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CONSIDERATION OF THE VACUUM OF QCD IN A COMPOSITE QUARK MODEL. Strange Hadrons

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1. Introduction

There are three basic approaches to the problem of mass splitting within the hadron multiplets. First, the nonrelativistic quark model where splitting is defined by the mass differences of composed quarks and quark mass dependence of Breit-Fermi potential /1/. Second, different versions of the bag model MIT /2/ explain this effect by the differences of current quark masses and dependence of the gluon exchange on quark flavours. The third one, most constructive, is based on using the QCD sum rules /3/. Here, the basic contribution to the mass splitting is given by the quark interaction with the QCD vacuum.

This paper is the continuation of the work /4/, where a new quark model of hadrons has been proposed. The model unified the most attractive features of these above-mentioned approaches. Namely it took into account both the confinement specific of composite models and the quark interaction with the QCD vacuum condencates dominating in the QCD sum rule method. The mass of hadrons containing massless u- and d - quarks was calculated in ref. /4/. In the present paper, the results of /4/ are generalized to the case of hadrons with nonzeros strangeness.

2. Mass formula

In the model /4/ the hadrons mass is defined as

$$M^2 = E^2 - \langle P^2 \rangle , \tag{1}$$

where E is the bag energy, $\langle P^2 \rangle$ is the contribution due to the center of mass motion. The bag energy

$$E = E_{kin} + \Delta E_g + \Delta E_{inst} + \Delta E_{vac} \tag{2}$$

consists of the quark kinetic energy E_{kin}, the one-gluon exchange potential ΔE_g, the quark interaction caused by instantons ΔE_{inst} and the interaction of quarks with long-wave functions ΔE_{vac}. The kinetic energy and the contributions due to the one-gluon exchange are calculated by using the standard MIT cavity perturbation theory /2,5/:

$$E_{kin} = N_0\,\varkappa_0/R + N_s\left[\varkappa_s^2 + (m_s R)^2\right]^{1/2}/R\,, \qquad (3)$$

$$\Delta E_g = \frac{0{,}35\,\alpha_s}{3R}\left[M_{00} + (1-0{,}13\,m_s R)M_{0s} + (1-0{,}25\,m_s R)M_{ss}\right]. \qquad (4)$$

In expressions (3), (4) N_0 is the number of light $(u\text{-}, d\text{-})$ quarks in hadrons, N_s is that of strange quarks, $\varkappa_0$ and $\varkappa_s$ are defined for $1S$ state from the solutions of the equation

$$\left[1 - mR - (\varkappa^2 + (mR)^2)^{1/2}\right] \operatorname{tg}\varkappa = \varkappa \qquad (5)$$

at $m=0$ and $m=m_s$ respectively, R is the bag radius, α_s is the quark-gluon coupling and the values of the matrix elements M are contained in Table I

Table I. Matrix elements of the one-gluon exchange

hadron	π	K	η	η'	ρ,ω	K^*	Φ	N	Λ	Σ	Ξ	Δ	Σ^*	Ξ^*	Ω
M_{00}	-6	0	-2	-4	2	0	0	-3	-3	1	0	3	1	0	0
M_{0s}	0	-6	0	0	0	2	0	0	0	-4	-4	0	2	2	0
M_{ss}	0	0	-4	-2	0	0	2	0	0	0	1	0	0	1	3

The inclusion of the quark interaction with the quark and gluon condensates is the most important new feature of the model /4/ in contrast with the bag model MIT /2/. Within the model /4/ the energy of interaction with the low-frequency component of vacuum fields is obtained by perturbation theory

$$\Delta E_{vac} = \langle\Phi|H_I|\Phi\rangle_{con}\,,\quad |\Psi\rangle = U(-\infty,0)\,|\Phi\rangle\,,$$

$$U(-\infty,0) = \sum_{n=0}^{\infty}\frac{(-i)^n}{n!}\int_{-\infty}^{0}dt_1\ldots\int_{-\infty}^{t_{n-1}}dt_n\,T\left[H_I(t_1)\ldots H_I(t_n)\right] \qquad (6)$$

with the interaction hamiltonian /4/

$$H_I = \frac{\omega}{2}(\bar{Q}\gamma^0 q + \bar{q}\gamma^0 Q) + \frac{i}{2}(\bar{q}\gamma^0\partial_0 Q - \partial_0\bar{Q}\gamma^0 q) -$$
$$-\frac{g}{4}(\bar{Q}\gamma^\mu\lambda^a q + \bar{q}\gamma^\mu\lambda^a Q)A_\mu^a - \frac{g}{2}\bar{q}\gamma^\mu\lambda^a q\,A_\mu^a\,, \qquad (7)$$

where ω is the one-particle quark energy $\omega = (\varkappa^2/R^2 + m^2)^{1/2}$, q, Q are the valence and vacuum quark wave functions, respectively, A_μ^a is the vacuum gluon field.

Evaluating (6) with the wave functions of mass quarks /2/

$$q(x) = \frac{\bar{N}_0}{\sqrt{4\pi}}\begin{pmatrix}\left(\frac{\omega+m}{\omega}\right)^{1/2} i\,j_0(\varkappa\frac{r}{R})\,U \\ \left(\frac{\omega-m}{\omega}\right)^{1/2}(\vec{\sigma}\hat{r})\,j_1(\varkappa\frac{r}{R})\,U\end{pmatrix} \qquad (8)$$

when U is the Dirac spinor,

$$\bar{N}_0^{-2} = R^3 j_0(\varkappa)\left[2\omega(\omega - \tfrac{1}{R}) + \tfrac{m}{R}\right]/\left[\omega(\omega-m)\right]$$

and preserving the contributions of only quark condensates /4/ we obtain:

$$\Delta E_{vac} = -N_0\frac{\pi}{24}\frac{\langle 0|\bar{u}u|0\rangle}{\varkappa_0 - 1}R^2 - N_s\frac{\pi}{12}\frac{\langle 0|\bar{s}s|0\rangle R^3 (y+a)^2 y}{\varkappa_s^2\left[2y(y-1)+a\right]} +$$
$$+\frac{\pi^2\langle 0|\bar{u}u|0\rangle^2 R^5}{1152\,\varkappa_0(\varkappa_0-1)^2}\left\{\tilde{M}_{00} + \frac{4y(y+a)^4\varkappa_0(\varkappa_0-1)^2}{\left[2y(y-1)+a\right]^2\varkappa^4}\left(\frac{\langle 0|\bar{s}s|0\rangle}{\langle 0|\bar{u}u|0\rangle}\right)^2\tilde{M}_{ss} +\right.$$
$$\left.+\frac{(\varkappa_0+y)(y+a)^2(\varkappa_0-1)}{\varkappa_s^2\left[2y(y-1)+a\right]}\frac{\langle 0|\bar{s}s|0\rangle}{\langle 0|\bar{u}u|0\rangle}\tilde{M}_{s0}\right\}. \qquad (9)$$

In expression (9) $\langle 0|\bar{u}u|0\rangle=\langle 0|\bar{d}d|0\rangle$ are the condensates of light u-, d- quarks, $\langle 0|\bar{s}s|0\rangle$ is the strange quark condensate, $y=\omega R$, $a=m_s R$ and the values of the coefficients M_{ij} are listed in Table 2.

Table 2. Coefficients $\tilde{M}_{ij}$

hadron	π	K	η	η'	ρ,ω	K^*	ϕ	N	Λ	Σ	Ξ	Δ	Σ^*	Ξ^*	Ω^-
$\tilde{M}_{00}$	4	0	4/3	8/3	4	0	0	12	4	4	0	12	4	0	0
$\tilde{M}_{0s}$	0	4	0	0	0	4	0	0	8	8	8	0	8	8	0
$\tilde{M}_{ss}$	0	0	8/3	4/3	0	0	4	0	0	0	4	0	0	4	12

The mass formula (1) contains also the interaction energy of valence quarks with the high-frequency component of vacuum fluctuations - instantons /4/

$$\Delta E_{inst}=-\langle\Phi|\mathcal{L}_{inst}|\Phi\rangle . \qquad (10)$$

Under the factorization supposition $\mathcal{L}_{inst}$ was shown in /6/ to be of the form

$$\mathcal{L}_{inst}=\sum_{i\neq j}^{u,d,s}\eta_{ij}\Big[\bar{q}_{iR}q_{iL}\bar{q}_{jR}q_{jL}+\frac{3}{32}\big(\bar{q}_{iR}\lambda^a q_{iL}\bar{q}_{jR}\lambda^a q_{jL}-\frac{3}{4}\bar{q}_{iR}\sigma_{\mu\nu}\lambda^a q_{iL}\bar{q}_{jR}\sigma^{\mu\nu}\lambda^a q_{jL}\big)\Big], \qquad (11)$$

where $q_{R,L}=\frac{1}{2}(1\mp\gamma_5)q$, $\eta_{ij}=\frac{4}{3}\pi^2\rho_c^2\, m^{*2}/(m_i^* m_j^*)$,

$$m^*=\frac{2\pi^2}{3}\langle 0|\bar{u}u|0\rangle\rho_c^2 , \quad m_i^*=m_i+m^* ,$$

ρ_c is the characteristic dimension of instantons in the QCD vacuum. The matrix elements (10) were evaluated in /7/. Their values in dependence on hadron states are equal to

$$\Delta E_\pi=-\frac{\lambda_0}{R^3} , \quad \Delta E_K=-\frac{\lambda_s}{R^3} , \quad \Delta E_\eta=\frac{\lambda_0-4\lambda_s}{3R^3} ,$$

$$\Delta E_\rho=\Delta E_\omega=\Delta E_{K^*}=\Delta E_\phi=0 , \quad \Delta E_{\eta'}=\frac{2}{3}\frac{\lambda_0+2\lambda_s}{R^3} ,$$

$$\Delta E_N=-\frac{3}{4}\frac{\lambda_0}{R^3} , \quad \Delta E_\Lambda=-\frac{\lambda_s+2\lambda_0}{4R^3} , \quad \Delta E_\Sigma=\Delta E_\Xi=-\frac{3}{4}\frac{\lambda_s}{R^3} , \qquad (12)$$

$$\Delta E_\Delta=\Delta E_{\Sigma^*}=\Delta E_{\Xi^*}=0 ,$$

where $\lambda_0=\pi\bar{N}_0^4 R^6\rho_c^2 I_0$, $\lambda_s=\lambda_0 m^* I_s/(m_s^* I_0)$,

$$I_0=\int_0^1 dx\, x^2\left[j_0^2(\varkappa_0 x)+j_1^2(\varkappa_0 x)\right]^2 , \qquad (13)$$

$$I_s=\frac{\bar{N}_s^2}{\bar{N}_0^2}\int_0^1 dx\, x^2\left[j_0^2(\varkappa_0 x)+j_1^2(\varkappa_0 x)\right]\times$$

$$\times\left[j_0^2(\varkappa_s x)\left(1+\frac{m_s}{\omega}\right)+j_1^2(\varkappa_s x)\left(1-\frac{m_s}{\omega}\right)\right],$$

$\bar{N}_{0,s}$ are the normalizations of the wave functions of massless and strange quarks respectively. From (12) and (13) it follows that the contribution of the interaction induced by instantons decreases with increasing quark mass /6,7/. It should be noted that this conclusion disagrees with the results of ref. /8/ where the isotopic differences of baryon masses have been considered.

The last term in (1) takes into account the center of masses of quark motion and is approximated by the expressions /9/:

$$\langle P^2\rangle\approx N_0\left(\frac{\varkappa_0}{R}\right)^2+N_s\left(\frac{\varkappa_s}{R}\right)^2 . \qquad (14)$$

As usual /2/, the bag radius is defined by the balance condition:

$$\frac{dM^2}{dR}=0 . \qquad (15)$$

The masses of hadrons containing u-, d-, s -quarks in $1S$ state were evaluated by formula (1) - (15). By analysing the mass spectrum the value of a strange quark mass is

$$m_s \simeq 220 \pm 30 \text{ MeV} . \qquad (16)$$

The values of other parameters were chosen as in /4/

$$\alpha_s = 0{,}7 \ , \ \rho_c = 2\ GeV^{-1} , \ \langle 0|\bar{u}u|0\rangle = \langle 0|\bar{s}s|0\rangle = -(250 MeV)^3 .$$

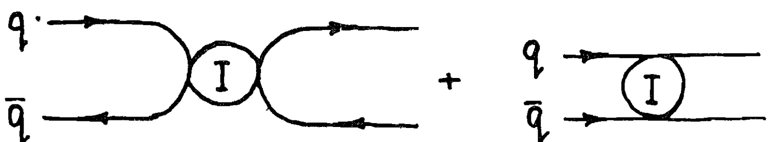
The results of calculations (with m_s = 220 MeV) are listed in Table 3.

As we see, the agreement with experiment is satisfactory. The discrepancy is shown only for η' -meson. In the next section we shall discuss the mechanism removing these defects.

3. Mixing pseudoscalar mesons

In calculating masses of the ground meson states there was supposed that the vector mesons are ideal mixed unitary states: $\Phi = \bar{s}s$, $\omega = \frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d)$. In contrast with the pseudoscalar sector, mixing of unitary octet with singlet was not taken into account. This is valid if in the vector channel the Okubo-Zweig-Iizuka (OZI) rule is fulfilled that forbids the transitions between the states with different quark flavours.

In /10,11/ the instanton mechanism of mixing was proposed and its self-consistence was proved. Accordingly, the diagrams contributive to mixing are



a) b)

Fig. I. Mixing diagrams: a) annihilation, b) scattering, I - instanton (anti-instanton).

Such contributions are equal to zero for vector mesons and the OZI rule is fulfilled in this channel. In the pseudoscalar channel these diagrams are nonvanishing (12-13) and cause π^0-η-η' mixing.

Table 3. Masses and radii of hadrons: M_0 is the hadron mass without instantons, ΔM_{inst} is the instanton correction.

hadron	M_0 (MeV)	ΔM_{inst} (MeV)	M_{theor} (MeV)	M_{exp} (MeV)	R(GeV^{-1})
π	591	-471	120	140	5.44
K	758	-310	448	498	5.08
η	805	-252	553	550	5.08
η'	699	540	1239	960	5.20
N	1133	-180	953	940	5.80
Λ	1298	-181	1117	1116	5.56
Σ	1309	-205	1169	1192	5.56
Ξ	1448	-160	1288	1315	5.32
ρ, ω	736	0	736	783 770	6.16
K^*	890	0	890	896	5.68
ϕ	1022	0	1022	1020	5.40
Δ	1228	0	1228	1236	6.04
Σ^*	1393	0	1393	1385	5.68
Ξ^*	1535	0	1535	1532	5.56
Ω^-	1665	0	1665	1672	5.44

First, consider η-η' mixing. In the basis

$$\eta_1=\frac{1}{\sqrt{3}}(\bar{u}u+\bar{d}d+\bar{s}s)\ ,\quad \eta_8=\frac{1}{\sqrt{6}}(\bar{u}u+\bar{d}d-2\bar{s}s)$$

the mass formula is

$$M=\begin{pmatrix} M_1 & M_{18} \\ M_{81} & M_8 \end{pmatrix}, \tag{17}$$

where M_1 and M_8 are the masses of η_1 and η_8, $M_{18}=\frac{\sqrt{2}}{3}(M_{\bar{u}u}-M_{\bar{s}s}+(\lambda_0-\lambda_s)/\bar{R}^3)$, $M_{\bar{u}u,\bar{s}s}$ are the masses of particles consisting of $u(\bar{u})$ or $s(\bar{s})$ quarks (antiquarks), $\bar{R}$ is the mean radius of η_1 and η_8.

By diagonalizing (17) with the help of transformation

$$\eta=\eta_8\cos\theta_p+\eta_1\sin\theta_p\ ,\quad \eta'=-\eta_8\sin\theta_p+\eta_1\cos\theta_p$$

we have for the singlet-octet mixing angle

$$tg\,2\theta_p=\frac{2M_{18}}{M_8-M_1}. \tag{18}$$

Note, as follows from our results, in the case of π-, η'-mesons the use of perturbation theory becomes unjustified, (Corrections become large). That is why for the mixing angle estimations we shall consider the instanton dimension ρ_c as an effective parameter which takes into account all orders of perturbation theory. In other words, for π-, η'-mesons the instanton energy contribution (12) enters into the minimization condition (15). The π-meson becomes massless, as is expected at $m_u=m_d=0$, and mass and radius of η'-meson are obtained

$$m_{\eta'}=m_1=996\ MeV\ ,\quad R_1=7{,}24\ GeV^{-1}. \tag{19}$$

Substituting (19) and the values $M_{\bar{u}u}$ = 591 MeV, $M_{\bar{s}s}$ = 907 MeV, R_8 = 5 GeV^{-1} obtained from (1-15) into formula (18), we find

$$\theta_p=14^\circ\ ,\quad f_{\eta'}/f_\eta=\frac{R_8}{R_1}\simeq 0{,}7\ , \tag{20}$$

where the relation /12/ of the decay coupling $f_{\eta,\eta'}$ with radii was used. The estimations (20) obtained describe well the experimental situation /13/.

Analogously, we can evaluate π^0-η-η' mixing that is due to mass difference $\Delta m=m_d-m_u$ /14,15/. Expanding (1) over u-, d- quark masses, we find the nondiagonal matrix elements

$$M_{\pi^0\eta}=-0{,}52\,\Delta m\ ,\quad M_{\pi^0\eta'}=-1{,}02\,\Delta m.$$

With Δm = 3.80 $\pm$ 1.57 MeV /16/ and θ_p =14^0 we have

$$M_{\pi^0\eta}=-4.24\pm1.75\ \text{MeV},\quad M_{\pi^0\eta'}=-0.98\pm0.40\ \text{MeV},$$

or for the mixing angles

$$\theta_{\pi^0\eta}=(-1.03\pm0.42)\cdot10^{-2}\ ,\quad \theta_{\pi^0\eta'}=(-0.12\pm0.05)\cdot10^{-2}. \tag{21}$$

This result (21) should be compared with that obtained by using PCAC /10,14/:

$$\theta_{\pi^0\eta}=-1.3\cdot10^{-2}\ ,\quad \theta_{\pi^0\eta'}=-0.1\cdot10^{-2}.$$

Conclusion

The model proposed in /4/ describes well both the mass differences within $SU_f(3)$ multiplets and mixing between them. These effects are due to dependence of the kinetic energy, one-gluon exchange energy, the interaction with long-wave vacuum fluctuations and the instanton potential on quark masses. The value obtained for the strange quark mass is in agreement with the results of current algebra /16/. At the same time in all versions of the MIT model /2/ the strange quark mass is predicted large.

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Дорохов А.Е., Кочелев Н.И. E2-86-355

Учет структуры вакуума КХД в составной кварковой модели. Странные адроны

Метод учета вакуумных конденсатов КХД в рамках составной кварковой модели обобщается на случай адронов, содержащих странные кварки. Выведена массовая формула для адронов. Из анализа спектра низших адронных состояний определена масса странного кварка. Вычислены углы смешивания псевдоскалярных мезонов.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Dorokhov A.E., Kochelev N.I. E2-86-355

Consideration of the Vacuum of QCD in a Composite Quark Model. Strange Hadrons

The method of inclusion of QCD vacuum condensates within the quark copmosite model is generalized to the case of hadrons containing strange quarks. The mass formula for such hadrons is obtained. The mass of strange quark is defined by analysing the energy spectrum of hadron ground states. The mixing angles of pseudoscalar mesons are estimated.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1986