV)^4. \quad (17)$$

Decay $K^0 \to 2\pi$ amplitudes that are due to $|\Delta I| = 1/2$ transitions take the form [13] :

$$A^{1/2}(K^0\to\pi^+\pi^-)=A^{1/2}(K^0\to\pi^0\pi^0)=\frac{G_F}{\sqrt{2}}s_1c_1c_3\cdot\frac{4m_u\,g\,F_{\varepsilon\pi\pi}(m_K)}{m_\varepsilon^2-m_K^2}$$

$$\cdot\exp i\delta_\varepsilon(m_K)\cdot\cos\delta_\varepsilon(m_K)\cdot\langle\varepsilon|Q_{|\Delta I|=1/2}|K^0\rangle=$$

$$=-i\times 3.0\times 10^{-7}\times\exp i\delta_{1/2}\ (\mathrm{GeV}), \qquad (18)$$

where $\delta_{1/2}=\delta_\varepsilon(m_K)=61.4^0$ is the amplitude $A^{1/2}(K^0\to 2\pi)$ phase.

In the standard parametrization amplitude decay $K\to 2\pi$ phases are parametrized by two phases δ_0 and δ_2 [14] that are determined by strong $\pi\pi$ -interaction in states with I=0 and I=2, respectively. The phase $\delta_{1/2}$ of the amplitude $A^{1/2}(K^0\to 2\pi)$ should be compared with the phase δ_0. However, there are experimental data only for the quantity: $(\delta_0-\delta_2)_{exp}=56.5\pm 3.0^0$ [15] that is extracted from experimental data on decays $K\to 2\pi$. That is why, taking into account that a value of δ_2 is small as compared to δ_0, it is possible to compare $\delta_{1/2}$ with $(\delta_0-\delta_2)$. It is easy to see that the theoretical value $\delta_{1/2}=61.4^0$ agrees with the experimental one.

Numerical values of amplitudes and partial widths of decays $K\to 2\pi$, that are due to $|\Delta I|=1/2$ transitions are presented in Table 2.

5. Discussion

The obtained theoretical values of decay $K_{L,S}\to 2\gamma$ and $K\to 2\pi$ amplitudes confirm the phenomenological rule $|\Delta I|=1/2$. The account of the QCD-interaction in effective Lagrangian (2) plays an essential role for strengthening $|\Delta I|=1/2$ transitions. The main contribution to matrix elements of $K^0\to X$ transitions, where $X=\pi^0,\eta,\eta'$ or $\varepsilon(700)$, comes from Q_6-operator matrix elements. The appearance of the Q_6-operator in effective Lagrangian (2) is due to a diagram of the "Penguin" type defined by the W-boson and the gluon exchange [16].

In the decays $K^0\to 2\pi$ the strengthening of $|\Delta I|=1/2$ transitions is due to the scalar meson $\varepsilon(700)$ exchange. The dominance of ε-meson is not surprised. In the CQL-model the ε-meson exchange plays an important role for describing many decays (for example, $\eta'\to\eta\pi\pi$, $\eta(\eta')\to 3\pi$) and such important low-energy characteristics as scattering lengths and polarizabilities [17].

It should be emphasized that matrix elements of four-quark operators calculated in the CQL-model do not contain new low-energy parameters. For describing strong low-energy interactions in kaon decays sufficient are three parameters: Λ =1.25 GeV, $m_d=m_u$=0.28 GeV and m_s=0.46 GeV. In our calculation the sole free parameter is a normalization point μ, or accordingly $\alpha_s(\mu)$. The appearance of a normalization point is due to the account of QCD-interactions for obtaining the effective Lagrangian of weak interactions. We choose $\alpha_s(\mu)=1$ which corresponds to μ=0.24 GeV. In this case theoretical values of kaon decay amplitudes can be agreed with experimental values within 30% accuracy.

In conclusion let us discuss decays $K\to 3\pi$. Since the energy-release of decays $K\to 3\pi$ is sufficiently small (of an order of 0.025 GeV per one decay particle), the soft-pion approach (low-energy limit) is a good approximation for their description. In the low-energy limit decay $K\to 3\pi$ amplitudes can be connected with decay $K\to 2\pi$ amplitudes [18] : $|A(K^+\to 2\pi^+\pi^-)|=(1/\sqrt{2}F_\pi)|A(K^0\to\pi^+\pi^-)|$, $|A(K^+\to 2\pi^0\pi^+)|=(\sqrt{2}/F_\pi)|A(K^0\to\pi^0\pi^0)|$ etc. In this approximation one obtains

$$\frac{\Gamma(K^+\to 2\pi^+\pi^-)}{\Gamma(K^0\to\pi^+\pi^-)}=\frac{1}{64\pi^2F_\pi^2m_K}\cdot\frac{1}{(m_K^2-4m_\pi^2)^{1/2}}\int_{4m_\pi^2}^{(m_K-m_\pi)^2}\frac{ds}{s^{1/2}}(s-4m_\pi^2)^{1/2}\cdot$$

$$\cdot[(m_K-m_\pi)^2-s]^{1/2}[(m_K+m_\pi)^2-s]^{1/2}=1.35\times 10^{-3},$$

$$\Gamma(K^+\to 2\pi^+\pi^-)=4\Gamma(K^+\to 2\pi^0\pi^+)=4.0\times 10^{-18}\ \mathrm{GeV}. \qquad (19)$$

The theoretical values of decay $K\to 3\pi$ amplitudes agreed with experimental values: $\Gamma(K^+\to 2\pi^+\pi^-)=(2.98\pm 0.02)\times 10^{-18}$ GeV and $\Gamma(K^+\to 2\pi^0\pi^+)=(0.92\pm 0.04)\times 10^{-18}$ GeV [1].

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Волков М.К., Иванов А.Н., Троицкая Н.И. E2-86-414
Киральная модель кварковых петель
и правило $\Delta I = 1/2$ в распадах каонов

Рассмотрены слабые распады каонов: $K \to 2\gamma$, $K \to 2\pi$ и $K \to 3\pi$. Дано теоретическое подтверждение феноменологического правила $\Delta I = 1/2$. Для описания слабых взаимодействий использован эффективный лагранжиан, найденный в стандартной модели Кобаяши - Маскавы с учетом КХД-взаимодействия. Слабые вершины имеют вид четырехкварковых операторов. Низко-энергетические матричные элементы четырехкварковых операторов вычислены в киральной модели кварковых петель /КМКП/. В КМКП амплитуды распадов определены контактными и полюсными диаграммами. Последние обусловлены обменом псевдоскалярными мезонами π^0, η и η' и скалярным мезоном ϵ /700/. В пределах точности модели /15÷20%/ вкладом контактных диаграмм можно пренебречь по сравнению с вкладом полюсных диаграмм. Основной вклад в низкоэнергетические матричные элементы переходов $K^0 \to X(X = \pi^0, \eta, \eta'$ или ϵ (700)) дают матричные элементы Пингвин-оператора Q_6. Теоретические значения амплитуд распадов $K \to 2\gamma$, $K \to 2\pi$ и $K \to 3\pi$ согласуются с экспериментальными с точностью до 30%.

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Volkov M.K., Ivanov A.N., Troitskaya N.I. E2-86-414
The Chiral Quark-Loop Model and the $\Delta I = 1/2$ Rule in Kaon Decays

$K \to 2\gamma$, $K \to 2\pi$ and $K \to 3\pi$ kaon decays are considered. A theoretical groundwork is given for the phenomenological $(\Delta I) = 1/2$ rule. For describing weak interaction the effective Lagrangian has been obtained in the standard Kobayashi-Maskawa model with the account of the QCD-interaction. Weak vertices take the form of four-quark operators. Low-energy four-quark matrix elements are calculated in the Chiral Quark-Loop Model (the CQL-model). In the CQL-model decay amplitudes of kaons are defined by contact and pole diagrams with exchange of pseudoscalar π^0, η, η' -mesons and scalar ϵ (700)-meson. The contribution of contact diagrams, as compared to pole diagrams, can be neglected within the CGL-model accuracy (15-20%). The main contribution to low-energy K^0-X transition matrix elements ($X = \pi^0$, η, η' and ϵ) comes from Penguin operator matrix elements. Theoretical values of decay $K \to 2\gamma$, $K \to 2\pi$ and $K \to 3\pi$ amplitudes are consistent with experimental data within 30% accuracy.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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