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QCD: A NEW VIEW ON AN OLD PROBLEM

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I. The QCD Problems

QCD has arisen[1] and has become fruitfully developed[2] as a field theory version of the quark-parton model[3,4] after the discovery of asymptotic freedom phenomenon[5] with the help of renormalization group method[6].

QCD has been constructed by analogy with QED, all physical consequences of which can be got from the first principles of symmetry and quantization. The main task for QCD up to now was foundation of its working hypotheses from the first principles. One can roughly separate these hypotheses into two parts related to regions of the low (I) and high (II) energies:

I. The hypothesis of the short-distance action of gluon forces, that concerns the PCAC (F_π) and hadron spectra (α').

II. The principle of the local quark-hadron duality (LQUD) and its modifications.

QCD inherits the principle LQHD from the Feynman naive parton model[3]. Feynman justified this principle with the help of the unitary condition $SS^+ = 1$ ($S = 1 + iT$)

$$\sum_h \langle i|T|h\rangle\langle h|T^*|j\rangle = 2\,\mathrm{Im}\langle i|T|j\rangle \cdot \qquad (1)$$

In the left-hand side of eq.(1) there is sum over the complete set of hadron physical states. For the calculation of the right-hand side of eq.(1) Feynman proposed that there is a quantum field theory for partons which do not contribute to the physical states on the left-hand side and for which the usual free propagators of perturbation theory are valid. This assumption allows interpretation of the inclusive-processes cross-sections in terms of the imaginary parts of the quark-parton diagrams and hence determination of the quark quantum numbers[3] (which are now the basis for the construction of unification theory).

Now the principle LQHD is the basis of the QCD-phenomelology[2,7]. The experimental momentum distributions of hadrons in the left-hand side of eq.(1) entirely reproduce the distributions of partons (quarks, antiquarks, gluons), whose dynamics is completely controlled by the right-hand side of eq.(1) (i.e. QCD-perturbation theory). Here the following paradox arises: in the right-hand side of eq.(1) one uses the free state of quarks and gluons in the mass shell regime and simultaneously one proposes that the quarks and gluons do not contribute to the observable physical states of the left-hand side of eq.(1). Just the absence of the quark and gluon states in the left-hand side of eq.(1) is called the confinement hypothesis.

The proof of confinement has to satisfy the principle of accordance with the parton model. (Any attempts to support the confinement with the help of quark propagator modification by removing their poles in the scaling region, simultaneously removes the very possibilities of the quark-parton interpretation of deep-inelastic processes).

The QCD hypotheses now are explained by the asymptotic freedom phenomenon[5]

$$\alpha(q^2) = \frac{1}{\beta \log\left(\frac{q^2}{\Lambda^2}\right)} \quad ; \quad \beta = \frac{1}{4\pi}\left(11 - \frac{2}{3} n_f\right), \qquad (2)$$

where n_f is the number of flavours of quarks, Λ is the dimensional parameter arising from the infrared boundary conditions for solutions of the renormalizable equation. This formula of asymptotic freedom defines the ranges of validity of perturbation theory as at $q = \Lambda$ the coupling constant is infinite. Attempts are made to bind Λ with the scale of the PCAC and hadron mass spectra.

At this stage the ideology of potential coninement arises. Its essence consists in the aspiration to get the confinement gluon propagator (or quark-quark potential) by an approximate summation of the Feynman diagrams by means of renormgroup equations or the Schwinger-Dyson ones[8].

As a model of such a confinement one proposes the following gluon propagator:

$$\frac{M^2}{q^4} \quad ; \quad M^2\,\delta^4(q) \;\; /9/ \quad ; \quad \left(V(r) = \alpha' r \,,\; \frac{r^2}{V}\right).$$

Further calculations on the basis of such a propagator are founded in the main on solutions of the Schwinger-Dyson or the Bethe-Salpeter equations of the type

$$\Sigma(p) = \frac{e^2}{(2\pi)^4 i} \int d^4 q \; \mathcal{D}_{\mu\nu}(q) \gamma_\mu \frac{1}{\hat{p} - \hat{q} - m - \Sigma(p-q)} \gamma_\nu \qquad (3)$$

and the results of the calculations are the hadron mass spectrum, condensates, F_π , etc.[8,9].

2. A New View on QCD

Remarkable success in constructing the consistent quantum gravity theory (superstring $E_8 \times E_8$)[10] gives reasons to recomprehend anew both solved and unsolved QCD problems. In particular, now one undertakes construction of a finite unification field theory (without divergences)[11] containing QCD as a part. It should be noted, in the theory without ultraviolet divergences the renormalization group equations turn into identities and have no any physical information including the asymptotic freedom.

The asymptotic freedom phenomenon in such theory may be only a consequence of the trivial summation of the Feynman diagrams in the "one-log" approximation

$$\alpha(q^2) = \frac{\alpha(M_s^2)}{1 + \beta\,\alpha(M_s^2) \log(q^2/M_s^2)} = \frac{1}{\beta \log(q^2/M_s^2)},$$

where M_s is the scale of the supersymmetry breaking in the ultrarelativistic region of the asymptotic "desert" ($M_s \sim 10^{15} m_p$). The parameter Λ

$$\Lambda^2 = M_s^2 \exp\left(-\frac{1}{\beta\alpha(M_s^2)}\right)$$

here really does not concern the infrared gluon interaction (as one proposes in the renormalization group version of the dimensional transmutation).

A new point of view forced us to find another infrared mechanism for justifying the QCD working hypotheses (I, II).

As has been shown in ref.[12], for the confinement of colour fields the infrared topological degeneration of the gauge (phase) factors are sufficient: due to the destructive interference of these factors the amplitudes with colour particles disappear and do not contribute to the left-hand side of eq.(1).

Here, we consider the dynamics of the infrared fields $\partial_i^2 A_j(x,t) = 0$ that is omitted in the canonical relativistic covariant method of quantization of gauge fields[13-15]. It is well-known that this method is based on the transverse commutation relation[15,16]

$$i\left[E_i^{T\,a}(x,t),\, A_j^{T\,b}(y,t)\right] = \delta^{ab}\, \delta_{ij}^{T}\, \delta^3(x-y)$$

$$\left(\delta_{ij}^{T} = \delta_{ij} - \partial_i \frac{1}{\partial_k^2} \partial_j \equiv \delta_{ij} + \frac{1}{4\pi} \frac{\partial}{\partial x_i} \int \frac{d^3 z}{|x-z|} \frac{\partial}{\partial x_j} \right) \qquad (4)$$

that is given on the function class

$$\partial_i^2 A_j^{T\,a} \neq 0 \quad ; \quad \int d^3x\, A_j^{T\,a}(x,t) = 0. \qquad (5)$$

The infrared dynamical fields

$$\partial_i^2 b_j^a(x,t) = 0 \qquad (6)$$

are omitted by the commutation relations (4). In QED this omission is physically justified, as these quantum fields are unobservable due to the finite energy arrangement resolution[12].

In QCD we have no such a justification. Moreover, the including of the gluon fields (6) may be justified by the nonlinearity of the theory and the strong coupling of fields in the infrared limit (that leads as a rule to collective excitation of infrared gluons correlated in the whole volume of the space they occupy, $V = \int d^3x$) There is a trivial generalization of the commutation relations (4) with the space-constant fields b_i^a included

$$i\left[E_i^a(x,t),\, A_j^b(y,t)\right] = \delta^{ab}\left[\delta_{ij}^{T}\, \delta^3(x-y) + \frac{\delta_{ij}}{V}\right].$$

where $A_i^a(x,t) = A_i^{T\,a}(x,t) + b_i^a(t)$. (7)

The most consistent canonical quantization of the theory

$$\mathcal{L} = -\frac{1}{4}\left(F_{\mu\nu}^a\right)^2 + \bar\psi i \gamma_\mu \nabla_\mu \psi - m\bar\psi\psi \; ; \; S = \int d^4x\, \mathcal{L}$$

$$\nabla_\mu = \partial_\mu + \hat{A}_\mu \; ; \; \hat{A}_\mu = g\frac{\lambda^a A^a}{2i} \; ; \; \hat{F}_{\mu\nu} = \partial_\mu \hat{A}_\nu - \partial_\nu \hat{A}_\mu + [\hat{A}_\mu, \hat{A}_\nu] \qquad (8)$$

with the constraint equation $\delta S / \delta A_0^a = 0$ has been made in ref.[13].

According to this paper the quantization of only the transverse fields A^T and the fields ψ leads to the effective potential for the fields

$$Z(b_i|0,0)=\int d^4A^a_\mu\, d\psi\, d\bar\psi\; \delta(\partial_i A^a_i)\,\det[\nabla_i(A_j+b_j)\partial_i]\times$$
$$\times \exp\{iS^l(A_0,A_i+b_i)+i\int d^4x(\bar\psi\eta+\bar\eta\psi)\}\Big|_{\eta=\bar\eta=0}=\exp\{iV\int dt[\tfrac{1}{2}\dot b^2+\varphi(b)]\}, \quad (9)$$

where

$$V\int dt\,\varphi(b)=-\int d^4x\,\tfrac{1}{4}(F^a_{ij}(b))^2-i\,\mathrm{tr}\log\det[\nabla_i(A+b)\partial_i]+\cdots \quad (10)$$

is the potential of the infrared fields (6), induced by all interactions in the whole volume V.

The quantization of the infrared fields in the limit of the infinite volume V can be reduced to the stochastization of the Green function generating functional

$$Z(\eta,\bar\eta)=\exp\{\tfrac{1}{2}M^2\Big(\frac{\partial}{\partial b^a_i}\Big)^2\}\Big[\frac{Z(b|\eta,\bar\eta)}{Z(b|0,0)}\Big]\Big|_{b=0}, \quad (11)$$

where M is the infrared dimensional transmutation parameter - the analog of the arrangement energy resolution in QED. (Recall that the old renormalization-group QCD parameter Λ was also defined by a nonperturbative interaction in the infrared region where the renormalization group method was invalid).

The relativistic covariant version of eq.(11) has the form[13]

$$Z^l(\eta,\bar\eta)=\exp\{-\frac{M^2}{2}\Big(\frac{\partial}{\partial b^a_\mu l}\Big)^2\}\Big[\frac{Z^l(b^l|\eta\bar\eta)}{Z^l(b^l|0,0)}\Big]\Big|_{b^l=0} \quad (12)$$

$$Z^l(b|\eta,\bar\eta)=\int d\bar\psi\, d\psi\, d^4A^a_\mu\;\delta(\partial^l_\mu A^{l}_\mu)\det(\nabla^l_\mu\partial^l_\mu)\times$$
$$\times\exp\{iS[A_\mu+b^l_\mu]+i\int d^4x(\bar\psi\eta+\bar\eta\psi)\}, \quad (13)$$

where $B^l_\mu=B_\mu-l_\mu(B_\nu l^\nu)$; $B^l=(b^l,A^l,\partial^l,\nabla^l)$; and l_μ is the time axis of quantization.

The possibilities of such a stochastization and its physical meaning can be seen on the simplest example of the Abelian theory given on the function class (6)

$$L=\int d^3x\Big[-\tfrac{1}{4}F^2_{\mu\nu}(b)-\frac{\mu^2}{2}b_i^2+b_i j_i(x,t)\Big]=$$
$$=\tfrac{1}{2}V[\dot b_i^2-\mu^2 b_i^2]+b_i\int d^3x\, j_i(x,t) \quad (14)$$

we introduced here the mass μ for the infrared regularization of the propagator of the quantum field b_i:

$$P_i=\frac{\partial L}{\partial\dot b_i}=V\dot b_i\;;\quad i[P_j,b_i]=\delta_{ji}\;;\quad \Big(i[\dot b_j,b_i]=\frac{\delta_{ij}}{V}\Big) \quad (15)$$

$$D_{ij}(t)=\frac{1}{i}\langle T(b_i(t)b_j(0))\rangle=\frac{\delta_{ij}}{2\pi V}\int dq_0\frac{e^{iq_0t}}{q_0^2-\mu^2+i\varepsilon}=i\frac{e^{i\mu|t|}}{2\mu V}. \quad (16)$$

We see that there is a limit of the infinite volume

$$V\to\infty,\ \mu\to 0\;;\quad 2\mu V=M^{-2}\neq 0 \quad (17)$$

with the nonzero propagator (16)

$$\lim_{\substack{V\to\infty\\ \mu\to 0}} D_{ij}(t)=iM^2\neq 0. \quad (18)$$

The evolution operator for the theory (14) has the form

$$\lim_{\substack{V\to\infty\\ \mu\to 0}}\langle e^{-iTH}\rangle=\exp\{-\frac{M^2}{2}\Big(\frac{\partial}{\partial b_i}\Big)^2\}\,e^{ib_iJ_i}\Big|_{b=0}\;;\quad J_i=\int d^4x\, j_i.$$

It is clear that in the limit (17) the propagator of the total field $A=A^T+b$ has the form of the sum of the usual transverse propagator and expression (18)

$$D_{ij}=\frac{1}{i}\langle T[A_i(x)A_j(0)]\rangle=D^T_{ij}(x)+iM^2 \quad (19)$$

or in the momentum representation:

$$\tilde D_{ij}=\Big(\delta_{ij}-q_i\frac{1}{q^2}q_j\Big)\frac{1}{q_\mu^2+i\varepsilon}+i(2\pi)^4\delta^4(q)M^2. \quad (20)$$

So, we have got one of the versions of the confinement propagator[9] that reflects the collective excitation of the infrared fields (6) in the whole space they occupy. In the light of this fact the attempts to get the confinement propagator by analytical calculation in the framework of the convention perturbation theory given only in the function class (5)/8,9/ look very doubtful.

For the generation functional of the Green functions for the Abelian theory with the commutation relations like (7) in the limit (17), we get the expression of the type of (11)

$$Z(\eta,\bar\eta) = \exp\left\{\frac{M^2}{2}\left(\frac{\partial}{\partial b_i}\right)^2\right\} Z(b|\eta,\bar\eta)|_{b=0}$$

$$Z(b|\eta\bar\eta)=\int d^4A\, d\bar\psi\, d\psi\, \delta(\partial_i A_i)\exp\left\{iS[A_0, A_i+b_i]+i\int d^4x[\bar\psi\eta+\bar\eta\psi]\right\}, \qquad (21)$$

where $S^V[A_\mu]$ is the usual QED action. As has been shown in ref./13/, the correct transformation properties of the operator formalism/15/ can be restored in terms of the functional integral if in it one explicitly takes into account the time dependent axis ℓ_μ of quantization

$$Z^{\ell}(\eta,\bar\eta) = \exp\left\{-\frac{M^2}{2}\left(\frac{\partial}{\partial b^{\ell}_\mu}\right)^2\right\}\int d^4A\, d\bar\psi\, d\psi\, \delta(\partial^{\ell}_\mu A^{\ell\mu}) \times$$

$$\times \exp\left\{iS^V[A_\mu+b^{\ell}_\mu] + i\int d^4x(\bar\eta\psi+\bar\psi\eta)\right\}|_{b=0},$$

where $A^{\ell}_\mu = A_\mu - \ell_\mu(\ell_\nu A^\nu)$.

If we neglect the interaction with the transverse fields, we can exactly calculate the fermion Green function and the correlator of two currents

$$G(p_0,\bar p=0)=\exp\left\{\frac{M^2}{2}\left(\frac{\partial}{\partial b_i}\right)^2\right\}[\hat p - eb_i\gamma_i - m]^{-1} = \frac{\hat p+m}{e^2M^2}\Big\{-1+$$

$$+\sqrt{\pi\delta}\, e^{\delta}[1-\Phi(\sqrt{\delta})]\Big\}; \quad \delta=\frac{m^2-p^2}{2e^2M^2}; \quad \Phi(x)=\frac{2}{\sqrt\pi}\int_0^x dt\, e^{-t^2} \qquad (22)$$

$$\langle j(q)\, j(-q)\rangle = \exp\left\{\frac{M^2}{2}\left(\frac{\partial}{\partial b_i}\right)^2\right\}\int d^4p\, tr\, \gamma_\mu[\hat p+\hat q-eb_i\gamma_i-m]^{-1}\gamma_\nu[\hat p-eb_i\gamma_i-m]^{-1}$$

$$= \int d^4p\, tr\, \gamma_\mu[\hat p-\hat q-m]^{-1}\gamma_\nu[\hat p-m]^{-1}. \qquad (23)$$

In the Abelian version of the collective excitation the analytical properties of the correlator (28) do not change but the Green function (22) loses its pole. Note that in the potential version of confinement the physical consequences of the propagator $\delta^4(q)$/9/ are obtained with the help of the Schwinger-Dyson equation of the type of eq. (3)

$$\Sigma(p) = -\hat p A(p^2) + B(p^2);$$

$$B(p^2) - \hat p A(p^2) = 3e^2M^2[\hat p(1+A)+(m+B)]^{-1}.$$

It is easy to convince oneself that the solution of this equation does not concern the exact expression (22). In QCD the Expression (12) leads to the infinite power series in momenta M^2/q^2, that disappears in limit $M^2\to 0$ or $q^2\to 0$. In this limit we get the usual QCD. The constant fields b_i ($\vec k^2 = 0$) take part only in hadronization of the colour fields in the low-energy region.

C o n c l u s i o n

From 1974 till 1984 the renormalization group idea of asymptotic freedom dominates in QCD. Constructively this idea consists in the introduction of the QCD parameter Λ as the infrared boundary condition of the renormalization group equation in the region where this equation is in valid. (In this sense the parameter Λ reflects the result of an infrared nonperturbative interaction denoted by dimensional transmutation).

The 1984 theoretical revolution led to consistent unification theories without ultraviolet divergences/10,11/ where the renormalization group equations became identities and lost their physical meaning. We can see in such a theory that the mysterious infrared dimensional transmutation is absent and the parameter Λ sooner reflects the ultraviolet scale of the supersymmetry breaking in the asymptotic desert region than the infrared nonperturbative interaction.

We suggested here the new infrared mechanism of dimensional transmutation that is omitted in the conventional approach and leads effectively to the stochastization of the Faddeev-Popov functional. We have proved the possibilities of such a stochastization in the Abelian version of the collective excitation and showed that the quantization of infrared fields ($\vec k^2 = 0$) leads to one of the versions of the "confinement propagator".

A c k n o w l e d g e m e n t

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КХД: новый взгляд на старые проблемы

КХД рассматривается в контексте построения самосогласованной единой теории поля без расходимостей, в которой ренорм-групповые методы не несут никакой физической информации. В качестве альтернативы явлению асимптотической свободы обсуждается механизм инфракрасной размерной трансмутации, который допускает гамильтонова формулировка квантования калибровочных теорий с явным решением уравнений связи. Показано, что каноническая неоднозначность квантования ведет к пропагатору конфайнмента и последний невозможно обосновать в классе функций обычной теории возмущений.

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QCD: a New View on an Old Problem

During the last ten years the renormalization group idea of asymptotic freedom has been dominating in QCD. The 1984 - theoretical revolution led to consistent unification theories without ultraviolet divergences where the renormalization equations became identities and lost their physical meaning. Here we consider QCD as a part of such a consistent theory and propose a new mechanism of the dimensional transmutation based on the collective excitation of infrared fields ($\overline{K}^2 = 0$) that are usually omitted in the canonical quantization. We show that the quantization of the infrared fields leads to one of the versions of the "confinement propagator".

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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