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POSSIBLE TESTS FOR MAJORANA NATURE OF HEAVY NEUTRAL FERMIONS PRODUCED IN POLARIZED e^+e^--COLLISIONS

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1. INTRODUCTION

In recent years considerable efforts have been made to find possible ways of going beyond the standard $SU_C(3) \times SU_L(2) \times U_Y(1)$ model. One of the most successful approaches is associated with the unification of all interactions based on supersymmetry [1]. In supersymmetric models heavy Majorana fermions appear inevitably. Thus, in the minimal supersymmetric model Majorana particles arise as superpartners of the photon, Z° and the two neutral Higgs bosons [2]. So, an observation of heavy Majorana fermions would be of a particular importance for unified theories.

In this paper we consider the production of two different Majorana fermions in a collision of polarized e^+ and e^-. We shall assume that the process is identified by observing a lepton pair (from the decay of the short-lived Majorana particle) and a large amount of "missing" momentum (taken away by the stable Majorana particles). The main question which we shall examine here is the following: how, using only the general principles of invariance and unitarity of the S-matrix, can one test the nature (Dirac or Majorana) of the produced particles? For unpolarized initial beams this question has been considered in ref. [3]. Here we shall show that investigations of the process, when the initial e^+e^- beams are polarized, considerably enlarge our possibilities to get information about the nature of the produced particles. In the minimal supersymmetric model the cross section of the process considered has been calculated for unpolarized e^+e^- initial beams in refs. [4-7] and for polarized beams in ref. [8].

In Sec. II an expression for the cross section for the sequential process of production and subsequent decay of a short-lived particle in the general case of arbitrarily polarized e^+ and e^--beams is obtained. In Sec. III we shall show that even when e^+ and e^- are polarized, measurements of the cross section and the relevant asymmetries do not allow one to distinguish between Majorana and Dirac pair production. Different relations between the asymmetries are obtained. Their experimental check would make it possible to examine CP and CPT invariance of the new (supersymmetric?) interaction. In Sec. IV relations between the energy distributions of the final leptons are obtained. It is shown that measurements of the energy distributions would allow one to answer whether the particles produced in e^+e^- annihilation are of a Majorana or a Dirac type.

2. GENERAL FORMALISM

Here general expressions for the cross sections and the spin-density matrix to be used further on will be derived.

Let us consider the production of two electrically neutral fermions (χ and χ') in the collision of polarized electrons and positrons:

$$e^+ + e^- \to \chi + \chi'. \qquad (1)$$

We shall assume that χ' is heavier than χ and that process (1) will be identified by an observation of a $\ell^+\ell^-$-pair from the decay:

$$\chi' \to \chi + \ell^+ + \ell^- . \qquad (2)$$

Thus, a signature for process (1) would be the lepton pair from decay (2) and a "missing" four-momentum (taken away by χ-particles).

First we shall calculate the cross section for process (1) with a subsequent decay of χ' in the general case of polarized initial e^+ and e^-. No assumptions specifying the interaction will be made.

The matrix element for the process

$$e^+ + e^- \to \chi' + \chi \qquad (3)$$
$$\hookrightarrow \chi + \ell^+ + \ell^-$$

can be written in the form

$$\langle f|S-1|i\rangle = N\bar{M}_2 \frac{\hat{q}_1 + iM}{q_1^2 + M^2 - iM\Gamma} \bar{u}(-p_2) M_1 u(p_1) \times \qquad (4)$$
$$\times (2\pi)^4 \delta(q_1 + q - p_1 - p_2).$$

Here p_1 and p_2 are the momenta of the initial particles, q is the momentum of χ produced in process (1), p_1', p_2' and q_2 are the momenta of the χ' decay products ℓ^-, ℓ^+ and χ, $q_1 = p_1' + p_2' + q_2$ is the momentum of χ', $M_1(q_1, q; p_1, p_2)$ and $M_2(p_1', p_2', q_2; q_1)$ are matrix elements of processes (1) and (2), M and Γ are the mass and total width of χ', $N = N_{p_1} N_{p_2} N_{q_2} N_q N_{p_1'} N_{p_2'}$ is a normalization factor ($N_p = \frac{1}{(2\pi)^{3/2}} \frac{1}{\sqrt{2p_0}}$).

The Dirac spinors $u(p_1)$ and $u(-p_2)$ are normalized as follows: $\bar{u}(p_1)u(p_1) = 2m$, $\bar{u}(-p_2)u(-p_2) = -2m$, m is the mass of the electron. Note that in eq.(3) only the spinors of the initial electron and positron are written explicitly and the instability of χ' is taken into account in a standard way adding $-iM\Gamma$ to M^2 in the denominator of the propagator of χ'.

As soon as $\Gamma \ll M$, the "narrow-width" approximation takes place:

$$\frac{1}{(q_1^2 + M^2)^2 + M^2\Gamma^2} \simeq \frac{\pi}{M\Gamma} \delta(q_1^2 + M^2). \qquad (5)$$

Also, the following relations hold:

$$\delta(q_1 + q - p_1 - p_2)\,\delta(q_1^2 + M^2) =$$
$$= \int \delta(p_1' + p_2' + q_2 - q_1)\,\delta(q_1 + q - p_1 - p_2)\,\delta(q_1^2 + M^2)\,dq_1 = \qquad (6)$$
$$= \int \delta(p_1' + p_2' + q_2 - q_1)\,\delta(q_1 + q - p_1 - p_2) \frac{d\vec{q}_1}{2q_{10}},$$

where $q_{10} = \sqrt{M^2 + \vec{q}_1^{\,2}}$. Using eqs.(4)-(6) we obtain the following expression for the differential cross section of process (3):

$$d\sigma = N^2 \frac{1}{j} \mathrm{Tr}(\bar{M}_2 \Lambda(q_1) M_1 \rho(p_1) \bar{M}_1 \rho(-p_2) \Lambda(q_1) M_2) \times$$
$$\times (2\pi)^4 \frac{\pi}{M\Gamma} \delta(p_1' + p_2' + q_2 - q_1)\,\delta(q + q_1 - p_1 - p_2) \frac{d\vec{q}_1}{2q_{10}} d\vec{q}\, d\vec{q}_2\, d\vec{p}_1'\, d\vec{p}_2'. \qquad (7)$$

Here $\Lambda(q_1) = \sum_r u^r(q_1)\bar{u}^r(q_1)$ is the projection operator,

$$j = \frac{1}{(2\pi)^6} \frac{\sqrt{(p_1p_2)^2 - m^4}}{p_{10}\,p_{20}} \quad \text{is the incident flux and}$$

$$\rho(p_1) = \sum_{r,r'} u^r(p_1)\bar{u}^{r'}(p_1)\rho^{(1)}_{rr'}, \quad \rho(-p_2) = \sum_{r,r'} u^r(-p_2)\bar{u}^{r'}(-p_2)\rho^{(2)}_{rr'} \qquad (8)$$

are the spin-density matrices of the initial particles ($\rho^{(1)}_{rr}$ is the probability that the helicity of the electron is r, ect.). Eq.(7) may be written in the form:

$$d\sigma = d\sigma_{\chi'}\, d\Gamma_{\chi'} \frac{q_{10}}{M\Gamma}. \qquad (9)$$

Here

$$d\sigma_{\chi'} = \frac{1}{4\sqrt{(p_1 p_2)^2 - m^4}} \mathrm{Tr}(M_1 \rho(p_1) \bar{M}_1 \rho(-p_2) \Lambda(q_1)) \times$$
$$\times (2\pi)^4 \delta(q_1 + q - p_1 - p_2) \frac{1}{(2\pi)^3} \frac{d\vec{q}_1}{2q_{10}} \frac{1}{(2\pi)^3} \frac{d\vec{q}}{2q_0} \tag{10}$$

is the differential cross section for process (1) for polarized e^+ and e^- and

$$d\Gamma_{\chi'} = \frac{1}{2q_{10}} \mathrm{Tr}(\bar{M}_2 \rho(q_1) M_2)(2\pi)^4 \delta(p_1' + p_2' + q_2 - q_1) \times$$
$$\times \frac{1}{(2\pi)^3} \frac{d\vec{p}_1'}{2p_{10}'} \frac{1}{(2\pi)^3} \frac{d\vec{p}_2'}{2p_{20}'} \frac{1}{(2\pi)^3} \frac{d\vec{q}_2}{2q_{20}} . \tag{11}$$

In the last expression

$$\rho(q_1) = \frac{\Lambda(q_1) M_1 \rho(p_1) \bar{M}_1 \rho(-p_2) \Lambda(q_1)}{\mathrm{Tr}(M_1 \rho(p_1) \bar{M}_1 \rho(-p_2) \Lambda(q_1))} \tag{12}$$

is the density matrix of χ' produced in process (1) for polarized e^+e^- beams. Eq.(11) implies that $d\Gamma_{\chi'}$ is the partial width of χ'-decay (2) with the density matrix of χ' given by eq.(12).

So, if the width of the unstable particle χ' is much less than its mass in the general case of polarized incident particles the cross section of the sequential process (3) is the cross section of process $e^+e^- \to \chi\chi'$ times the partial decay rate of χ' with polarization determined in the former process. Recall that by definition Γ is the total decay width of χ' in its rest frame, so $M\Gamma/q_{10}$ is the total decay width of χ' in a frame, in which the momentum of χ' equals $\vec{q}$, $d\Gamma_{\chi'}$ is its decay rate in the same frame.

Note that eq.(9) is a direct consequence of the fact that the amplitude of process (3) is proportional to $\sum_r (\bar{M}_2 u^r(q_1))(\bar{u}^r(q_1) \bar{u}(-p_2) M_1 u(p_1))$. A generalization of this result for an arbitrary process in which one or several unstable particles are produced is straightforward. Equation (9) for unpolarized and transversely polarized initial particles has been obtained in refs./9,10/. However, the authors of these papers have used a complicated spin-algebra which makes it difficult to understand the physical meaning of this equation.

We shall finish with the following remarks:

1. Making use of eq.(10) the density matrix $\rho(q_1)$ may be written in the standard form:

$$\rho(q_1) = \sum_{r,r'} u^r(q_1) \bar{u}^{r'}(q_1) \rho_{rr'} , \tag{13}$$

where

$$\rho_{rr'} = \frac{\bar{u}^r(q_1) M_1 \rho(p_1) \bar{M}_1 \rho(-p_2) u^{r'}(q_1)}{\sum_r \bar{u}^r(q_1) M_1 \rho(p_1) \bar{M}_1 \rho(-p_2) u^r(q_1)} . \tag{14}$$

Evidently, $\rho_{rr} \geq 0$; $\sum \rho_{rr} = 1$. The quantity ρ_{rr} is the probability for χ' to be produced in a state with helicity r. Also, we have:

$\mathrm{Tr}\,\rho(q_1) = 2M.$

2. The density matrix of a spin 1/2 particle has the following general form:

$$\rho(q_1) = \frac{1}{2}(1 + i\gamma_5 \gamma\xi) \Lambda(q_1) , \tag{15}$$

where

$$\xi_\alpha = \frac{\mathrm{Tr}\, i\gamma_5 \gamma_\alpha \rho}{\mathrm{Tr}\rho} \tag{16}$$

is the polarization vector ($\xi \cdot q_1 = 0$). Making use of the identity

$$\Lambda(q_1) i\gamma_5 \gamma_\alpha \Lambda(q_1) = 2M(\delta_{\alpha\beta} + \frac{q_{1\alpha} q_{1\beta}}{M^2}) \Lambda(q_1) i\gamma_5 \gamma_\beta \tag{17}$$

and eqs.(10), (14) and (15) we obtain the following expression for the polarization ξ_α:

$$\xi_\alpha = (\delta_{\alpha\beta} + \frac{q_{1\alpha} q_{1\beta}}{M^2}) \frac{\mathrm{Tr}\, i\gamma_5 \gamma_\beta M_1 \rho(p_1) \bar{M}_1 \rho(-p_2) \Lambda(q_1)}{\mathrm{Tr} M_1 \rho(p_1) \bar{M}_1 \rho(-p_2) \Lambda(q_1)} . \tag{18}$$

3. The density matrix $\rho(-p_2)$ is

$$\rho(-p_2) = -C \rho^T(p_2) C^{-1}. \tag{19}$$

Here $\rho(p_2)$ is the density matrix of the positron and C is the charge conjugation matrix for which:

$$C\gamma_\alpha^T C^{-1} = \gamma_\alpha . \tag{20}$$

Eq.(19) is readily obtained from eq.(8) using the relations

$$u^r(-p_2) = C(\bar{u}^r(p_2))^T , \qquad \bar{u}^r(-p_2) = -(u^r(p_2))^T C^{-1} ,$$

where $u^r(p_2)$ is a spinor, which describes a positron with a momentum p_2 and a helicity r. For the density matrix of the positron we have

$$\rho(p_2, \xi_2) = \frac{1}{2}(1 + i\gamma_5\gamma\xi_2)\Lambda(p_2), \quad (21)$$

where ξ_2 is the positron polarization vector $(\xi_2 \cdot p_2 = 0)$. From eqs.(19)-(21) it follows that

$$\rho(-p_2, \xi_2) = -\frac{1}{2}(1 + i\gamma_5\gamma\xi_2)\Lambda(-p_2). \quad (22)$$

In our case $p_{10} \gg m$, $p_{20} \gg m$, where p_{10} and p_{20} are the energies of the electron and positron in the c.m.s. We shall give now an approximate expression for the electron and positron density matrices in this case. Let us denote by $\vec{\xi}^{\,o}_1$ the polarization vector of the electron in its rest frame. The longitudinal (along its momentum) $\vec{\xi}_{\parallel}(1)$, transverse $\vec{\xi}_{\perp}(1)$ and time $\xi_0(1)$ components of the polarization 4-vector in the lab.system are respectively

$$\vec{\xi}_{\parallel}(1) = \frac{\xi^o_{\parallel}(1)p_{10}}{m}\vec{k}_1, \quad \vec{\xi}_{\perp}(1) = \vec{\xi}^{\,o}_{\perp}(1), \quad \xi_0(1) = \xi^o_{\parallel}(1)\frac{|\vec{p}_1|}{m}, \quad (23)$$

where $\vec{k}_1 = \frac{\vec{p}_1}{|\vec{p}_1|}$. From eq.(23) we obtain

$$\vec{\xi}_{\parallel}(1) \simeq \xi^o_{\parallel}(1)\frac{\vec{p}_1}{m}\left(1 + \frac{m^2}{2\vec{p}_1^{\,2}}\right), \quad \xi_0(1) \simeq \xi^o_{\parallel}(1)\frac{p_{10}}{m}\left(1 - \frac{m^2}{2p_{10}^2}\right). \quad (24)$$

Using eq.(24) up to terms linear in m/p_{10}, we find the following approximate expression for the electron density matrix /11/

$$\rho(p_1, \xi^o(1)) \simeq \frac{1}{2}(1 - \xi^o_{\parallel}(1)\gamma_5 + i\gamma_5\vec{\gamma}\cdot\vec{\xi}^{\,o}_{\perp}(1))\Lambda(p_1). \quad (25)$$

Analogously, for the positron spin-density matrix we have:

$$\rho(-p_2, \xi^o(2)) \simeq -\frac{1}{2}(1 + \xi^o_{\parallel}(2)\gamma_5 + i\gamma_5\vec{\gamma}\cdot\vec{\xi}^{\,o}_{\perp}(2))\Lambda(-p_2), \quad (26)$$

where $\xi^o_{\parallel}(2)$ and $\vec{\xi}^{\,o}_{\perp}(2)$ are the longitudinal and transverse polarizations of the positron in the rest frame.

3. POLARIZATION CHARACTERISTICS OF THE PROCESS $e^+ + e^- \to \chi + \chi'$ AND POSSIBLE TESTS OF CP AND CPT - INVARIANCE

Consider the production of two Majorana particles χ and χ' in a collision between polarized electrons and positrons. The

matrix element for the process may be written as

$$\langle \chi'(q_1,s_1)\chi(q,s)|S-1|e^-(p_1,r_1)e^+(p_2,r_2)\rangle = $$
$$= iN\bar{u}^{r_2}(-p_2)M_1(q_1,s_1,q,s;p_1,p_2)\,u^{r_1}(p_1)(2\pi)^4\delta(q_1+q-p_1-p_2), \quad (27)$$

where q_1 and s_1 specify the momentum and helicity of χ', etc. From CPT-invariance and unitarity of the S-matrix (up to terms of an order of the fine structure constant α) we obtain:

$$M_1(q_1,s_1,q,s;\ p_1,p_2) = \eta_{CPT}\,\gamma_5\bar{M}_1(q_1,-s_1,q,-s;\ p_2,p_1)\gamma_5, \quad (28)$$

where η_{CPT} is a phase factor. Note that in the right-hand side of this equation p_2 is the momentum of the electron and p_1 is that of the positron. From eq.(28) one finds:

$$\sum_{s_1,s}\mathrm{Tr}\,M_1(q_1,s_1,q,s;p_1,p_2)\rho(p_1,\lambda_1,\vec{\xi}^{\perp}_1)\bar{M}_1(q_1,s_1,q,s;p_1,p_2)\rho(-p_2,\lambda_2,\vec{\xi}^{\perp}_2) = $$
$$= \sum_{s_1,s}\mathrm{Tr}\,M_1(q_1,s_1,q,s;p_2,p_1)\rho(p_2,-\lambda_2,-\vec{\xi}^{\perp}_2)\bar{M}_1(q_1,s_1,q,s;p_2,p_1)\rho(-p_1,-\lambda_1,-\vec{\xi}^{\perp}_1), \quad (29)$$

where the density matrices $\rho(p,\lambda,\vec{\xi}^{\perp})$, $\rho(-p,\lambda,\vec{\xi}^{\perp})$ are given by equations (25) and (26), respectively, and $\lambda_i = \xi^o_{\parallel}(i)$, $i = 1.2$. From eqs. (10) and (29) the following relation for the differential cross section of process (1) in the c.m.s. is obtained:

$$\sigma_{\lambda_1\vec{\xi}^{\perp}_1,\lambda_2\vec{\xi}^{\perp}_2}(\vec{k}',\vec{k}) = \sigma_{-\lambda_2-\vec{\xi}^{\perp}_2,-\lambda_1-\vec{\xi}^{\perp}_1}(\vec{k}',-\vec{k}), \quad (30)$$

where $\vec{k} = \frac{\vec{p}_1}{|\vec{p}_1|}$, $\vec{k}' = \frac{\vec{q}_1}{|\vec{q}_1|}$. Note that in both sides of this equation the first (second) argument is a unit vector in the direction of momentum of χ' (of the electron), and the first (second) subindex is the polarization of the electron (positron). Relation (30) is illustrated in figs.(1a) and (1b).

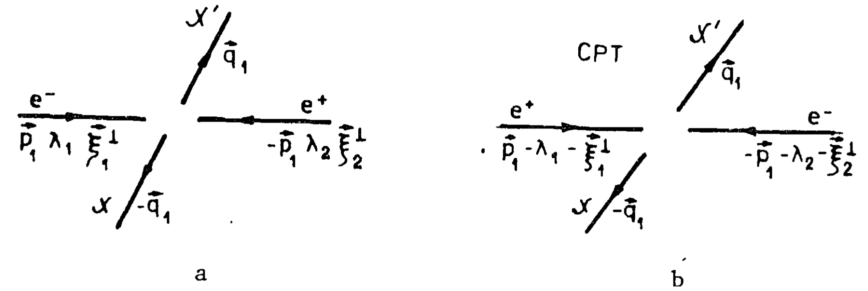

Fig. 1.

First we shall consider longitudinally polarized electrons and positrons*. From eq.(30) we get:

$$\sigma_{\lambda_1,\lambda_2}(\theta) = \sigma_{-\lambda_2,-\lambda_1}(\pi-\theta). \tag{31}$$

Here θ is the angle between the momenta of the electron and χ' in the c.m.s., the first (second) subindex denotes the longitudinal polarization of the electron (positron).

The differential cross section $\sigma_{\lambda_1,\lambda_2}(\theta)$ has the following general form:

$$\sigma_{\lambda_1,\lambda_2}(\theta) = \sigma_0(\theta) + \lambda_1\sigma_-(\theta) + \lambda_2\sigma_+(\theta) + \lambda_1\lambda_2\sigma_{-+}(\theta). \tag{32}$$

Evidently, the terms linear in λ_1 and λ_2 appear only when parity in process (1) is violated. Substituting eq.(32) into eq.(31) we find:

$$\sigma_0(\theta) = \sigma_0(\pi-\theta), \tag{33}$$

$$\sigma_+(\theta) = -\sigma_-(\pi-\theta), \tag{34}$$

$$\sigma_{-+}(\theta) = \sigma_{-+}(\pi-\theta). \tag{35}$$

Relations (33)-(35) are a direct consequence of the assumption that χ' and χ are Majorana particles. The first one has been obtained and discussed in detail in ref.[3]. We see that in the case of polarized incident particles additional relations arise.

To determine $\sigma_+(\theta)$ and $\sigma_-(\theta)$, it is enough to measure the cross section of process (1) when either of the beams is polarized. Indeed, let us define the asymmetries:

$$A_-(\theta) = \frac{\sigma_{\lambda,0}(\theta) - \sigma_{-\lambda,0}(\theta)}{\sigma_{\lambda,0}(\theta) + \sigma_{-\lambda,0}(\theta)}\,\frac{1}{\lambda}, \quad A_+(\theta) = \frac{\sigma_{0,\lambda}(\theta) - \sigma_{0,-\lambda}(\theta)}{\sigma_{0,\lambda}(\theta) + \sigma_{0,-\lambda}(\theta)}\,\frac{1}{\lambda}. \tag{36}$$

With the help of eqs.(32) and (36) we obtain:

$$A_-(\theta) = \frac{\sigma_-(\theta)}{\sigma_0(\theta)}, \qquad A_+(\theta) = \frac{\sigma_+(\theta)}{\sigma_0(\theta)}. \tag{37}$$

From eqs.(33), (34) and (37) one gets:

$$A_+(\theta) = -A_-(\pi-\theta). \tag{38}$$

To determine $\sigma_{-+}(\theta)$, it is necessary to measure the cross-section of process (1) when both initial beams are polarized. Let us difine the asymmetry:

*Longitudinally polarized e^+e^- beams are expected to be obtained at SLC[12].

$$A_{-+}(\theta) = \frac{1}{\lambda_1\lambda_2}\,\frac{\sigma_{\lambda_1,\lambda_2}(\theta) - \sigma_{\lambda_1,-\lambda_2}(\theta) - \sigma_{-\lambda_1,\lambda_2}(\theta) + \sigma_{-\lambda_1,-\lambda_2}(\theta)}{\sigma_{\lambda_1,\lambda_2}(\theta) + \sigma_{\lambda_1,-\lambda_2}(\theta) + \sigma_{-\lambda_1,\lambda_2}(\theta) + \sigma_{-\lambda_1,-\lambda_2}(\theta)}. \tag{39}$$

From eqs.(35) and (39) one finds

$$A_{-+}(\theta) = A_{-+}(\pi-\theta). \tag{40}$$

So, if χ' and χ are Majorana particles, from CPT-invariance and unitarity of the S-matrix (to the lowest order of α) it follows that the differential asymmetries obey relations (38) and (40). Now we can readily obtain also relations between the integral characteristics of process (1). Upon integration of eqs.(33)-(35) over θ from θ_0 (a value of the scattering angle fixed by the experimental set-up) to $\pi/2$ it follows:

$$\sigma_0^F = \sigma_0^B, \quad \sigma_+^{F,B} = -\sigma_-^{F,B}, \quad \sigma_{-+}^F = \sigma_{-+}^B, \tag{41}$$

where

$$\sigma_\pm^F = 2\pi\int_{\theta_0}^{\pi/2}\sigma_\pm(\theta)\sin\theta\,d\theta, \quad \sigma_\pm^B = 2\pi\int_{\pi/2}^{\pi-\theta_0}\sigma_\pm(\theta)\sin\theta\,d\theta. \tag{42}$$

The $\sigma_0^{F,B}$ and $\sigma_{-+}^{F,B}$ are defined in a similar way.

The quantities $\sigma_+^{F,B}$, $\sigma_-^{F,B}$ and $\sigma_{-+}^{F,B}$ may be found if one measures the asymmetries $A_+^{F,B}$, $A_-^{F,B}$ and $A_{-+}^{F,B}$ defined analogously to eqs.(36) and (39). For example,

$$A_+^{F,B} = \frac{\sigma_{0,\lambda}^{F,B} - \sigma_{0,-\lambda}^{F,B}}{\sigma_{0,\lambda}^{F,B} + \sigma_{0,-\lambda}^{F,B}}\,\frac{1}{\lambda}. \tag{43}$$

We have:

$$A_+^{F,B} = \frac{\sigma_+^{F,B}}{\sigma_0^{F,B}}, \quad A_-^{F,B} = \frac{\sigma_-^{F,B}}{\sigma_0^{F,B}}, \quad A_{-+}^{F,B} = \frac{\sigma_{-+}^{F,B}}{\sigma_0^{F,B}}. \tag{44}$$

From eq.(41) it follows that

$$A_-^{F,B} = -A_+^{B,F}\ ; \quad A_{-+}^F = A_{-+}^B. \tag{45}$$

Finally, for the total cross section of process (1) from eq.(32) we obtain

$$\sigma_{\lambda_1,\lambda_2} = \sigma_0 + \lambda_1\sigma_- + \lambda_2\sigma_+ + \lambda_1\lambda_2\sigma_{-+}, \tag{46}$$

where according to eq.(34):

$$\sigma_+ = -\sigma_-. \tag{47}$$

From eq.(47) one has

$$A_+ = -A_-, \tag{48}$$

where the integral asymmetries A_+ and A_- are defined in a similar way as eq.(36).

The main relation (31) and all relations between the measurable asymmetries (eqs.(38), (40) and (48)) which follow from (31) are a direct consequence of the assumption that χ and χ' are Majorana particles. Would it be possible to check this assumption by investigating these relations experimentally? To answer this question, consider the processes:

$$e^+ + e^- \to N' + \bar{N}\,, \qquad e^+ + e^- \to \bar{N}' + N\,, \qquad (49,50)$$

where N' and N are electrically neutral Dirac particles*. Suppose the processes (49) and (50) are identified by an observation of a lepton pair from the decays $N' \to N + \ell^+ + \ell^-$ and $\bar{N}' \to \bar{N} + \ell^+ + \ell^-$. The number of events with a lepton pair and "missing" momentum in this case would define the sum of the cross-sections of processes (49) and (50):

$$\sigma^{N'+\bar{N}'}_{\lambda_1,\lambda_2}(\theta) = \sigma^{N'}_{\lambda_1,\lambda_2}(\theta) + \sigma^{\bar{N}'}_{\lambda_1,\lambda_2}(\theta)\,, \qquad (51)$$

where θ is the angle between the momenta of the electron and N' or $\bar{N}'$. CPT-invariance and unitarity of S-matrix imply

$$\sigma^{N'}_{\lambda_1,\lambda_2}(\theta) = \sigma^{\bar{N}'}_{-\lambda_2,-\lambda_1}(\pi - \theta). \qquad (52)$$

So, the following relation for $\sigma^{N'+\bar{N}'}_{\lambda_1,\lambda_2}$ should hold:

$$\sigma^{N'+\bar{N}'}_{\lambda_1,\lambda_2}(\theta) = \sigma^{N'+\bar{N}'}_{-\lambda_2,-\lambda_1}(\pi - \theta). \qquad (53)$$

Comparing eqs.(31) and (53) we come to the conclusion that the cross section for a production of a pair of Dirac particles in a collision of polarized e^+ and e^- obeys the same symmetry relation as the corresponding cross section for a production of a pair of Majorana particles**.

Thus if the production of two neutral fermions in e^+e^- annihilation is detected by an observation of $\ell^+\ell^-$ pair and "missing" momentum, by measuring the cross section of the process in the general case of polarized initial beams it is impossible to distinguish the production of a pair of Majorana particles

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*A possible example is N and N' being a heavy and a light neutrino in a theory with lepton mixing.

**Note that this conclusion refers to the general case of longitudinally as well as transversely polarized beams.

from the production of a pair of Dirac particles*. Suppose now that the interaction responsible for process (1) is CP-invariant. In this case the following relation for matrix M_1, defined by eq.(27), holds:

$$M_1(q_1, s_1, q, s; p_1, p_2) = \eta_{CP}\,\gamma_4 C M_1^T(q_1', -s_1, q', -s; p_2', p_1')\, C^{-1}\gamma_4\,, \qquad (54)$$

where $p' = (-\vec{p}, ip_0)$ and η_{CP} is a phase factor. Equation (54) leads to the following relation

$$\sigma_{\lambda_1\vec{\xi}_1^{\perp},\lambda_2\vec{\xi}_2^{\perp}}(\vec{k}', \vec{k}) = \sigma_{-\lambda_2\vec{\xi}_2^{\perp},-\lambda_1\vec{\xi}_1^{\perp}}(-\vec{k}', \vec{k}). \qquad (55)$$

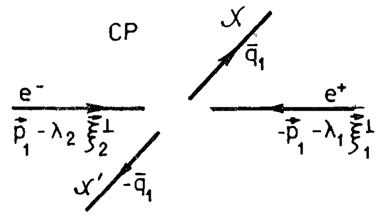

Fig. 1c.

This is illustrated in Figs. 1a and 1c. From eq.(55) for longitudinally polarized e^+ and e^- eq.(31) follows. Let us emphasize that if CP-invariance holds, eq.(31) is an exact one. This is true both for Majorana and Dirac final particles. If it would be established that relations between the measurable asymmetries (such as eqs.(38), (40), (45) and (48)) that follow from eq.(31) do not take place, this would mean that the new (supersymmetric?) interaction is not CP-invariant. What can one say about CPT-invariance under these circumstances? From the above discussions it is clear that if deviations from the relations obtained appear to be small ($\lesssim O(\alpha)$), no conclusions about CPT-invariance can be made. If a considerable ($> O(\alpha)$) violation of these relations would be observed this would mean that CPT-invariance is violated.

Suppose CPT-invariance holds. To check CP-invariance in this case it is necessary to investigate process (1) with transversely polarized e^+ and/or e^-. For example, consider $\lambda_1 = \lambda_2 = 0$, $\vec{\xi}_2^{\perp} = 0$, $\vec{\xi}_1^{\perp} \neq 0$. In this case the cross section has the following general form:

$$\sigma_{\vec{\xi}_1^{\perp},0} = \sigma_0(\theta) + \sigma_{1k'}(\theta)\,\vec{\xi}_1^{\perp}\cdot\vec{k}' + \sigma_{1n}(\theta)\,\vec{\xi}_1^{\perp}\cdot\vec{n}\,, \qquad (56)$$

---

*If the produced neutral particles may decay into a pair of different leptons (for example, $e^+\mu^-$ or $e^-\mu^+$) then identification of the process by observing such a pair would, in principle, allow one to distinguish between a Majorana pair production and a Dirac pair production. This follows from the fact that, for example, the decay rates of $N' \to N + e^+ + \mu^-$ are not equal in the general case to the decay rates of $\bar{N}' \to \bar{N} + e^+ + \mu^-$ (We thank Dr. F.Niedermayer, who draw our attention to this possibility).

where $\vec{n} = \dfrac{\vec{k} \times \vec{k}'}{|\vec{k} \times \vec{k}'|}$ is a vector perpendicular to the production plane. It is easy to see that CP-invariance implies

$$\sigma_{1n}(\theta) = 0. \tag{57}$$

Let the vector $\vec{\xi}_1^{\perp}$ be perpendicular to the production plane $\vec{\xi}_1^{\perp} = \xi_1^{\perp}\vec{n}_0$, where $\vec{n}_0$ is a unit fixed vector perpendicular to $\vec{k}$ and $\vec{k}'$. For the left-right asymmetry we have

$$A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R}\,\frac{1}{\xi_1^{\perp}}, \tag{58}$$

where σ_L is the cross section for production of χ' to the left $(\vec{n} = \vec{n}_0)$ and σ_R is the cross section for production of χ' to the right $(\vec{n} = -\vec{n}_0)$. From eqs.(56) and (58) we get:

$$A_{LR} = \sigma_{1n}. \tag{59}$$

So, if the asymmetry A_{LR} does not turn out to vanish, this would signify CP-violation.

4. ENERGY DISTRIBUTIONS OF THE FINAL LEPTONS AND POSSIBLE TESTS FOR MAJORANA NATURE OF χ AND χ'

Now we shall show that the study of the energy distributions of the final leptons makes it possible to distinguish the case of Majorana pair production from Dirac pair production (process (1) from processes (49) and (50)). In accordance with eqs.(9)-(11) the probability to observe, at an angle θ in the c.m.s., ℓ^- with energy E and ℓ^+ with energy E′ is given by the following expression:

$$d\sigma_{\lambda_1,\lambda_2}(\theta;\, E, E') = \sigma_{\lambda_1,\lambda_2}(\theta)\, d\Omega\, W_{\chi'}(E, E')\, dE\, dE'. \tag{60}$$

Here $W_{\chi'}(E, E') = d\Gamma/\Gamma_0$ is the partial decay width of process (2) for an unpolarized Majorana particle χ'* (Γ_0 is the total decay width of χ' in its rest frame). Hereafter the first (second) argument of the function $W_{\chi'}(E,E')$ will denote the energy of $\ell^-(\ell^+)$.

*Evedently the energy distributions of ℓ^+ and ℓ^- do not depend on the polarization vector of χ'.

CPT-invariance and unitarity of the S-matrix implies that

$$W_{\chi'}(E, E') = W_{\chi'}(E', E). \tag{61}$$

From eqs.(60) and (61) it follows that

$$d\sigma_{\lambda_1,\lambda_2}(\theta;\, E, E') = d\sigma_{\lambda_1,\lambda_2}(\theta;\, E', E). \tag{62}$$

On the other hand from eqs.(31) and (60) we obtain:

$$d\sigma_{\lambda_1,\lambda_2}(\theta;\, E, E') = d\sigma_{-\lambda_2,-\lambda_1}(\pi - \theta;\, E, E'). \tag{63}$$

It is easy to prove that relations (62) and (63) are not true for processes (49) and (50). Indeed, when Dirac particles are produced, we have:

$$d\sigma^{N'+\bar{N}'}_{\lambda_1,\lambda_2}(\theta;\, E, E') = [\,\sigma^{N'}_{\lambda_1,\lambda_2}(\theta)\, W_{N'}(E, E') + \sigma^{\bar{N}'}_{\lambda_1,\lambda_2}(\theta)\, W_{\bar{N}'}(E, E')]\, d\Omega\, dE'\, dE, \tag{64}$$

where, according to CPT-invariance,

$$W_{N'}(E, E') = W_{\bar{N}'}(E', E). \tag{65}$$

In the general case $\sigma^{N'}_{\lambda_1,\lambda_2}(\theta) \neq \sigma^{\bar{N}'}_{\lambda_1,\lambda_2}(\theta)$ and $W_{N'}(E, E') \neq W_{\bar{N}'}(E, E')$ and the measurable quantity $\sigma^{N'+\bar{N}'}_{\lambda_1,\lambda_2}(\theta; E, E')$ does not obey relations (62). Thus, experimental verification of relations (62) and (63) would allow one to check whether the lepton $\ell^+\ell^-$ pair originates from the decay of a Majorana or a Dirac particle. From eqs.(62) and (63) one can easily obtain a number of relations specific for production of Majorana particles for the integral quantities. We have

$$d\sigma^{(+)}_{\lambda_1,\lambda_2}(E) = d\sigma^{(-)}_{\lambda_1,\lambda_2}(E), \tag{66}$$

$$d\sigma^{(\pm)}_{\lambda_1,\lambda_2}(E) = d\sigma^{(\pm)}_{-\lambda_2,-\lambda_1}(E), \tag{67}$$

where the function

$$d\sigma^{(-)}_{\lambda_1,\lambda_2}(E) = [\,\int \sigma_{\lambda_1,\lambda_2}(\theta)\, d\Omega \int W(E, E')\, dE'\,]\, dE$$

describes the energy distribution of ℓ^- from the decay of χ'. Let us define the asymmetry

$$A_{\lambda_1,\lambda_2} = \frac{N^-_{\lambda_1,\lambda_2} - N^+_{\lambda_1,\lambda_2}}{N^-_{\lambda_1,\lambda_2} + N^+_{\lambda_1,\lambda_2}}, \tag{68}$$

where $N^{-}_{\lambda_1,\lambda_2}$ ($N^{+}_{\lambda_1,\lambda_2}$) is the number of events in which the energy of ℓ^- is higher (lower) than the energy of ℓ^+. For example, we have:

$$N^{-}_{\lambda_1,\lambda_2} = \int \sigma_{\lambda_1,\lambda_2} d\Omega \int_{m_\ell}^{E_{max}} dE \int_{m_\ell}^{E} W(E, E') dE' . \qquad (69)$$

It is easy to show using eq.(62) that for Majorana particles we have:

$$A_{\lambda_1,\lambda_2} = 0 . \qquad (70)$$

For the Dirac pair production in the general case $A_{\lambda_1,\lambda_2} \neq 0$.

Also, it is not hard to convince ourselves that for the Dirac pair production the following relations take place:

$$d\sigma^{(+)}_{\lambda_1,\lambda_2}(E) = d\sigma^{(-)}_{-\lambda_2,-\lambda_1}(E) , \qquad (71)$$

$$N^{+}_{\lambda_1,\lambda_2} = N^{-}_{-\lambda_2,-\lambda_1} . \qquad (72)$$

Evidently, for nonpolarized beams (or when $\lambda_1 = -\lambda_2$) relations (66) and (71) coincide, eq.(67) becomes an identity and eq.(72) implies that $A_{0,0} = 0$. So, by comparing the integral energy distributions or by measuring the asymmetry (68) we can get information about the nature of particles produced in process (1) only if polarized e^+ and/or e^- beams are used. In all the relations obtained above integration over the whole solid angle had been carried out. In conclusion we shall also write down the relations between the spectra of final leptons produced in the forward and/or backward hemisphere characteristic of the Majorana particle production. We have:

$$d\sigma^{F,B(+)}_{\lambda_1,\lambda_2}(E) = d\sigma^{F,B(-)}_{\lambda_1,\lambda_2}(E) , \qquad (73)$$

$$d\sigma^{F(\pm)}_{\lambda_1,\lambda_2}(E) = d\sigma^{B(\pm)}_{-\lambda_2,-\lambda_1}(E) . \qquad (74)$$

Here $d\sigma^{F(\pm)}_{\lambda_1,\lambda_2}(E)$ describes the energy distribution of $\ell^+(\ell^-)$ from the decay of χ' produced in the forward hemisphere, etc. Note that eq.(74) for $\lambda_1 = \lambda_2 = 0$ has been obtained earlier in ref.[3].

Thus, studying the spectra of final leptons one would have the possibility to get information about the nature of the neutral particles produced in e^+e^- annihilation.

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Биленький С.М., Христова Е.Х., Неделчева Н.П. E2-86-353

Рождение тяжелых нейтральных фермионов в столкновении поляризованных e^+ и e^- и возможные тесты их майорановской природы

Рассмотрен процесс рождения двух разных тяжелых майорановских /суперсимметричных?/ фермионов в столкновении поляризованных e^+ и e^-. Предполагается, что процесс наблюдается по регистрации $\ell^+\ell^-$-пары и "недостающего" импульса. Получено общее выражение для сечения процессов такого типа в случае произвольно поляризованных начальных частиц. Основной вопрос, который исследуется, состоит в следующем: измерение каких величин позволило бы отличить процесс рождения пары майорановских фермионов от процесса рождения пары дираковских фермионов? Показано, что путем измерения сечения рождения отличить эти процессы невозможно даже в случае поляризованных начальных частиц. Сравнение спектров ℓ^+ и ℓ^- могло бы позволить ответить на вопрос о природе родившихся фермионов. Рассмотрение основано на общих принципах инвариантности и унитарности. Предлагаются возможные способы проверки CP и CPT-инвариантности, основанные на сравнении поляризационных характеристик процесса.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Possible Tests for Majorana Nature of Heavy Neutral Fermions Produced in Polarized e^+e^--Collisions

Production of two different heavy Majorana (supersymmetric?) fermions in collisions of polarized e^+ and e^- beams is considered on the basis of general invariance principles. It is assumed that the process is identified by an observation of a lepton $\ell^+\ell^-$ pair and a large amount of "missing" energy. A general expression for the cross section of processes of this type is obtained for arbitrarily polarized initial beams. It is shown that comparison of the energy distributions of the final ℓ^+ and ℓ^- would allow one to gain information about the nature (Dirac or Majorana) of the produced fermions. Possible tests for CP and CPT-invariance based on measurements of the spin asymmetries are suggested.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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