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INFRARED ASYMPTOTICS OF PERTURBATIVE QCD. VERTEX FUNCTIONS

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1. Introduction

The present paper is devoted to a study of the infrared (IR) asymptotics of the vertex functions (form factors) within the framework of perturbative QCD. The solution of this problem is necessary both for a quantitative description of hard processes and for a consistent proof of the factorization theorem for hadron-hadron reactions. Earlier /1/ numerous calculations were made for the colour singlet quark form factor in the lowest orders of perturbation theory (PT). They demonstrated that the leading (double-logarithmic) IR asymptotics of the QCD amplitudes is given by the exponential of the one-loop contribution. The proof of this property in higher orders of PT was given in ref. /2/. However, even the first attempts to calculate the nonleading IR asymptotics /3/ have shown that the problem is far from being trivial. The expression for the two-loop correction to the quark form factor contains nonleading IR logarithms having the same order of magnitude as the leading ones included in the exponential and proportional to the β -function coefficient $\frac{11}{24}C_F N$. Thus, there arises the problem of finding the nonleading IR asymptotics in higher orders of PT.

In this paper we demonstrate (using as an example the colour singlet quark form factor) how the gauge properties of QCD fix the IR asymptotics of amplitudes. We obtain also a renormalization-group (RG) equation, the solution of which completely describes the IR behavior of the process.

In sect. 2 the infrared singularities of the form factor amplitude for two specific kinematics are shown to factorize into a separate factor. The properties of the IR factors obtained are considered in sect. 3. There we also compare it with analogous QED expressions. In sect. 4 we formulate the RG equation for the IR asymptotics of the form factor and discuss its general properties.

2. Factorization of the Infrared Singularities

Consider the amplitude corresponding to the colour singlet (e.g., electromagnetic) quark form factor. It is characterized by the following kinematic invariants: Q^2 -momentum transfer squared, p^2 and q^2 squared 4-momenta of initial and final state quarks, ($p^2 - m^2$) and ($q^2 - m^2$) -virtualities (offshellnesses) corresponding to them. (In some gauges there appear also gauge condition related scale, e.g. $(pn)^2/n^2$ in the axial gauge $n_\mu A^a_\mu(x) = 0$). In what follows, we consider two specific cases of symmetric kinematics:

a) $p^2 - m^2 = q^2 - m^2 = 0$, $Q^2 > m^2$ - the on-mass-shell quark form factor,

b) $p^2 - m^2 = q^2 - m^2 = -\mu m$, $\mu \leqslant m$, $Q^2 > m^2$ - the near-mass-shell quark form factor.

Using the well-known methods of analysis of asymptotics of Feynman diagrams[4] it is easy to establish that the contributions responsible for the leading $1/Q^2$ contributions to the form factor have the structure shown in fig. 1a. The three types of subgraphs, viz.: hard (H), collinear (J_p and J_q) and soft (S) are characterized by the momenta ascribed to its internal lines. It is convenient to introduce the center-of-mass reference frame in which the momentum k_μ is specified by the components: $k_\mu = (k_+, k_-, \vec{k}_T)$*). In particular, by definition

$$p_\mu = \left(p_+, \frac{p^2}{2p_+}, \vec{0}\right) \qquad q_\mu = \left(\frac{q^2}{2q_-}, q_-, \vec{0}\right) \tag{1}$$

and let in the frame chosen the relation $p_+ > p_-$, $q_- > q_+$ be fulfilled.

The hard block (H) describes the short-distance interaction with momenta

$$k_\mu \gtrsim \sqrt{Q^2}. \tag{2}$$

It has two quark external lines (twist 1) and an arbitrary number of longitudinally polarized gluon lines (twist 0) with momenta collinear to that of one of the two quark lines. (In a gauge from the contour gauge class[5] the gluon potential A^a_μ is a linear functional of the field strength $G^a_{\mu\nu}$ and its twist effectively equals 1. The hard block in this case has no external gluon lines for the lowest twist contribution).

The collinear blocks J_p and J_q of diagram 1a describe the jets of quarks and gluons whose momenta are collinear to the vectors

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*)Here the light cone variables $k_\pm = \frac{1}{\sqrt{2}}(k_0 \pm k_3)$ are introduced.

p_μ or q_μ , respectively. The momentum k_μ of any particle of the jet J_p , e.g., has the following components

$$k_\mu = (1, \lambda^2, \lambda)\sqrt{Q^2}, \quad k^2 = \lambda^2 Q^2, \tag{3}$$

where $\lambda^2 \sim p^2/Q^2$ is a small parameter determining the virtuality of the collinear quarks and gluons.

According to the dimensional counting of refs.[4,6] for the soft block S there are two IR singular regions in the momentum space corresponding to

a) the regime of (homogeneously) soft momenta

$$k_\mu = (\lambda, \lambda, \lambda)\sqrt{Q^2}, \tag{4}$$

where $\lambda^2 \sim (p^2 - m^2)/Q^2 = (q^2 - m^2)/Q^2$ is the boundary scale separating infrared and collinear regimes;

b) the Glauber regime of the IR gluons[6]:

$$k_\mu = (\lambda^2, \lambda^2, \lambda)\sqrt{Q^2}, \tag{5}$$

where $\lambda^2 \sim (p^2 - m^2)/Q^2$; i.e., the gluon in this case has essentially the transverse vanishing momentum.

Our task now is to study the contribution of the infrared regimes (4), (5) to the form factor amplitude. To this end we perform two transformations of the original diagram 1a. First, we pick out the effects due to the radiation of collinear gluons from the hard block (fig. 1b). Then within the resulting diagram we factorize out the contribution of the soft subprocess S (fig. 1c). We use here the following properties:

a) the radiation of a collinear gluon from the hard block is damped by powers of p^2/Q^2 provided that the gluon potential has effectively twist 1 (or can be represented as a linear functional of the strength tensor);

b) to yield a singular contribution to the amplitude, a soft gluon with momentum k_μ satisfying eq.(4) must be emitted in a quark-gluon or three-gluon vertices inside the collinear blocks, while the four-gluon vertices are damped by, at least, the first power of k_μ [4,6].

In region (4) such a triple vertex insertion reduces (up to $O(k)$ terms) to the original propagator of the collinear quark or gluon (with momentum p_μ) multiplied by a scalar factor $2p_\mu$ where μ is the polarization index of the soft gluon. Hence, the radiation of the soft gluons from the collinear block J_p is damped by powers of ($p^2 - m^2$) if the gluon momentum satisfies eq.(4) and the gluon potential is subjected to the axial gauge condition

$$p_\mu A^a_\mu(k) = 0 . \qquad (6)$$

Besides, as it is shown in ref./5/, in the axial gauge (6) there exists a linear relation between the potential and strength tensor, and, consequently, the fulfillment of eq.(6) is sufficient to damp simultaneously both collinear (eq.(2)) and infrared (eq.(4)) regimes of the gluon momentum*.

Consider now fig. 1a and sum over all possible insertions into the hard block vertices of the gluons collinear to the external momentum p_μ. As is shown in Appendix the result of such a summation is the following modification of the hard block propagators. The quark propagator in the lowest twist approximation becomes.

$$S_0(x-y) \to \hat{E}_p(x,\infty)\, S_0(x-y)\, \hat{E}^+_p(y,\infty), \qquad (7)$$

where

$$\hat{E}_p(x,\infty) = P\exp\big(ig\int_0^\infty ds\, e^{-\epsilon s} p_\mu \hat{A}_\mu(x+ps)\big), \quad \hat{A}_\mu = A^a_\mu \lambda^a, \ \epsilon \to 0 \qquad (8)$$

is the gauge transformation operator for the axial gauge (6)/5/. The modification of the gluon propagator in the general case may be written as

$$D_0^{\mu\nu}(x-y) \to \tilde{E}_p(x,\infty)\, \tilde{D}^{\mu\nu}(x-y)\, \tilde{E}^+_p(y,\infty), \qquad (9)$$

where $\tilde{E}_p$ results from eq.(8) if one uses there the matrices of the adjoint (gluon) representation of the gauge group instead of the fundamental (quark) one. Similarly, one has for the "ghost" propagator

$$D(x-y) \to \tilde{E}_p(x,\infty)\, \tilde{D}(x-y)\, \tilde{E}^+_p(y,\infty). \qquad (10)$$

If the original gauge field A^a_μ is taken in a "ghost-free" gauge, then the function $\tilde{D}^{\mu\nu}$ in the r.h.s. of eq.(9) will coincide with $D_0^{\mu\nu}$. Otherwise, $\tilde{D}^{\mu\nu}$ and $\tilde{D}$ are the free gluon and ghost propagators, respectively, taken in the gauge resulting from the original one by transformation (A.9), e.g., an α-gauge transforms into the background field gauge (see Appendix). Taking into account eqs.(8)-(10) and also the possibility to insert a collinear gluon into a three-gluon vertex inside the hard block one observes that all the P-exponentials (8) related to internal vertices of the H-block are cancelled/7/ and in some cases the gauge condition for the hard-block related gluon field A^a_μ is modified. Uncancelled remains the exponential $\hat{E}_p(x,\infty)$ corresponding to the external (with respect to H) quark with momentum collinear to p_μ. Graphically, this is depicted in fig. 1b by a dashed line connected with gluon lines* the momenta of which, by construction, are in the collinear regime (eq. (2)).

* Peculiarities of the Glauber regime (5) will be considered at the end of this section.

The gluons collinear to the external momentum q_μ and radiated from H can be treated in a similar way with two natural modifications in eqs.(7)-(10). First, p_μ must be substituted by q_μ. Second, the propagators $D_0^{\mu\nu}$ and D in the l.h.s. of eqs.(9),(10) are substituted by $\tilde{D}^{\mu\nu}$ and $\tilde{D}$, while in the r.h.s. of these equations there appear free propagators $\tilde{\tilde{D}}^{\mu\nu}$ and $\tilde{\tilde{D}}$ taken in a "doubly" transformed gauge. As a result, there appears the P-exponential $\hat{E}^+_q(x,\infty)$ (see fig. 1b).

After performing these transformations (valid only for the leading $1/Q^2$ asymptotics) one picks out the hard block contribution $m_{hard}(Q^2/\mu_c^2)$ depending only on large transferred momentum Q^2 and the scale μ_c separating short and long distances. In the lowest twist approximation the function m_{hard} is gauge invariant/4,7/. Hence it does not depend on the above-mentioned change of the gauge in eqs.(9), (10). The remaining part of the form factor amplitude contains a mixed combination of collinear and soft blocks. It may be viewed as a diagram describing the propagation of the jets J_p, J_q in the "external" field of IR gluons belonging to the eqs. (4), (5). Incorporate now expressions (A.3) and (A.8) for propagators of a collinear subprocess in the "external" field of the IR gluons.

Note that the transformed potential $\hat{A}^v_\mu$ present in these expressions satisfies the axial gauge condition (6). Hence, the amplitude for the emission of a gluon in the homogeneously soft region (4) is damped. As for the Glauber region of momenta (5) (to be studied in

*A more detailed discussion of properties of the P-exponentials is given in ref./8/.

more detail in a separate paper), for it the same is true provided that the original field $\hat{A}_\mu$ was not obliged to satisfy the axial gauge condition $n_\mu \hat{A}_\mu(x) = 0$ with a gauge vector n_μ having

$$\vec{n}_T = 0 \tag{11}$$

in the frame we chose. Thus, just as in the preceding case, all the effects due to the emission of the IR gluons by collinear quarks, gluons and ghosts are accumulated in two P -ordered exponentials appearing at the end-points of the corresponding propagator.

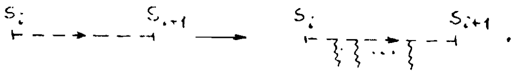
In addition, there exists a possibility of inserting the gluons into the dashed lines (see diagram 1b) of the PT -expansion for the P -ordered exponential:

$$\hat{E}_p(x,\infty) = \sum_{n=0}^{\infty} (ig)^n \int_0^\infty ds_1 e^{-\epsilon s_1} \ldots \int_0^\infty ds_n e^{-\epsilon s_n} \theta(s_2 - s_1) \ldots \theta(s_n - s_{n-1}) \cdot (p_\mu \hat{A}_\mu(x+ps_1)) \ldots (p_\nu \hat{A}_\nu(x+ps_n)) \tag{12}$$

to which there corresponds the factor $\theta(s_{i+1} - s_i)$. After summation over all possible insertions this factor is modified in the following way

$$\theta(s_{i+1} - s_i) \to \hat{E}_p(x+ps_{i+1},\infty)\, \theta(s_{i+1} - s_i)\, \hat{E}_p^+(x+ps_i,\infty) \tag{13}$$

or, graphically,



Following the method of ref. /7/ it is easy to observe once more the cancellation of all the factors $\hat{E}_p$, $\tilde{E}_p$ and $\hat{E}_q$, $\tilde{E}_q$ (taken in appropriate representations) associated with the internal vertices of the collinear subgraphs J_p and J_q, respectively. The uncancelled are left the factors $\hat{E}_p^+(\xi,\infty)$, $\hat{E}_p(0,\infty)$, $\hat{E}_q(\eta,\infty)$, $\hat{E}_q^+(0,\infty)$ associated with the end-points of the collinear subgraphs. The ultimate result - the diagram with contributions of all subgraphs factorized - is shown in fig. 1c.

Introduce the notation for the amplitudes related to the collinear $\mathcal{M}_{coll(p)}(\frac{\mu_c}{\mu_{IR}}, \frac{p^2}{\mu_c^2})$ and soft $\mathcal{M}_{soft}(\frac{\mu_{IR}^2}{p^2-m^2}, \frac{\mu_{IR}^2}{q^2-m^2})$ subprocesses, where μ_{IR} is the parameter splitting the collinear and soft regimes of the gluon momentum. It should be noted that $\mathcal{M}_{hard}$, $\mathcal{M}_{coll(p)}$

and $\mathcal{M}_{soft}$ are matrices with respect to spinor and colour indices. However, from the colour singlet nature of the external potential and the structure of diagram 1c it follows that they are also colour singlets. Thus, the final factorized expression for the quark form factor amplitude may be written in the following forms, depending on kinematics:

a) for $p^2 = q^2 = m^2$ we have

$$\mathcal{M} = \bar{v}(q) \mathcal{M}_{coll(q)}(\frac{\mu_c}{\mu_{IR}}, \frac{q^2}{\mu_c^2}) \mathcal{M}_{hard}(\frac{Q^2}{\mu_c^2}) \mathcal{M}_{coll(p)}(\frac{\mu_c}{\mu_{IR}}, \frac{p^2}{\mu_c^2}) u(p) \mathcal{M}_{soft}(\frac{\mu_{IR}}{\lambda}, \frac{Q^2}{m^2}) + O(\frac{1}{Q^2}),$$

where λ has the meaning of the IR regularization parameter (e.g., fictitious gluon mass), and

$$\mathcal{M}_{soft}(\frac{\mu_{IR}}{\lambda}, \frac{Q^2}{m^2}) = \langle 0 | T\, \hat{E}_q^+(0,\infty)\, \hat{E}_p(0,\infty) | 0 \rangle_{IR}. \tag{15}$$

The notation $\langle 0|\ldots|0\rangle_{IR}$ indicates that the integration region for the S -subprocess is confined to regimes (4), (5).

b) For $p^2 - m^2 = q^2 - m^2 = -\mu m$ the amplitude taken in the momentum representation is

$$\mathcal{M} = \int d^4\xi\, e^{-ip\xi} \int d^4\eta\, e^{iq\eta}\, \overline{\mathcal{M}}_{coll(q)}(\frac{\mu_c}{\mu_{IR}}, \mu_c \eta)\, \mathcal{M}_{hard}(\frac{Q^2}{\mu_c^2})\, \overline{\mathcal{M}}_{coll(p)}(\frac{\mu_c}{\mu_{IR}}, \mu_c \xi) \cdot \mathcal{M}_{soft}(\mu_{IR}\xi, \mu_{IR}\eta), \tag{16}$$

where

$$\mathcal{M}_{soft}(\mu_{IR}\xi, \mu_{IR}\eta) = \langle 0 | T\, \hat{E}_q(\eta,\infty)\, \hat{E}_q^+(0,\infty)\, \hat{E}_p(0,\infty)\, \hat{E}_p(\xi,\infty) | 0 \rangle_{IR}.$$

Now we pick the free-quark propagator out of the collinear subprocess amplitude

$$\overline{\mathcal{M}}_{coll(p)}(\frac{\mu_c}{\mu_{IR}}, \mu_c \xi) = \int \frac{d^4k}{(2\pi)^4} e^{ik\xi} S_0(k)\, \mathcal{M}_{coll(p)}(\frac{\mu_c}{\mu_{IR}}, \frac{k^2}{\mu_c^2}).$$

The next step is to introduce the α -representation for $S_0(k)$. Proceeding analogously to ref. /9/ (see § 13) we find

$$\mathcal{M} = \frac{1}{\hat{q}-m} \mathcal{M}_{coll(q)}(\frac{\mu_c}{\mu_{IR}}, \frac{q^2}{\mu_c^2}) \mathcal{M}_{hard}(\frac{Q^2}{\mu_c^2}) \mathcal{M}_{coll(p)}(\frac{\mu_c}{\mu_{IR}}, \frac{p^2}{\mu_c^2}) \frac{1}{\hat{p}-m} \cdot \int_0^\infty d\sigma (p^2-m^2) e^{i\sigma(p^2-m^2)} \int_0^\infty d\sigma' (q^2-m^2) e^{i\sigma'(q^2-m^2)} \mathcal{M}_{soft}(-2p\sigma\mu_{IR}, 2q\sigma'\mu_{IR}), \tag{17}$$

where

$$\mathcal{M}_{soft}(-2p\sigma\mu_{IR}, 2q\sigma'\mu_{IR}) = \langle 0|T\hat{E}_q(2\sigma' q,\infty)\hat{E}_q^+(0,\infty)\hat{E}_p(0,\infty)\hat{E}_p^+(-2\sigma p,\infty)|0\rangle_{IR} \cdot \quad (18)$$

Note that eq. (18) describes the IR asymptotics of diagram 1a with nonamputated external quark legs. It is necessary, therefore, to pick out of eq.(17) additional IR singularities associated with these legs. Only then eq.(17) will correspond to asymptotics of the quark form factor.

From eqs.(15), (17) it follows that all the IR singularities of diagram 1a in the kinematics we are interested in are contained in the exponentials path-ordered along the contours 2a,b and averaged over the PT vacuum.

3. Renormalization of the IR Factor

Consider now the dependence of the IR factors obtained in eqs. (15), (18) on the dimensional parameters. For the contour averages (15), (18) there are two of them: the length of the contour (fig. 2) and the parameter μ_{IR} separating collinear and soft regimes of the momenta in the original diagram 1a. It is apparent from fig.2 that the contour length is inversely proportional to the IR regulator (fictitious gluon mass for fig. 2a and quark virtuality for fig. 2b) and, hence, it determines the maximal wave lengths (softness) of the IR gluons belonging to the S subprocess. At the same time, for the contour integrals (15), (18) $1/\mu_{IR}$ serves as a scale the minimal wave lengths~(maximal momenta)$^{-1}$ of the gluons, i.e. μ_{IR} is the cut-off for these integrals. Thus, the IR asymptotics of the quark form factor is related to the UV properties of the contour integrals studied in refs.[10,11].

The infrared factor (15) depends on two-dimensional parameters μ_{IR} and λ introduced via the regularization schemes for the UV and IR singularities, respectively. The integration contour in eq. (15) (see fig. 2a) is closed at infinite and it is smooth everywhere except for the point 0 where there is a cusp characterized by the external angle γ defined in the Minkowski space

$$\operatorname{ch}\gamma = \frac{(pq)}{\sqrt{p^2q^2}} \cdot \quad (19)$$

Hence, the renormalization properties of eq.(15) (and, at the same time, its dependence on μ_{IR}) are described by the following RG equation[11]:

$$\left(\mu_{IR}\frac{\partial}{\partial\mu_{IR}} + \beta(g)\frac{\partial}{\partial g} + \Gamma_{cusp}(g,\gamma)\right)\mathcal{M}_{soft}\left(\frac{\mu_{IR}}{\lambda}, \frac{Q^2}{m^2}\right) = 0 \, , \quad (20)$$

where $\Gamma_{cusp}(g,\gamma)$ is the gauge-invariant cusp anomalous dimension depending on the only characteristics of the contour: the cusp angle[10,11]. We present here our two-loop result for Γ_{cusp} in the MS scheme obtained in ref.[8]:

$$\begin{aligned}\Gamma_{cusp}(g,\gamma) = {} & \frac{\alpha_s}{\pi}C_F(\gamma\operatorname{cth}\gamma - 1) \\ & + \left(\frac{\alpha_s}{\pi}\right)^2 C_F N\Big[\frac{1}{2} + \left(\frac{67}{36} - \frac{\pi^2}{24}\right)(\gamma\operatorname{cth}\gamma - 1) - \operatorname{cth}\gamma\int_0^\gamma dx\, x\operatorname{cth}x \\ & + \operatorname{cth}^2\gamma\int_0^\gamma dx\, x(\gamma - x)\operatorname{cth}x - \frac{1}{2}\operatorname{sh}2\gamma\int_0^\gamma dx\frac{x\operatorname{cth}x - 1}{\operatorname{sh}^2\gamma - \operatorname{sh}^2 x}\ln\frac{\operatorname{sh}\gamma}{\operatorname{sh}x}\Big] \, .\end{aligned} \quad (21)$$

Hence forth we use the asymptotic form of Γ_{cusp} in the limit of large cusp angles $\gamma \gg 1$ (or $Q^2 \gg m^2$)[8]:

$$\Gamma_{cusp}(g,\gamma) = \ln\frac{Q^2}{m^2}\sum_{n=1}^{\infty}\left(\frac{\alpha_s}{\pi}\right)^n C_n a_n + O\left(\ln^0\frac{Q^2}{m^2}\right)$$
$$a_1 = 1 \, , \quad a_2 = \frac{67}{36} - \frac{\pi^2}{12} \, , \quad (22)$$

where C_n is a "maximally non-Abelian"[12] colour factor in the n-th order of the PT expansion over α_s , and a_n are some numerical factors (in the lowest orders of PT $C_n = C_F N^{n-1}$).

Solving eq.(20) we use the boundary condition on $\mathcal{M}_{soft}\left(\frac{\mu_{IR}}{\lambda}, \frac{Q^2}{m^2}\right)$[5]:

$$\mathcal{M}_{soft}\left(\frac{\mu_{IR}}{\lambda}, \frac{Q^2}{m^2}\right) = 1 \quad \text{for} \quad \mu_{IR} = \lambda \quad (23)$$

with account of which we find

$$\mathcal{M}_{soft}\left(\frac{\mu_{IR}}{\lambda}, \frac{Q^2}{m^2}\right) = \exp\left(-\int_\lambda^{\mu_{IR}}\frac{dt}{t}\Gamma_{cusp}(g(t),\gamma)\right) \, . \quad (24)$$

In the kinematics $p^2 - m^2 = q^2 - m^2 = -\mu m \cdot$ the integration contour (fig. 2b) is not closed, and this leads to new UV singularities in eq.(18) compared to eq. (24). The RG equation (20) must be substituted in this case by the following equation[11]:

$$\left(\mu_{IR}\frac{\partial}{\partial\mu_{IR}} + \beta(g)\frac{\partial}{\partial g} + 2\Gamma_{end}(g) + \Gamma_{cusp}(g,\gamma)\right)\mathcal{M}_{soft}(-2\sigma p\mu_{IR}, 2\sigma' q\mu_{IR}) = 0 \quad (25)$$

that contains the additional contribution to the anomalous dimension, the end-point anomalous dimension. In the Feynman gauge and MS -scheme it is equal to

$$\Gamma_{end}(g) = -\frac{\alpha_s}{2\pi}C_F - \frac{7}{48}\left(\frac{\alpha_s}{\pi}\right)^2 C_F N + \ldots \qquad (26)$$

(see ref.[12]). To solve eq.(25), one should know the relevant boundary condition. Note, that the contour shown in fig. 2b has a finite length, and relation (23) does not hold for it. Our calculation of the infrared factor (18) to the order α_s in the MS -scheme and Feynman gauge gives*

$$\mathcal{M}_{soft}(-2\sigma p\mu_{IR}, 2\sigma' q\mu_{IR}) = 1 - \frac{\alpha_s}{\pi}C_F\Big\{(\gamma\,\mathrm{cth}\,\gamma - 1)\ln(-\mu_{IR}^2 e^{2\gamma_E}\sigma\sigma'\sqrt{p^2q^2}) - \frac{1}{2}\gamma\,\mathrm{cth}\,\gamma\ln(-\mu_{IR}^2 e^{2\gamma_E}(p\sigma+q\sigma')^2) + 2\,\mathrm{cth}\,\gamma\int_0^{\gamma}dx\,x\,\mathrm{cth}\,x - \mathrm{cth}\,\gamma\int_{-\eta}^{\gamma-\eta}dx\,x\,\mathrm{cth}\,x - 2\Big\}, \qquad (27)$$

where $\gamma_E = 0.5772\ldots$ and the η angle is defined by the relation

$$\frac{\mathrm{sh}(\gamma-\eta)}{\mathrm{sh}\,\eta} = -\sqrt{\frac{p^2\sigma^2}{q^2\sigma'^2}}\,.$$

In the expression for diagram 1a (eq.(17)) one deals with the integral of eq.(27) over the parameters σ, σ', and the main contribution is due to regions close to the points $\sigma = \sigma' = |p^2-m^2|^{-1} = |q^2-m^2|^{-1}$

$$\mathcal{M}_{soft}\left(\frac{\mu_{IR}^2 p^2}{(p^2-m^2)^2}, \frac{Q^2}{p^2}\right) = \int_0^\infty d\sigma(p^2-m^2)e^{i\sigma(p^2-m^2)}\int_0^\infty d\sigma'(q^2-m^2)e^{i\sigma'(q^2-m^2)}\mathcal{M}_{soft}(-2\sigma p\mu_{IR}, 2\sigma' q\mu_{IR})$$
$$= 1 - \frac{\alpha_s}{\pi}C_F\left[\left(\frac{1}{2}\gamma\,\mathrm{cth}\,\gamma - 1\right)\ln\frac{\mu_{IR}^2 p^2}{(p^2-m^2)^2} - 2 + 2\,\mathrm{cth}\,\gamma\int_0^{\gamma/2}dx\,x\,\mathrm{cth}\,x\right]. \qquad (28)$$

By an appropriate choice of the renormparameter μ_{IR} one can minimize the magnitude of $\mathcal{M}_{soft}$. So, let us introduce the concept of the effective length of the contour 2b[14]. Define it to be equal to the inverse of μ_{IR} for which

$$\mathcal{M}_{soft}\left(\frac{\mu_{IR}^2 p^2}{(p^2-m^2)^2}, \frac{Q^2}{p^2}\right)\Big|_{\mu_{IR}=1/L_{eff.}} = 1\,. \qquad (23)$$

From eq.(23) it follows that for the contour 2a one has

$$L_{eff.} = 1/\lambda \qquad (30)$$

*In addition to the IR asymptotics of the form factor we are interested in, eq.(18) takes into account also the additional IR singularities of the external quark propagators and that is why it is gauge-dependent.

to all orders of PT. For the contour 2b we find from eq.(28) the value $L_{eff.}$ in the one-loop approximation (and in Feynman gauge):

$$L_{eff.} = \frac{\sqrt{p^2}}{|p^2-m^2|}\exp\left(\frac{\mathrm{cth}\,\gamma\int_0^{\gamma/2}dx\,x\,\mathrm{th}\,x - 1}{\frac{1}{2}\gamma\,\mathrm{cth}\,\gamma - 1}\right). \qquad (31)$$

This relation can be considerably simplified in the limit $Q^2 \gg p^2$

$$L_{eff.} = \frac{1}{|p^2-m^2|}\left(Q^2p^2\right)^{1/4}. \qquad (32)$$

Taking into account the higher orders of PT in expansion (28) of eq.(31) will modify eq. (32) by the α_s -corrections. However, from purely dimensional considerations it follows that the effective length of the contour 2b to all orders of PT is a function of the cusp angle γ and it linearly depends on the dimensional characteristics of the contour $\sigma \sim \sigma' \sim |p^2-m^2|^{-1}$ (see eqs.(18),(28)), i.e.:

$$L_{eff.} = \frac{\sqrt{p^2}}{|p^2-m^2|}f(\gamma)\,, \quad f(\gamma) \underset{\gamma\to\infty}{=} \left(\frac{Q^2}{p^2}\right)^{1/4} + O(\alpha_s)\,. \qquad (33)$$

With account of the boundary condition (2a) we find the solution of the RG equation (25) and substitute it into eq.(28):

$$\mathcal{M}_{soft}\left(\frac{\mu_{IR}^2p^2}{(p^2-m^2)^2}, \frac{Q^2}{p^2}\right) = \exp\left\{-\int_{1/L_{eff.}}^{\mu_{IR}}\frac{dt}{t}\left(\Gamma_{cusp}(g(t),\gamma) + 2\Gamma_{end}(g(t))\right)\right\}. \qquad (34)$$

Observe that eq.(34) satisfies the following infrared RG equation

$$\left(L_{eff.}\frac{\partial}{\partial L_{eff.}} + \beta(g)\frac{\partial}{\partial g} + \Gamma_{cusp}(g,\gamma) + 2\Gamma_{end}(g)\right)\mathcal{M}_{soft}\left(\frac{\mu_{IR}^2p^2}{(p^2-m^2)^2}, \frac{Q^2}{p^2}\right) = 0\,. \qquad (35)$$

4. Renormalization Group Equations for the Infrared Asymptotics

Amplitude (14) for the on-shell quark form factor contains all the IR singularities in the factor $\mathcal{M}_{soft}(\frac{\mu_{IR}}{\lambda}, \frac{Q^2}{m^2})$. Using its explicit form (eq.(24)) one can obtain

$$\frac{d\ln\mathcal{M}}{d\ln\lambda} = \frac{d\ln\mathcal{M}_{soft}(\frac{\mu_{IR}}{\lambda}, \frac{Q^2}{m^2})}{d\ln\lambda} = \Gamma_{cusp}(g(\lambda),\gamma)\,. \qquad (36a)$$

Thus, the IR asymptotics of the form factor in this kinematics is described by the RG equation

$$\left(-\lambda\frac{\partial}{\partial\lambda}+\beta(g)\frac{\partial}{\partial g}+\Gamma_{cusp}(g,\gamma)\right)\mathcal{M}\left(\frac{\lambda^2}{m^2},\frac{Q^2}{m^2}\right)=0. \tag{36b}$$

The validity of eqs. (36) is supported by calculations of the leading IR asymptotics in the lowest orders of PT/15/ corresponding to the first term of the α_s expansion for Γ_{cusp} in eq.(21). The nonleading two-loop IR asymptotics calculated in ref./16/ completely agrees in two limiting cases $\gamma\to\infty$ $(Q^2\gg m^2)$ and $\gamma\to 0$ $(Q^2\ll m^2)$ with the limiting values of the two-loop Γ_{cusp} from eq.(21).

In the case when quark momenta are slightly off-shell ($p^2-m^2=$ $=q^2-m^2=-\mu m,\ \mu\leqslant m$) all the IR singularities of amplitude (17) are contained in the factor $\mathcal{M}_{soft}$ (eq.(34)). However, to get the form factor amplitude from eq. (17), one should extract external free quark propagators from $\mathcal{M}$:

$$\mathcal{M}\to\overline{\mathcal{M}}=(\hat{q}-m)\,\mathcal{M}\,(\hat{p}-m)$$

and subtract from $\overline{\mathcal{M}}$ the additional IR singularities of the non-amputated propagators $S(p)$ and $S(q)$ (see the footnote to eq. (27)):

$$\overline{\mathcal{M}}\to\mathcal{M}_F=\overline{\mathcal{M}}/\mathcal{M}_{amp.},\quad \mathcal{M}_{amp}=-(p^2-m^2)\frac{dS(p)}{d(p^2-m^2)}\Big|_{p^2-m^2=-\mu m}.$$

Differentiating $\overline{\mathcal{M}}$ with respect to μ

$$\frac{d\ln\overline{\mathcal{M}}}{d\ln\mu}=\frac{d\ln\mathcal{M}_{soft}\left(\frac{\mu^2p^2}{(p^2-m^2)^2},\frac{Q^2}{p^2}\right)}{d\ln\mu}$$

using the explicit form of eq.(34) and incorporating the dependence of the effective length $L_{eff.}$ on μ ($L_{eff.}\sim 1/\mu$, see eq.(33)) we get

$$\frac{d\ln\overline{\mathcal{M}}}{d\ln\mu}=\Gamma_{cusp}\left(g(1/L_{eff.}),\gamma\right)+2\Gamma_{end}\left(g(1/L_{eff.})\right). \tag{37}$$

To find an equation for $\mathcal{M}_F$, we utilize the results of our analysis of the IR properties of the quark propagator $S(p)$ in Feynman gauge/14/. All the IR singularities of the quark propagator can be extracted into a separate factor given by a PT-vacuum -averaged P -exponential, path-ordered along the straight-line

segment between points 0 and $2p\mu|p^2-m^2|^{-1}$. As a result, $\mathcal{M}_{amp}$ satisfies an equation analogous to eq. (37)/14/:

$$\frac{d\ln\mathcal{M}_{amp}}{d\ln\mu}=2\Gamma_{end}\left(g(1/L^{(s)}_{eff.})\right), \tag{38}$$

where $L^{(s)}_{eff.}$ is the effective length of the above contour equal to

$$L^{(s)}_{eff.}=\frac{\sqrt{p^2}}{|p^2-m^2|}c_1=\frac{1}{\mu}c_1\ ,\quad c_1=\exp(2+O(\alpha_s)).$$

Uniting now eqs.(37) and (38) we get finally our equation for the quark form factor:

$$\frac{d\ln\mathcal{M}_F}{d\ln\mu}=\Gamma_{cusp}\left(g\left(\tfrac{1}{L_{eff.}}\right),\gamma\right)+2\Gamma_{end}\left(g\left(\tfrac{1}{L_{eff.}}\right)\right)-2\Gamma_{end}\left(g\left(\tfrac{1}{L^{(s)}_{eff.}}\right)\right). \tag{39}$$

Substituting into the r.h.s. of this equation the quantities defined earlier we obtain

$$\frac{d\ln\mathcal{M}_F}{d\ln\mu}=\frac{\alpha_s(\mu)}{\pi}C_F(\gamma\,\mathrm{cth}\gamma-1)+\left(\frac{\alpha_s(\mu)}{\pi}\right)^2C_FN\Big[\frac{1}{2}+\frac{11}{3}\mathrm{cth}\gamma\int_0^{\gamma/2}dx\,x\,\mathrm{th}x$$
$$+\left(\frac{67}{36}-\frac{\pi^2}{24}\right)(\gamma\,\mathrm{cth}\gamma-1)-\mathrm{cth}\gamma\int_0^{\gamma}dx\,x\,\mathrm{cth}x+\mathrm{cth}^2\gamma\int_0^{\gamma}dx\,x(\gamma-x)\,\mathrm{cth}x \tag{40}$$
$$-\frac{1}{2}\mathrm{sh}2\gamma\int_0^{\gamma}dx\,\frac{x\,\mathrm{cth}x-1}{\mathrm{sh}^2\gamma-\mathrm{sh}^2x}\ln\frac{\mathrm{sh}\gamma}{\mathrm{sh}x}\Big]+O(\alpha_s^3(\mu)).$$

Our equation (40), obtained in the MS -scheme, differs (by $\left(\frac{\alpha_s}{\pi}\right)^2C_FN\left[-\frac{23}{18}(\gamma\,\mathrm{cth}\gamma-1)+\frac{5}{24}\gamma\,\mathrm{th}\frac{\gamma}{2}\right]$) from the results of ref./17/ obtained within the MOM-scheme.

Consider the $Q^2\gg m^2$ limit of eq.(39):

$$\frac{d\ln\mathcal{M}_F}{d\ln\mu}=\frac{\alpha_s\left(\mu\left(\frac{m^2}{Q^2}\right)^{1/4}\right)}{\pi}C_F\ln\frac{Q^2}{m^2}+\left(\frac{\alpha_s\left(\mu\left(\frac{m^2}{Q^2}\right)^{1/4}\right)}{\pi}\right)^2C_FN\left[\frac{67}{36}-\frac{\pi^2}{12}\right]\ln\frac{Q^2}{m^2}$$
$$+O\left(\alpha_s^3,\ln^0\frac{Q^2}{m^2}\right). \tag{41}$$

and note the two properties of eq.(41): the nontrivial argument of the effective coupling constant and absence (in the first two orders of PT) of the logarithmic contributions proportional to $\ln^2\frac{Q^2}{m^2}$ and higher powers of $\ln\frac{Q^2}{m^2}$. On the other hand, two-loop calculations performed in refs./17,18/ produce a slow-convergent expansion

$$\frac{d\ln\mathcal{M}_F}{d\ln\mu}=\frac{\alpha_s(\mu)}{\pi}C_F\ln\frac{Q^2}{m^2}+\frac{11}{24}\left(\frac{\alpha_s(\mu)}{\pi}\right)^2C_FN\ln^2\frac{Q^2}{m^2}+O\left(\alpha_s^2\ln\frac{Q^2}{m^2},\alpha_s\ln^0\frac{Q^2}{m^2}\right). \tag{42}$$

Comparing eqs. (41) and (42) we conclude that in higher orders of PT the $\alpha_s^n(\mu)\ln^k\frac{Q^2}{m^2}$ -terms $(1<k\leqslant n)$ of expansion (42) are propor-

tional to the β -function coefficients and their role is to modify the argument of the coupling constant.

These important properties of eq.(41) are preserved in all orders of PT. Indeed, from eqs.(39) and (36) it follows that the behaviour of $\frac{d \ln m_F}{d \ln \mu}$ and $\frac{d \ln m}{d \ln \lambda}$ for $Q^2 \gg m^2$ is determined by the asymptotics of the cusp anomalous dimension (see eq.(22)). Substituting eq.(22) in eq. (39) we find, for example, that

$$\frac{d \ln m_F}{d \ln \mu} = \sum_{n=1}^{\infty} \left(\frac{\alpha_s(1/L_{eff.})}{\pi}\right)^n c_n a_n \ln \frac{Q^2}{m^2} + O\left(\ln^0 \frac{Q^2}{m^2}\right), \tag{43}$$

i.e., that the expansion parameter in eq.(43) is $\alpha_s(1/L_{eff.})$. Eq.(43) can be immediately generalized for QED, where $c_1 = 1$, $c_n = 0$ for $n \geq 2$, $\alpha(1/L_{eff.}) \simeq \alpha(0)$ and eq.(43) reproduces the well-known exponentiation property of the one-loop IR asymptotics /19/. For QCD such an exponentiation is absent, but one can conclude from eq.(43) that truncating the PT series in the r.h.s. of eq.(43) one neglects in its solution only the terms of an order of $O(\alpha_s(1/L_{eff.}))$ having "maximally non-Abelian" colour structure .

5. Conclusions

In the present paper we studied the infrared asymptotics of the colour singlet quark form factor within the framework of perturbative QCD. The use of the gauge properties of QCD enabled us to perform the factorization procedure for contirubions of vaious subprocesses and to single out all the IR singularities of the process into a universal IR factor. It has been shown that this factor is given by a PT-vacuum-averaged P-exponential, path-ordered along some definite contour of fig. 2 type. The specific form of the contour is unambiguosly fixed by kinematics of external quark lines. We demonstrated that there exists a one-to-one correspondence between UV and IR singularities of the contour averages present in the expression for the IR factors. This fact enabled us, first, to prove the applicability of the RG methods to the IR problem under study and to formulate the appropriate RG equations, the solution of which describes the whole IR asymptotics of the form factor and, second, it enabled us to demostrate that the anomalous dimensions of the infrared RG equations coincide with an appropriate combination of the

cusp and end-point anomalous dimensions of contour averages, the two-loop properties of which were studied earlier in refs. /8,12/. Third, this approach to the IR problem allows one to find an arbitrary nonleading IR asymptotics of the form factor, modifying in a nontrivial way the argument of the coupling constant in the lowest orders of PT and estimating here the magnitude of the PT terms not taken into account. In particular, we reproduced the existing results for the quark form factor and supplemented in higher orders of PT on the basis of the performed one-loop calculations.

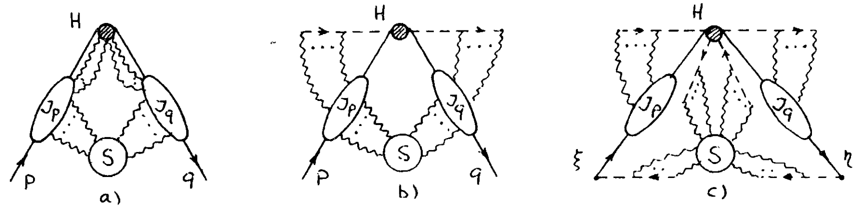


Fig. 1a) Feynman diagram for color singlet quark form factor, containing hard (H), collinear (J_p and J_q) and soft (S) subgraphs.

b) Diagram Fig. 1a) with factorized contribution of hard subgraph.

c) Diagram corresponding to totally factorized form factor amplitude.

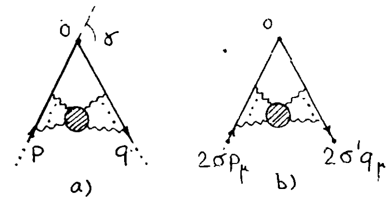


Fig. 2. The integration contours in the Minkowski space determined the infrared asymptotics of the quark form factor for on-shell (a) or off-shell (b) external momenta.

APPENDIX

Here we consider the properties of the propagators of quarks, gluons and "ghosts" in an external gluonic field, defining them as solutions of the Dyson-Schwinger equations illustrated graphically in fig. 3.

The quark propagator $S(x,y;A)$ in the external gluonic field $\hat{A}_\mu$ satisfies the following equation:

$$(i\partial_\mu\gamma^\mu + g\hat{A}_\mu\gamma^\mu - m)\,S(x,y;A) = -\delta(x-y). \tag{A.1}$$

We shall look for its solution in the form

$$S(x,y;A) = \hat{E}_\rho(x,\infty)\,\tilde{S}(x,y;A)\,\hat{E}^+_\rho(y,\infty), \tag{A.2}$$

where $\hat{E}_\rho(x,\infty)$ is defined in eq.(8). After substituting eq.(A.2) into eq.(A.1) we get an equation for $\tilde{S}$:

$$(i\partial_\mu\gamma^\mu + g\hat{A}^U_\mu\gamma^\mu - m)\,\tilde{S}(x,y;A) = -\delta(x-y), \tag{A.3}$$

where

$$\hat{A}^U_\mu(x) = \hat{E}^+_\rho(x,\infty)\left(\hat{A}_\mu(x) + \frac{i}{g}\partial_\mu\right)\hat{E}_\rho(x,\infty)$$

is a gauge-transformed gluonic potential satisfying the axial gauge condition $\rho_\mu\hat{A}^U_\mu = 0$ /5/. Hence, if $S(x,y;A)$ is the propagator of a quark participating in the hard or collinear subprocess J_ρ., then the contribution of those terms in the solution of eq.(A.3) which are proportional to $\hat{A}^U_\mu(k)$ with momenta k_μ in the collinear or IR regimes, respectively, has a power-law suppression. Thus, in the lowest-twist approximation we have

$$\tilde{S}(x,y;A) = S_0(x-y)\left[1 + O(1/Q^2)\right]. \tag{A.4}$$

Substituting eq.(A.4) into eq.(A.2) we reproduce eq.(7).

The gluon propagator $D^{\mu\nu}(x,y;A)$ in the external gluonic field $\hat{A}_\mu$ satisfies the equation:

$$\left(\Pi_{\mu\nu}(x,A) + \frac{1}{\alpha}\Delta_\mu(x)\Delta_\nu(x)\right)D_{\nu\rho}(x,y;A) = -g_{\mu\rho}\delta(x-y),\ \alpha\to 0, \tag{A.5}$$

where

$$\Pi^{ab}_{\mu\nu}(x;A) = g_{\mu\nu}(D^2)^{ab} - \frac{1}{2}[D_\mu, D_\nu]^{ab}_+ - \frac{3}{2}g f^{abc}G^c_{\mu\nu}$$

$$D^{ab}_\mu = \partial_\mu\delta^{ab} + g f^{acb}A^c_\mu, \quad G_{\mu\nu} = \frac{1}{g}[D_\mu, D_\nu]$$

and the gluonic field satisfies the gauge condition

$$\Delta_\mu(x)\,\hat{A}_\mu(x) = 0. \tag{A.6}$$

Solution of eq.(A.5) is looked for in the form

$$D_{\mu\nu}(x,y,A) = \tilde{E}_\rho(x,\infty)\,\tilde{D}_{\mu\nu}(x,y,A)\,\tilde{E}^+_\rho(y,\infty). \tag{A.7}$$

Then $\tilde{D}_{\mu\nu}$ is a solution of the equation

$$\left(\Pi_{\mu\nu}(x;A^U) + \frac{1}{\alpha}\tilde{\Delta}_\mu(x)\tilde{\Delta}_\nu(x)\right)\tilde{D}_{\nu\rho}(x,y;A) = -g_{\mu\rho}\delta(x-y),\ \alpha\to 0, \tag{A.8}$$

where the following notation is introduced:

$$\tilde{\Delta}_\mu(x) = \tilde{E}^+_\rho(x,\infty)\,\Delta_\mu(x)\,\tilde{E}_\rho(x,\infty). \tag{A.9}$$

Comparing eqs.(A.5) and (A.8), we conclude that $\tilde{D}_{\nu\rho}$ is the gluonic propagator in the gauge $\tilde{\Delta}_\mu\hat{A}_\mu = 0$ in the external field of the gluons $\hat{A}^U_\mu$ such that $\rho_\mu\hat{A}^U_\mu = 0$. Hence, in the lowest twist approximation (in a complete analogy with eq.(A.4)) the solution of eq. (A.8) is

$$\tilde{D}^{\mu\nu}(x,y;A) = \tilde{D}^{\mu\nu}_0(x-y)\left[1 + O(1/Q^2)\right], \tag{A.10}$$

where $\tilde{D}^{\mu\nu}_0$ is a free gluonic propagator in the gauge

$$\tilde{\Delta}_\mu(x)\hat{A}_\mu(x) = 0 \tag{A.11}$$

obtained from eq.(A.6) via transformation (A.9). In the class of the simplest gauge conditions (A.6) (with $\Delta_\mu(x)$ not depending on $\hat{A}_\mu$) it can be shown that the absence of "ghosts" in this gauge implies that

$$\Delta_\mu(x) = \tilde{\Delta}_\mu(x), \quad \tilde{D}^{\mu\nu}_0(x-y) = D^{\mu\nu}_0(x-y). \tag{A.12}$$

In the α -gauge, e.g., $\Delta_\mu(x) \equiv \partial_\mu$, and with account of eq.(A.9), the modified gauge condition (A.11) has the following form:

$$\left(\partial^\mu - ig\tilde{B}^\mu(x)\right)\tilde{A}_\mu(x) = 0,\ \tilde{B}^\mu(x) = \frac{i}{g}\tilde{E}^+_\rho(x,\infty)\partial^\mu\tilde{E}_\rho(x,\infty) \tag{A.13}$$

i.e., it is the background field gauge.

If gauge (A.6) is not a ghost free one, then the ghost propagator $D(x,y;A)$ (see fig. 3) in the external gluonic field $\hat{A}_\mu$ satisfies the equation

$$\Delta_\mu(x)\tilde{D}_\mu(x)D(x,y;A) = -\delta(x-y). \tag{A.14}$$

Using the ansatz

$$D(x,y;A) = \tilde{E}_\rho(x,\infty)\, \tilde{D}(x,y;A)\, \tilde{E}^{+}_\rho(y,\infty) \qquad (A.15)$$

we obtain the equation for $\tilde{D}$:

$$\tilde{\Delta}_\mu(x)\, \tilde{D}_\mu(x)\, \tilde{D}(x,y;A) = -\delta(x-y) . \qquad (A.16)$$

The solution of this equation is the ghost propagator corresponding to the modified gauge (A.11) in the external gluonic field $\hat{A}_\mu$. Hence, in the lowest twist approximation we get

$$\tilde{D}(x,y;A) = \tilde{D}(x-y)\left[1 + O(1/Q^2)\right],$$

where $\tilde{D}(x-y)$ is the ghost propagator in gauge (A.11).

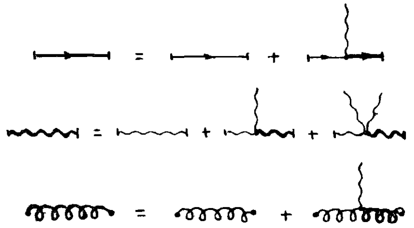


Fig. 3. Graphical Dyson-Schwinger equation for quark, gluon and "ghost" propagators in the external gluon field, respectively.

R e f e r e n c e s

1. Cornwall J.M., Tiktopoulos G., Phys.Rev.Lett. 1975, 35, 338; Belokurov V.V., Usyukina N.I., Teor.Mat.Fiz. 1979, 41, 157; 1980, 45, 171.
2. Frenkel J., Taylor J.C., Nucl.Phys. 1976, B116, 185; Cornwall J.M., Tiktopoulos G., Infrared behaviour of nonabelian gauge theories II, UCLA/76/TEP/22; Kinoshita T., Ukawa A., Phys.Rev. 1977, D16, 332.
3. Poggio E.C., Pollak G., Phys.Lett. 1977, 71B, 135; Smilga A.V., Nucl. Phys. 1979, B161, 449.
4. Sterman G., Phys.Rev. 1978, D17, 2773; Ellis R.K. et al., Nucl.Phys. 1979, B152, 285; Radyushkin A.V., Particles and Nuclei, 1983, 14, 58.

5. Ivanov S.V., Korchemsky G.P., Phys.Lett. 1985, 154B, 197; Ivanov S.V., Korchemsky G.P., Radyushkin A.V., JINR preprint E2-85-595, Dubna, 1985.
6. Collins J.C., Soper D.E., Sterman G., Phys.Lett 1984, 134B, 263; Bodwin G.T., Phys.Rev. 1985, D31, 2616.
7. Efremov A.V., Radyushkin A.V., Teor.Mat.Fiz., 1980, 44, 327.
8. Korchemsky G.P., Radyushkin A.V., JINR preprint E2-85-779, Dubna, 1985.
9. Popov V.N., Functional integral in quantum field theory, Moscow, Atomizdat, 1976.
10. Polyakov A.M., Nucl.Phys. 1980, B164, 171; Aref'eva I.Ya., Phys.Lett., 1980, 93B, 347; Potsenko V.S., Vergeles S.N., Nucl.Phys. 1980, B169, 527; Brandt R.A., Neri F., Sato M.-A., Phys.Rev. 1981, D24, 879.
11. Brandt R.A. et al., Phys.Rev. 1982, D26, 3611.
12. Aoyama S., Nucl.Phys. 1982, B194, 513.
13. Gatheral J.G.M., Phys.Lett. 1984, 133B, 90; Frenkel J., Taylor J.C., Nucl.Phys. 1984, B246, 231.
14. Korchemsky G.P., Radyushkin A.V., JINR preprint E2-85-901, Dubna, 1985.
15. Frenkel J. et al., Phys.Lett. 1976, 64B, 211; Nucl.Phys. 1977, B121, 58; Rafael de E., Korthals-Altes C., Phys.Lett. 1976, 62B, 320.
16. Frenkel J., Frenkel M.-L., Taylor J.C., Nucl.Phys. 1977, B124, 268.
17. Sitenko Yu.A., Yad.Fiz., 1978, 27, 1098.
18. Poggio E.C., Phys.Lett. 1977, 68B, 347.
19. Sudakov V.V., ZhETF, 1956, 30, 87; Yennie D.R., Frautchi S.C., Suura H., Ann. of Phys. 1961, 13, 379; Grammer G., Yennie D.R., Phys.Rev. 1973, D8, 4332; Korthals-Altes C.P., Rafael de E., Nucl.Phys. 1976, B106, 237; Mueller A.H., Phys.Rev. 1979, D20, 2037; Collins J.C., Phys.Rev. 1980, D22, 1478.

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Инфракрасная асимптотика пертурбативных КХД. Вершинные функции

Изучена инфракрасная асимптотика синглетного по цвету кваркового формфактора в рамках пертурбативной КХД. Найдена ее связь с перенормировочными свойствами контурных интегралов от калибровочного потенциала. Показано, что инфракрасная асимптотика формфактора поглощается новыми нелокальными объектами - экспонентами, упорядоченными вдоль контура и усредненными по вакууму теории возмущений. Вид контура однозначно фиксируется кинематикой процесса. Установлена связь ультрафиолетовых и инфракрасных особенностей соответсвующих контурных интегралов и на ее основе сформулировано ренормгрупповое уравнение для инфракрасной асимптотики кваркового формфактора. Решения этого уравнения сравниваются с результатами вычислений в низших порядках теории возмущений.

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Infrared Asymptotics of Perturbative QCD. Vertex Functions

The infrared asymptotics of the colour singlet quark form factor are investigated within the framework of perturbative QCD. The deep connection between infrared problem and renormalization properties of contour averages is found. It is demonstrated that the infrared asymptotics of the form factor are accumulated by new nonlocal objects (the path ordered exponentials) averaged over the perturbation theory vacuum. The form of the contours is uniquely fixed by kinematics of the process under consideration. The one-to-one correspondence of the ultraviolet and infrared singularities of corresponding contour integrals is established and the renormalization group equation for the infrared asymptotics of the quark form factor is formulated. The solution of the equation is compared with the results of the lowest perturbation theory calculations.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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