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HAMILTONIAN FORMULATION OF GAUGE THEORIES WITH AN EXPLICIT SOLUTION OF THE CONSTRAINT EQUATION

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Introduction

For a rigid quantization of gauge theories the generalized Hamiltonian formulation (GHF) is used /1,2,3/. It allows one to carry out the quantization procedure without an explicit solution of the constraint equations and gauge conditions (by a common opinion, the gauge-theory constraints cannot explicitly be solved).

During the last several years, in the gauge field theory a set of problems has been faced, whose solution depends on the gauge choice: for example, computation of the fermionic Green functions, quantization of the theories with chiral anomalies, interpretation of the gauge ambiguities etc.

In this paper we have tried to solve these problems in the framework of the standard hamiltonian formulation (HF) after an explicit solution of the constraint equations.

In section 1 the free electromagnetic field is used to present the quantization procedure proposed. Section 2 is devoted to computation of the fermionic Green function in QED_{3+1}. In section 3 the quantization of the chiral Schwinger model is considered. In section 4 the same method is applied to quantization of non-Abelian theory.

1. Canonical quantization of the free electromagnetic field

Dirac's quantization of the free electromagnetic field

$$\mathcal{L}(x) = -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} = \frac{1}{2}(\partial_0 A_i - \partial_i A_0)^2 - \frac{1}{2} B_i^2$$

$$S = \int d^4x\, \mathcal{L}(x) \tag{1}$$

consists in the following (see, for example ref. /3/). First, one determines the canonical momenta and Poisson brackets

$$\pi_\mu = \frac{\partial \mathcal{L}}{\partial(\partial_0 A^\mu)} = F_{0\mu}\, ; \quad -i\{\pi_\mu, A_\nu\} = g_{\mu\nu}\,\delta^3(\vec{x}-\vec{y}) \tag{2}$$

and finds out the primary ($\pi_0 = F_{00} = 0$) and secondary ($\partial_i \pi_i = 0$) constraints which are present in the Lagrangian itself. In the case under consideration the secondary constraints coincide with the Gauss law:

$$\frac{\delta S}{\delta A_0} = 0 \quad \Longrightarrow \quad \partial_i^2 A_0 = \partial_i \partial_0 A_i \, . \tag{3}$$

Then, one supplements these constraints with the gauge equations (for example, the Coulomb gauge ones $A_0 = 0$ and $\partial_i A_i = 0$), so that the whole set $\{ \varphi_1 = \pi_0 \; ; \; \varphi_2 = \partial_i \pi_i \; ; \; \varphi_3 = A_0 \; ; \; \varphi_4 = \partial_i A_i \}$ would form a second-class constraint system with nondegenerate Poisson-brackets matrix $\{ \varphi_\alpha , \varphi_\beta \} = C_{\alpha\beta} \; ; \; \det C \neq 0$. The next step is to determine independent variables with the help of the inverse matrix $C^{-1}_{\alpha\beta}$: $A' = A - \{ A, \varphi_\alpha \} C^{-1}_{\alpha\beta} \varphi_\beta$. As a result, one gets in the Coulomb gauge $\partial_i A_i^T = 0$, $A_0^T = 0$ nonlocal commutational relations

$$i [E_i^T (\vec{x}, t), A_j^T (\vec{y}, t)] = \delta_{ij}^T \delta^3 (\vec{x} - \vec{y})$$

$$\delta_{ij}^T = \left(\delta_{ij} - \partial_i \frac{1}{\partial^2} \partial_j \right) \; ; \; E_i^T = \pi_i^T = \delta_{ij}^T \dot{A}_j \, . \tag{4}$$

It is important to note the specific position the Coulomb gauge takes, i.e., a transition to another gauge, for example $A_3 = 0$, is connected with noncanonical transformations and leads to noncanonical commutation relations (the difficulties in quantization in other gauges are considered, for example, in paper /4/).

In the Dirac scheme described above the conserved quantities constructed with the canonical energy-momentum tensor do not lead to a correct transformation law for the fields A^T . As has been shown by Schwinger /5/, the crucial point for restoring the relativistic covariance of the transverse variables A^T is the nonlocality of the commutation relations (4)*) and the choice of the gauge-invariant energy-momentum tensor (Belinfante tensor)

$$T^S_{\mu\nu} = F_{\mu\lambda} F^\lambda{}_\nu - g_{\mu\nu} \mathcal{L} . \tag{5}$$

This tensor differs from the canonical one by a full derivative $\partial_\lambda \chi_{\lambda\mu\nu}$ of the antisymmetric in μ, ν tensor $\chi_{\lambda\mu\nu}$. Together with the Schwinger term $\partial_i \delta^3 (\vec{x} - \vec{y})$ in the commutator (4) it restores the correct transformation properties of the transverse fields $A_\mu^T = (0, A_i^T)$.

*) The arguments for using local commutation relations $i [E_i (\vec{x}, t), A_j (\vec{y}, t)] = \delta_{ij} \delta^3 (\vec{x} - \vec{y})$ in the Coulomb gauge are by no means convincing.

$$\delta A_\mu^T = i \varepsilon_K [M^S_{0K} , A_\mu^T] = \delta_L^0 A_\mu^T + \partial_\mu \Lambda \, , \tag{6}$$

where $\delta_L^0 A_\mu^T$ being the ordinary Lorentz transformation, and

$$\Lambda = \frac{1}{\partial^2} \varepsilon_K \dot{A}_K^T$$

$$M^S_{0K} = \int d^3 x \left(t \, T^S_{0K} - x_K T^S_{00} \right) . \tag{7}$$

The physical meaning of transformations (6) is that the transverse fields A^T are changed together with the time-axis $\ell_\mu^0 = (1, 0, 0, 0)$ rotations $\ell \cdot x = \ell_\mu^0 x^\mu = t$. The transition to another axis $\ell_\mu = \ell_\mu^0 + \delta_L^0 \ell_\mu^0$ simultaneously changes the gauge. Thus, the transverse relativistically covariant fields considered by Schwinger, /5/ practically do not correspond to any fixed gauge choice and this is in fact, the proof of their relativistic covariance. It is hard to realize in the Dirac quantization approach the specific role of the transverse variables A^T : the nonstandard transformation properties, non-correspondence to a fixed gauge in different Lorentz systems, the necessity of nonlocal commutation relations (4). At the same time all these properties are trivial consequences of the quantization scheme based on the explicit solution of the constraint equations.

Indeed, let us consider Lagrangian (1) and tensor (5) on the explicit solutions of the constraint (3) .

$$A_0 (A_i) = - \frac{1}{4\pi} \int d^3 x \frac{\partial_i \partial_0 A_i}{| \vec{x} - \vec{y} |} \, . \tag{8}$$

Because of their gauge invariance, expressions (1) and (5) depend only on the variables

$$A_i^T (A) = \delta_{ik}^T A_k$$

$$E_i^T = \partial_0 A_i - \partial_i A_0 (A) = \delta_{ij}^T \dot{A}_j = \partial_0 A_i^T \, , \tag{9}$$

which are connected with the initial fields A_K in a nonlocal way.

The fields (9) describe only two independent degrees of freedom. This is a result of their invariance under gauge transformations of the initial fields

$$A_i^\lambda(\vec{x},t) = A_i(\vec{x},t) + \partial_i\lambda(\vec{x},t)$$

$$A_0(A_i^\lambda) = A_0 + \partial_0\lambda$$

$$A_i^T(A_i+\partial_i\lambda) = A_i^T(A_i)\ ;\quad \partial_i A_i^T \equiv 0 \qquad (10)$$

To summarize: the HF-method for quantization of gauge fields consists in solving explicitly the constraint equations (the Gauss law)

$$\frac{\delta S}{\delta A_0} = 0$$

and applying the gauge-invariance principle in constructing the energy-momentum tensor and the canonical variables.

The only ambiguity in such an approach is in the time-axis $x\cdot\ell^0 = t$ choice. Removing of this ambiguity is the subject of Section 2.

2. Proof of the relativistic covariance of the fermion Green function in QED.

The transverse variables which appear naturally in solving the constraint equations are convenient in calculating some tangible physical effects. For example, the Lamb shift corrections $O(\alpha^6)$ are calculated only by the use of these variables. On the other hand, just for them the renormalization procedure is not held because of the absence of manifest relativistic covariant expression for the electron Green function /6,7/.

For constructing such an expression, we shall apply to the electrodynamics

$$\mathcal{L}(x) = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar\Psi(i\gamma^\mu\nabla_\mu - m)\Psi$$

$$\nabla_\mu = \partial_\mu + ieA_\mu \qquad (11)$$

the HF-quantization method. The Hamiltonian, momentum and other conserved quantities $H^S = \int d^3x\, T^S_{00}$; $P^S_k = \int d^3x\, T^S_{0k}$; M^S_{0k} will be obtained with the help of the Belinfante tensor

$$T^S_{\mu\nu} = F_{\mu\lambda}F^\lambda{}_\nu + i\bar\Psi\gamma^\nu\nabla_\mu\Psi - g_{\mu\nu}\mathcal{L} + \qquad (12)$$
$$+\frac{i}{2}\partial^\lambda\left\{\bar\Psi\left[\frac{1}{2}[\gamma_\lambda,\gamma_\nu]\gamma_\mu + \gamma_\lambda g_{\nu\mu} - \gamma_\nu g_{\lambda\mu}\right]\Psi\right\}.$$

Let us choose the time axis $\ell^0_\mu = (1,0,0,0)$. On the Gauss-equation solution

$$\frac{\delta S}{\delta A_0} = 0 \quad \Longrightarrow \quad A_0(A_i) = \frac{1}{\partial_k^2}(\partial_i\partial_0 A_i - j_0)$$

the Lagrangian and the Belinfante tensor are expressed only in terms of nonlocal gauge-invariant variables

$$\hat A_i^I(A_k) = v_I(\hat A_i + \partial_i)v_I^{-1} \equiv \hat A_i^T = ie\,\delta^T_{ij}A_j$$

$$\Psi^I = v_I\Psi = \Psi^T, \qquad (13)$$

where

$$\hat A_\mu = ieA_\mu\ ;\quad A^T_\mu = \left(-\frac{1}{\partial^2}j_0\,,\ A_i^T\right)$$

$$v_I = \exp\left\{\int^t dt'\frac{1}{\partial_k^2}\partial_i\partial_{0'}\hat A_i\right\} = \exp\left\{\frac{1}{\partial_k^2}\partial_i\hat A_i\right\}. \qquad (14)$$

Now the Hamiltonian takes the form

$$H = \int d^3x\left[\frac{1}{2}F_{0i}^{T2} - \frac{1}{2}(\varepsilon_{ijk}\partial_j A_k^T)^2 - i\bar\Psi^T\nabla_i(A^T)\Psi^T\right]. \qquad (15)$$

Commutational relations

$$\{\Psi^T_\alpha(\vec x,t),\ \Psi^{T+}_\beta(\vec y,t)\} = \delta_{\alpha\beta}\,\delta^3(\vec x-\vec y)$$

$$i[\dot A_i^T(\vec x,t),\ A_j^T(\vec y,t)] = \delta^T_{ij}\,\delta^3(\vec x-\vec y)$$

together with the Belinfante tensor (12) lead to the following transformation laws for the operators Ψ^T, A^T_μ /5/ under the Lorentz transformation with parameters ε_k:

$$\delta\Psi^T = i\varepsilon_k[M^S_{0k},\Psi^T] = \delta^0_L\Psi^T - ie\Lambda\Psi^T$$

$$\delta A^T_\mu = i\varepsilon_k[M^S_{0k},\ A^T_\mu] = \delta^0_L A^T_\mu + \partial_\mu\Lambda$$

$$\Lambda = \varepsilon_k\frac{1}{\partial_i^2}(\dot A_k^T + \partial_k A_0^T), \qquad (16)$$

where δ^0_L is the ordinary Lorentz transformation.

Transformations (16) have been obtained for the first time in paper /8/ (see also /9/) but the role of the Belinfante tensor has not been realized there.

In the HF-approach transformations (16) reproduce the classical transformations of nonlocal invariant variables (13).

Now we are ready to prove the relativistic covariance of the electron Green function

$$(2\pi)^4\delta^4(p-q)\,iG(p)=\int d^4x\,d^4y\,e^{ipx-iqy}\langle 0|T\Psi^T(x)\bar{\Psi}^T(y)|0\rangle, \qquad (17)$$

where Ψ^T, $\bar{\Psi}^T$ are field operators in the Heisenberg representation. In the one-loop approximation $G(p)$ is written as

$$G(p)=G_0(p)+G_0(p)\Sigma(p)G_0(p)+O(\alpha^4)$$

$$\Sigma(p)=\int\frac{(dq)}{q_\mu^2+i\varepsilon}\left[\delta_{ij}^T(q)\gamma_i\,\underline{G}\,\gamma_j+\gamma_0\,\underline{G}\,\gamma_0\frac{q_\mu^2}{\vec{q}^{\,2}}\right],$$

where

$$(dq)=\frac{e^2d^4q}{(2\pi)^4}\ ;\ \delta_{ij}^T(q)=\delta_{ij}-q_i\frac{1}{\vec{q}^{\,2}}q_j\ ;\ \underline{G}=G_0(p-q)=\frac{1}{\hat{p}-\hat{q}-m}.$$

In the electron rest frame $p_\mu=(p_0,\vec{p}=0)$ the self-energy operator may be written as /10/

$$\Sigma(p_\mu)=\int\frac{(dq)}{q_\mu^2+i\varepsilon}\,\frac{2}{\hat{p}-\hat{q}+m}-\int\frac{(dq)}{\vec{q}^{\,2}}\gamma_0\frac{1}{\hat{q}+m}\gamma_0=$$

$$=\frac{\alpha}{4\pi}\left[m(3\mathcal{D}+4)-\mathcal{D}(\hat{p}-m)\right]+\Sigma_R(p_\mu), \qquad (18)$$

where

$$\Sigma_R(p_\mu)=\frac{\alpha}{4\pi}(\hat{p}-m)^2\left\{\frac{\hat{p}+m}{p^2}\left[\ln\left(\frac{m^2-p^2}{m^2}\right)\right]\left[1+\frac{\hat{p}(\hat{p}-m)}{2p^2}\right]-\frac{\hat{p}}{2p^2}\right\}$$

($\mathcal{D}=\frac{1}{\varepsilon}-\gamma_E+\ln 4\pi$; ε is the dimensional regularisation parameter).

One has to take into account the additional diagrams which are induced by the operator Λ when passing to another Lorentz system (Fig.1). These diagrams, in fact, correspond to a change of the gauge

$$q_iA_i^T(q)=0\ \Longrightarrow\ \left(q_\mu-p'_\mu\frac{q\cdot p'}{p'^2}\right)A_\mu^T(q)=0$$

$$p'_\mu=(p'_0,\vec{p}\,'\neq 0). \qquad (19)$$

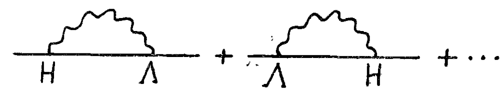

Fig.1. The additional diagrams that must be taken into account when passing to another Lorentz frame.

For the moving frame $p'_\mu=(p'_0,\vec{p}\,'\neq 0)$ we shall obtain expression (18) again but with $\hat{p}=p_0\gamma_0$ and p_0^2 changed to $\hat{p}'$, and p'^2_μ, respectively /10/.

Thus, we are convinced that in the canonical quantization approach with an explicit solution of the constraint equation the expression for the electron self-energy is relativistically covariant, has no infrared divergences and allows a renormalization with subtractions on the mass shell. That means: the problem stated at the beginning of this section is solved for the transverse variables.

The difficulties in describing the electron self-energy in papers /6,7/ were caused by treating the transverse variables as fixed (noncovariant)-gauge ones. That is why in these papers the covariant transformation laws (16) could not been taken into account. Therein had been made a nonphysical choice of the time axis $\ell_\mu=(1,0,0,0)$ for the Green function of the moving electron $p'_\mu=(p'_0,\vec{p}\,'\neq 0)$. The point is that the time axis fixes the self-Coulomb-field component and must be chosen to insure the coincidence of the velocities of the electron itself and its Coulomb field (Fig.2). Elsewhere there arise relativistic noncovariance, infrared divergencies and difficulties with the renormalizations. The time-axis choice in the rest frame of the electron is essential only in calculating the one-particle Green function and its physical residues $\lim_{\hat{p}\to m}(\hat{p}-m)G(p)=1$. All other scattering amplitudes do not depend on the time-axis choice on the mass shell, just as in the Dirac approach they do not depend on the gauge choice.

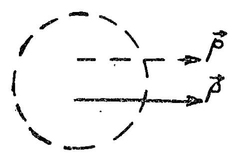

Fig.2. The time-axis choice in computation of the electron Green function (dashed line denotes the electron self Coulomb field).

3. The chiral Schwinger model

The chiral Schwinger model with left-handed fermions

$$\mathcal{L}(x) = i\,\psi_L^+ \partial_+ \psi_L - e\,\psi_L^+ A_+ \psi_L + \frac{1}{2}\left(\partial_+ A_- - \partial_- A_+\right)^2$$

$$x_\pm = t \pm x \; ; \quad \partial_\pm = \frac{1}{2}\left(\partial_0 \pm \partial_1\right)$$

has recently been considered from several points of view /11/ in connection with the chiral anomalies problem. However, the results for the mass spectrum are not in agreement with each other. Here, we shall use the HF-method for quantization of the model. Following the results of the previous section we choose x_+ as a time axis (the direction along the "quark" trajectory). The explicit solution of the constraint equation $\delta S/\delta A_+ = 0$ leads to the following nonlocal variables which are analogous to those in (9), (13):

$$A_-^I = e^{-i\omega}\left(A_- + \frac{i}{e}\partial_-\right)e^{i\omega} \equiv 0$$

$$\psi_L^I = e^{i\omega}\psi_L\,, \quad \omega = -\frac{e}{\partial_-}A_- \qquad (20)$$

Here, as in the usual Schwinger model, the bosonization allows one to find the mass spectrum: it contains only one massive scalar particle with $m = e/\sqrt{\pi}$, which is a fermions bound state.

However, if the initial (not gauge invariant) fields ψ_L are considered as basic quantized fields, there appears a kinetic term for the longitudinal field ω in the effective action of the model as a consequence of the axial anomaly

$$S_\omega = \int d^2x\, \frac{(\partial_\mu \omega)^2}{2\pi}.$$

The action S_ω is entirely quantum one, the physical reason for its appearance being the Dirac sea /12/. In such a consideration one finds one more (already massless) particle in the model spectrum.

4. Canonical quantization of non-Abelian theory with an explicit solution of the constraint equations

Let us apply the HF-quantization method*) to the non-Abelian theory with a Lagrangian

$$\mathcal{L}(x) = -\frac{1}{4}F_{\mu\nu}^a F^{\mu\nu a} + i\,\bar{\psi}\gamma_\mu \nabla\psi$$

$$\nabla_\mu = \partial_\mu + \hat{A}_\mu\,, \quad \hat{A}_\mu = g\frac{\tau^a A_\mu^a}{2i}$$

$$\hat{F}_{\mu\nu} = \partial_\mu \hat{A}_\nu - \partial_\nu \hat{A}_\mu + [\hat{A}_\mu, \hat{A}_\nu]. \qquad (21)$$

The Belinfante tensor has the same form as (12) up to renotations which are specific for the non-Abelian theory (see, for example /5/). Consider $\mathcal{L}$ and $T^S_{\mu\nu}$ on the explicit solutions of Gauss equation

$$\nabla_i^2 A_0^a = j_0^a + \nabla_i \partial_0 A_i^a$$

$$\left(\nabla_i^{ab} = \delta^{ab}\partial_i + g\,\varepsilon^{abc}A_i^c\,;\; j_\mu^a = g\,\bar{\psi}\gamma_\mu \frac{\tau^a}{2}\psi\right).$$

Here we are faced with a problem analogous to that of gauge ambiguities /15/.

Formally, we may write the Gauss equation solution and nonlocal variables like (13) in the form

$$A_0^a = \frac{1}{\nabla_i^2}j_0^a + a^a(A_k)\;;\quad a^a(A_k) = \frac{1}{\nabla_i^2}\nabla_k \partial_0 A_k \qquad (22)$$

$$\hat{A}_i^I(A) = v_I\left(\hat{A}_i + \partial_i\right)v_I^{-1}$$

$$\psi^I = v_I\,\psi\,, \qquad (23)$$

where the matrix $v_I = v_I(\hat{A})$ satisfies the following equation:

$$\partial_0 v_I = v_I\,\hat{a}\;:\quad v_I = T\exp\left[\int^t dt'\,\hat{a}\right]. \qquad (24)$$

The transformation properties of the matrix v_I, that follow from eqs. (24), ensure the invariance of variables (23) under gauge transformations of the initial fields /14/

$$\left.\begin{aligned}\hat{A}_i^g &= g\,(\hat{A}_i + \partial_i)\,g^{-1}\\ \psi^g &= g\,\psi\end{aligned}\right\}\quad v_I(A^g) = v_I(A)\,g^{-1}$$

*) First attempt to quantize a non-Abelian theory with explicit solution of the constraints has been performed in paper /13/ (see, aslo /14/ and the literature therein).

and the fulfilling of the identities

$$\nabla_i \partial_0 A_i^{\,I} \equiv 0$$

$$\int^t dt' \, \nabla_i (A^I) \partial_{0'} A_i^{\,I} \equiv 0 . \tag{25}$$

This means we have got rid of all unphysical degrees of freedom. Formal relations (22) - (25) are meaningful in the sense of perturbative series expansions, if there do not exist zeros of the operator ∇_i^2. Otherwise the matrix v_I is determined up to a left multiplication by a gauge factor u which satisfies the zero mode equations /14/

$$\nabla_i^2 u^{-1} \partial_0 u = 0 \; ; \; \int^t dt' \nabla_i^2 u^{-1} \partial_{0'} u = 0 . \tag{26}$$

Thus, the gauge ambiguities problem in HF-approach appears as a dynamical problem of the full solution of Gauss equation. (Nontrivial examples of zero-mode solutions of (26) leading to a colour confinement due to the destructive interference of the phase factors u have been considered in papers /14,16/). Here we shall construct an operator-level canonical quantization of a non-Abelian gauge theory after assuming only trivial solutions of eqs. (26) ($u = 1$).

It is easily seen that variables (23) with identities (25) cannot be canonical ones satisfying commutational relations (4), (15). The transition to the invariant canonical variables like (15) can be performed by the help of a usual change of the variables

$$\hat{A}_i^T = v_T(A^I)(\hat{A}_i^I + \partial_i) v_T^{-1}(A^I) \; ; \; \psi^T = v_T(A^I)\psi^I \tag{27}$$

so that the new variables A_i^T satisfy the transversality condition

$$\partial_i \hat{A}_i^T = 0 = v_T [\partial_i \hat{A}_i^I - \nabla_i (v_T^{-1} \partial_i v_T)] v_T^{-1} . \tag{28}$$

The explicit form of the transition matrix $v_T(A^I)$ is given by the solution of (28)

$$v_T(A^I) = \exp\left\{ \frac{1}{\nabla_i \partial_i} \partial_k \hat{A}_k^I \right\} \tag{29}$$

(here the zero modes of the operator $\nabla_i \partial_i$ are omitted again). This matrix is not degenerated, so the number of components of variables A_i^T and A_i^I is one and the same. Note, that

the $\{A_i^T, \psi^T\}$ variables are also invariant under gauge transformations of the initial fields A_i, ψ.

In terms of the variables (27) with nonlocal commutational relations

$$i [\dot{A}_i^{aT}(\vec{x},t), A_j^{bT}(\vec{y},t)] = \delta^{ab} \delta_{ij}^T \delta^3(\vec{x}-\vec{y})$$

$$\{\psi^{+T}(\vec{x},t), \psi^T(\vec{y},t)\} = \delta^3(\vec{x}-\vec{y})$$

the Lagrangian and the Belinfante tensor coincide with the ones in Schwinger's paper /5/ where the relativistic covariance of the non-Abelian transverse fields has been proved.

Nonlocal variables (27) are equivalent to the Schwinger variables /5/ and do not correspond to any fixed gauge choice in all the Loretz frames. In each new system with a time axis along the unit vector ℓ_μ a new transversality condition will be fulfilled

$$[\partial_\mu - \ell_\mu (\partial . \ell)] A_\mu^T = 0 . \tag{30}$$

This is what the relativistic covariance of the canonical quantization with an explicit solution of the constraint equation consists in. In particular, for the fermionic Green function in the one-loop approximation, we obtain expressions analogous with (17), (18) up to the corresponding changes in coupling constant. Following paper /17/ and with the help of $\{A^T, \psi^T\}$ - variables we obtain for the Green functions the generating functional

$$W^T[\eta^T, \bar{\eta}^T] = \int d\bar{\psi}^T d\psi^T d^2E_i^T d^2A_i^T \exp\{ i \int d^4x [\psi^{T+} \dot{\psi}^T + \dot{A}_k^T E_k^T - T_{00}^S(A^T, \psi^T) + \bar{\eta}^T \psi^T + \bar{\psi}^T \eta^T]\} = \tag{31}$$

$$= \int d\bar{\psi} d\psi d^4A_\mu \, \delta(\partial_i A_i) \, \mathrm{Det}(\nabla_i \partial_i) \exp\{ i \int d^4x (\mathcal{L} + \bar{\eta}^T \psi + \bar{\psi} \eta^T \}. \tag{32}$$

This form is usually used to motivate different heuristic methods of covariant quantization /2/ and allows one to pass to any special gauge condition, thus proving the equivalence of GHF and HF approaches. However, expression (32) does not represent entirely the operator-quantization specifics in terms of variables (27), in particular, their covariant transformation laws, explicitly written in paper /5/. The reason is that in eq. (32) the transverse variables have become the ordinary Coulomb-gauge variables in the Dirac approach with all the problems of noncovariant computation of Green's functions (17) /5,7/. The covariant properties of the transverse variables become natural after taking into account the explicit dependence of the functional (32) on the time axis ℓ_μ.

To do this, we pass from the gauge (31) to the Lorentz gauge $\partial_\mu A^\mu = 0$ using the transformation matrix $v_L^{\ell}(A_\mu)$:

$$v_L^{\ell}(A_\mu) = \exp\left\{ \frac{1}{\nabla^{\ell}\cdot\partial^{\ell}} (\ell\cdot\partial)(\ell\cdot\hat{A}) \right\}$$

$$\nabla_\mu^{\ell} = \nabla_\mu - (\ell\cdot\nabla)\ell_\mu \; ; \; \partial_\mu^{\ell} = \partial_\mu - (\ell\cdot\partial)\ell_\mu . \tag{33}$$

Then, the generating functional (32) takes the equivalent form

$$W^{T\ell}[\eta^{T}, \bar{\eta}^{T}] = \int d\psi\, d\bar{\psi}\, d^4 A_\mu\, \delta(\partial_\mu A^\mu)\, \mathrm{Det}(\nabla_\mu \partial^\mu) \cdot$$

$$\cdot \exp\left\{ i \int d^4x \left[\mathcal{L} + \bar{\eta}^{T}(v_L^{\ell}(A_\mu))\psi + \bar{\psi}(v_L^{\ell}(A_\mu))^{-1}\eta^{T} \right] \right\}, \tag{34}$$

where an explicit dependence on the time axis ℓ_μ is present. On the mass shell all scattering amplitudes except for the one-particle one (i.e. the Green function residue) do not depend on the matrix v_L^{ℓ} and on the time axis. However, the additional diagrams induced by $v_L^{\ell}(A_\mu)$ play a crucial role in restoring the correct analytical properties of the one-particle Green function, i.e. in obtaining an expression similar to (18) (as is known, the naive form of the Lorentz gauge leads to incorrect analytical properties of the fermionic Green function). Thus the canonical quantization with an explicit solution of the constraint equations (and with gauge-invariant conserved quantities and variables) comes out to be not only a possible strong fondation for heuristic quantization methods with a different choice of gauge-conditions. It also gives unique values (answer) for such physical quantities which depend on the gauge choice in the ordinary calculation approaches. Note that expression (18) cannot be reproduced in any of the relativistic gauges.

The gauge invariant consideration (33), (34) of the analytical properties of the fermionic Green function may be used to formulate a confinement criterion, consisting in the absence of the quark Green function poles /14,16/.

Conclusions

We have shown that the explicit solution of the constraint equations in gauge theories (HF-method) leads to the quantization scheme considered by Schwinger /5/. However, there are two differences from this scheme: the HF-transverse variables are not postulated (they are related in a nonlocal way with the initial ones) and are invariant under gauge transformations of the latter. Because of the nonlocality the classical and quantum systems have the same transformation properties and are relativistically invariant. The transverse-variable covariance means that they follow the time axis rotations and their gauge coincides with the Coulomb one $\partial_i A_i = 0$ only in one fixed Lorentz frame $\ell^{0}_{\mu} x^\mu = t$.

This fact was crucial for proving the relativistic covariance of the theory in terms of these transverse variables on the Feynman diagram level and for its renormalization.

In particular the electron relativistically covariant Green function with correct (from a physical point of view) analytical properties has been computed. A physical principle for the time-axis choice has also been established. This method allowed one to quantize in a unique way the chiral Schwinger model. In the HF-quantization the gauge ambiguity for the non-Abelian theory appears as a dynamical problem of consideration of the infrared solution of the constraints.

We have shown that canonical quantization with an explicit solution of the constraint equations is a good candidate for a relativistically invariant check-point for various heuristic quantization methods and for removing the ambiguities connected with the choice of gauge-conditions.

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Илиева Н.П., Нгуен Суан Хан, Первушин В.Н. E2-86-283
Гамильтонова формулировка калибровочных теорий с явным решением уравнения связи

В теории калибровочных полей существуют задачи, решения которых зависят от выбора калибровки, например: вычисления фермионных функций Грина, квантование теорий с аномалиями, учет калибровочных неоднозначностей, инфракрасная асимптотика неабелевых теорий. Для решения этих задач полезно иметь физический принцип, с помощью которого можно ограничить произвол в выборе калибровки. В работе предлагается гамильтонова формулировка калибровочных теорий с явным решением уравнения связи и рассматриваются несколько примеров, свидетельсвующих, что указанный принцип можно определить из самой динамики.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1986

Ilieva N.P., Nguyen Suan Han, Pervushin V.N. E2-86-283
Hamiltonian Formulation of Gauge Theories with an Explicit Solution of the Constraint Equation

There are problems in the gauge-field theory whose solution depends on the gauge choice. Such problems, for example, are the computation of the fermionic Green functions, quantization of anomalous theories, gauge ambiguities, infrared asymptotics of non-Abelian theories, etc. So, a physical principle restricting the arbitrariness in the gauge choice would be plausible for their consideration. In the present paper we propose a Hamiltonian formulation of gauge theories with an explicit solution of the constraint equations and consider some examples which show the role of the dynamics in determining the above principle.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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