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EFFECTIVE MESON LAGRANGIANS AND SKYRMION PHYSICS FROM QUARK DYNAMICS

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I. Introduction

It is generally accepted that QCD is the underlying theory of strong interactions. Yet, as far as low energy hadron dynamics is concerned, the evidence for this is mainly qualitative and comes largely from considerations of QCD in the limit of large numbers of colours N. In fact a detailed analysis of QCD-diagrams in the 1/N expansion resulted in the conjecture that QCD is equivalent to an effective chiral field theory involving only mesons [1,2]. Such an idea is in agreement with the well known conclusions of the current algebra era that low-energy properties of hadrons are successfully described by effective chiral Lagrangians. Moreover, it was of great importance to recognize that realistic effective Lagrangians should respect the Ward identities following from the global symmetries of QCD including chiral anomalies.

An important step towards the construction of the effective hadron Lagrangian has recently been provided by Witten [3] who worked out the topological structure of the anomalous Wess-Zumino action [4] and showed its connection to Skyrme's pioneering work [5] where baryons are considered as topological solitons ("skyrmions") of a non-linear chiral meson Lagrangian. Witten's work also renewed interest in the construction of complete low-energy effective Lagrangians including vector and axial-vector mesons, deduced in accord with the general principles dictated by QCD [6]. The ultimate purpose is, of course, to derive the full meson Lagrangian directly from QCD by integrating out the quark and gluon fields. Since the corresponding bosonization program is however extremely complicated in four dimensions due to the non-abelian character of the gluon interactions, it seems reasonable to investigate simpler "superconductor" quark models of the Nambu-Jona-Lasino /NJL/ [7-10] type. Modelling the important property of spontaneous breakdown of chiral symmetry in QCD, superconductor quark Lagrangians are expected to represent "certain approximations" to QCD in the low-energy region.

The purpose of this talk is to illustrate the path-integral bosonization of a superconductor quark model using a convenient exponential parametrization of the chiral field [9]. We shall show that the resulting effective meson Lagrangian includes not only the common kinetic and interaction terms but also quartic derivative terms of the Skyrme-type as well as the gauged Wess-Zumino term. Moreover, we shall review the consequences of the arising new "fine-structure" derivative terms for meson physics as well as for the problem of the skyrmion stability and the N-N potential.

II. The effective chiral hadron Lagrangian

II.1 Definitions

Let us consider the following superconductor quark Lagrangian of strong interactions which is invariant under global colour SU(N) symmetry

$$\mathcal{L} = \bar q (i\partial\!\!\!/ - m_0) q + 2G_1 \left\{ (\bar q \tfrac{\tau_0}{2} q)^2 + (\bar q i\gamma^5 \tfrac{\vec\tau}{2} q)^2 \right\} - 2G_2 \sum_{i=0}^{3} \left\{ (\bar q \gamma^\mu \tfrac{\tau_i}{2} q)^2 + (\bar q \gamma^\mu \gamma^5 \tfrac{\tau_i}{2} q)^2 \right\}, \qquad (1)$$

where q denotes the quark spinor (summation over colour indices is understood) and τ_i are the generators of the flavour group SU(2), $\tau_0 \equiv \mathbb{1}$. G_1 and G_2 are universal quark coupling constants with dimension $(\text{length})^2$. In (1) we admitted explicit breaking of chiral SU(2) x SU(2) symmetry by a bare quark mass m_0. To bosonize the theory (1) we follow the standard procedure /8,9/ and introduce collective meson fields in the generating functional of Green's functions corresponding to (1). The action becomes then bilinear in the quark fields and quark integration becomes trivial. This leads to the following effective meson Lagrangian*)

$$\mathcal{L}(M,V,A) = -\frac{1}{4G_1} tr (M-m_0)^+ (M-m_0) - \frac{1}{4G_2} tr (V_\mu^2 + A_\mu^2) - iN\, tr' \ln i D\!\!\!\!/ . \qquad (2)$$

where

$$i D\!\!\!\!/ = i(\partial\!\!\!/ + V\!\!\!/ + A\!\!\!/ \gamma^5) - (P_R M + P_L M^+) \qquad (3)$$

is the Dirac operator with collective fields M,V and A and $P_{R,L} = \frac{1}{2}(1 \pm \gamma^5)$ are chiral right/left projectors. The last term in (2) is due to the quark determinant $(\mathrm{Det}\, i D\!\!\!\!/)^N = \exp N\, \mathrm{Tr} \ln i D\!\!\!\!/$, and we used the matrix notation

*) We use the notation $\mathrm{Tr} = \int d^4x\, tr'$, $tr' = tr_\gamma tr$, where tr denotes the flavour trace.

$$V_\mu = -i(\omega_\mu \tfrac{\tau_0}{2} + \vec\rho_\mu \tfrac{\vec\tau}{2}), \quad A_\mu = -i(A_{0\mu} \tfrac{\tau_0}{2} + \vec A_{1\mu} \tfrac{\vec\tau}{2})$$

$$M = S + iP = s \tfrac{\tau_0}{2} + i \vec p \tfrac{\vec\tau}{2} . \qquad (4)$$

Here (ω_μ , $\vec\rho_\mu$) and ($A_{0\mu}$, $\vec A_{1\mu}$) denote the isosinglet and isotriplet vector and axial vector fields, respectively. It is convenient to introduce the following polar decomposition of the complex matrix M which comprises the scalar (S) and pseudoscalar (P) fields

$$M(x) = \sigma(x)\, U(x), \qquad (5)$$

where the chiral field $U(x)$ is parametrized by the (would be) Goldstone fields of pions

$$U(x) = e^{2i\pi(x)/F}, \quad \pi(x) = \vec\pi(x) \tfrac{\vec\tau}{2} \qquad (6)$$

with F being the (bare) pion decay constant, and $\sigma(x)$ is the field of the σ-particle.

The effective meson field theory defined by eq. (2) has been studied in the limit of a large number of colours, $N \to \infty$ ($G_i N$ fix)/9/. In this limit the meson functional integral is dominated by the stationary point $U = 1$, $V_\mu = A_\mu = 0$, $\sigma_0 \equiv m$ with m being the total (constituent) quark mass which is determined by a Schwinger-Dyson equation. The main task is then to evaluate the quark determinant in (2). In this context it is worth remarking that the modulus of the quark determinant $|\mathrm{Det}\, i D\!\!\!\!/| \equiv (\mathrm{Det}\, i D\!\!\!\!/ (i D\!\!\!\!/)^+)^{1/2}$ is invariant under chiral transformation of the Dirac operator $i D\!\!\!\!/ \to i D\!\!\!\!/_\omega = \omega\, i D\!\!\!\!/\, \omega$, $\omega = \exp(-i \vec\alpha \cdot \vec\tau \gamma^5)$, but not its phase which is related to the gauged Wess-Zumino action.

II.2 Extended Skyrme Lagrangian

The modulus of the quark determinant is conveniently calculated by using the proper time regularization

$$\ln |\mathrm{Det}\, i D\!\!\!\!/|^N = \frac{N}{2} \mathrm{Tr} \ln D\!\!\!\!/^+ D\!\!\!\!/ = \qquad (7)$$

$$= -\frac{N}{2}\int_{1/\Lambda^2}^{\infty}\frac{d\tau}{\tau}\,\mathrm{Tr}\, e^{-\not{D}^{+}\not{D}\tau}$$

and performing a heat kernel expansion which essentially yields a derivative expansion of $|\,\mathrm{Det}\, i\not{D}\,|$. To facilitate the discussion let us for a moment discard the V,A and σ' fields ($\sigma=\sigma_0+\sigma'$). Introducing the left group current $L_\mu=(\partial_\mu U)U^+$ and discarding a sixth order derivative term, we obtain, after some field renormalizations, the following extended Skyrme Lagrangian /9/

$$\mathcal{L} = \mathcal{L}^{(2)} + \mathcal{L}^{(4)} + \mathcal{L}_{SB}\,. \tag{8}$$

Here

$$\mathcal{L}^{(2)} = -\frac{F_\pi^2}{4}\,\mathrm{tr}\, L_\mu L^\mu \qquad (F_\pi \sim 93\ \mathrm{MeV}) \tag{9}$$

is the kinetic term of the chiral field,

$$\mathcal{L}^{(4)} = \frac{N}{384\pi^2}\,\mathrm{tr}\left\{[L_\mu, L_\nu]^2 + 2(L_\mu L^\mu)^2 - 4(\partial_\mu L^\mu)^2\right\} \tag{10}$$

are fourth order derivative ("Skyrme") terms and

$$\mathcal{L}_{SB} = \frac{1}{4}F_\pi^2 M_\pi^2\,\mathrm{tr}\,(U+U^+-2) \tag{11}$$

is a chiral symmetry breaking term.
The first term in the curly bracket in (10) is precisely the term originally introduced by Skyrme /5/ to stabilize the baryon as a soliton. If we identify its coefficient $1/32e^2$ defined in ref. 3 with $(1/32)\,N/12\pi^2$ we find for N=3 $e=\sqrt{12/N}\,\pi=2\pi$ to be compared with the value e =5.45 obtained from fitting the nucleon and delta masses /3/. Note that the structure of the quartic derivative terms in (10) and their numerical coefficients are model-independent and uniquely determined by the heat kernel expansion. Our results are in agreement with other approaches /11,12/ where analogous investigations of fermion determinants have been performed. The fact that the coefficients in (10) are independent of the cut-off Λ and of the quark masses is very reminiscent of anomaly-related effective interactions. Indeed, the quartic derivative terms in (10) have independently been derived from direct integration of the so-called "non-topological" part of the chiral anomaly /12/. It is, however, worth mentioning that our approach leads to a logarithmically divergent coefficient (if $\Lambda\to\infty$) of the kinetic meson term and not

to a quadratic divergence. The renormalized pion decay constant is then $F_\pi = O(m\sqrt{\ln\frac{\Lambda}{m}})$ and not $F_\pi = O(\tilde{\Lambda})$ yielding a larger cut-off $\Lambda \gtrsim 1$ GeV /8,9/ to be compared with the lower value $\tilde{\Lambda} = 2\pi F_\pi/\sqrt{N} \sim 340$ MeV of other works /11,12/ *).

For completeness, let us also quote the Noether current associated with the extended Skyrme Lagrangian (8)

$$J^i_{\mu L} = J^{(2)i}_{\mu L} + J^{(4)i}_{\mu L}\,, \tag{12}$$

$$J^{(2)i}_{\mu L} = i\,\frac{F_\pi^2}{2}\,\mathrm{tr}\,\frac{\tau_i}{2}L_\mu\,,$$

$$J^{(4)i}_{\mu L} = \frac{i}{8e^2}\,\mathrm{tr}\Big([L^\nu,\tfrac{\tau_i}{2}][L_\mu,L_\nu] - \{\tfrac{\tau_i}{2},L_\mu\}L^\nu L_\nu - 2\big(\tfrac{\tau_i}{2}\partial_\mu - [\tfrac{\tau_i}{2},L_\mu]\big)\partial^\nu L_\nu\Big).$$

The right current $J^i_{\mu R}$ is obtained from (12) by substituting $L_\mu \to -R_\mu = -U^+\partial_\mu U$. Finally, the vector and axial vector fields are easily included into (8) by replacing $\partial_\mu U$ by the $SU(2)$-covariant derivative $\nabla_\mu U$

$$\partial_\mu U \to \nabla_\mu U \equiv (\partial_\mu + [V,*] + \{A,*\})\,U \tag{13}$$

and adding some non-minimal terms /12/. This yields, for example,

$$\mathcal{L}^{(4)} \to \tilde{\mathcal{L}}^{(4)} = \frac{N}{96\pi^2}\,\mathrm{tr}\Big\{\tfrac{1}{2}\nabla_\mu U(\nabla_\nu U)^+\nabla^\mu U(\nabla^\nu U)^+ - \big(\nabla_\mu U(\nabla^\mu U^+)\big)^2 + \nabla_\mu^2 U(\nabla_\nu^2 U)^+ + 2F^L_{\mu\nu}\nabla^\mu U(\nabla^\nu U)^+ + 2F^R_{\mu\nu}(\nabla^\mu U)^+\nabla^\nu U + F^L_{\mu\nu}U F^{R\mu\nu}U^+\Big\}$$

with the notation

$$F^R_{\mu\nu} = \partial_\mu A^R_\nu - \partial_\nu A^R_\mu + [A^R_\mu, A^R_\nu],\ \text{etc.} \tag{14}$$

II.3 Chiral anomaly and gauged Wess -Zumino action

The gauged WZ-action is related to the phase of the quark determinant. In fact, it is obtained from integrating the variational equation /9/

*) Λ denotes here the cut-off of the proper time regularization (7) whereas the cut-off scale $\tilde{\Lambda}$ introduced in the regularization scheme of ref. 12 is a bound on the eigenvalues of the Dirac operator.

$$i N \delta [\mathrm{Im}\, \mathrm{Tr} \ln i \not{D}_\omega] = - \mathrm{Tr}(\delta\omega\,\omega^+ + \omega^+ \delta\omega) G(V^\omega, A^\omega) \quad (15)$$

between $\omega = 1$ and $\omega = U^{1/2}$ where G(V,A) is the minimal Bardeen anomaly[13] and $i\not{D}_\omega$, V^ω, A^ω are the chirally rotated Dirac operator and vector or axial vector fields, respectively. The integration of the chiral anomaly is easily performed using differen- tial geometric methods and yields precisely the gauged WZ-action[14]. Remember that the WZ-action leads to interesting physical consequen- ces and is responsible in particular for the so-called "anomalous" decays and reactions like $\pi^0 \to 2\gamma$, $\omega \to 3\pi$, $K^+ \to \pi^+\pi^- e^+\nu$ or $K^+K^- \to 3\pi$, $\pi^+\pi^- \to \eta\pi^+\pi^-$, etc. Moreover, its presence in the Lagrangian for skyrmion physics ensures that the solution must be quantised as a fermion if N is odd, and as a boson if N is even[3].

III. Physical applications

III.1. Meson sector

a) $\pi\pi$, πK scattering

Let us write the Lagrangian $\mathcal{L}^{(4)}$ in the form

$$\mathcal{L}^{(4)} = \mathcal{L}_Q + \mathcal{L}_T , \qquad (10')$$

where

$$\mathcal{L}_Q = \frac{1}{32e^2} \mathrm{tr}[L_\mu, L_\nu]^2 + \frac{\gamma}{32e^2} (\mathrm{tr}\, L_\mu L^\mu)^2 \qquad (16)$$

is the sum of Skyrme's original commutator term and a "non-Skyrme" or "symmetric" term ($\gamma_{th} = 1$). Moreover

$$\mathcal{L}_T = - \frac{1}{\Lambda_T^2} \mathrm{tr}(\partial_\mu L^\mu)^2 = \frac{1}{\Lambda_T^2} \mathrm{tr}\{\partial^2 U \partial^2 U^+ - (L_\mu L^\mu)^2\} \qquad (17)$$

is the so-called "tachyonic" term ($\Lambda_T^2 \equiv 96\pi^2/N$)*). The

*) This term leads to the appearance of a tachyonic state in the pion propagator. The natural range of validity of the derivative expansion is therefore expected to be $\langle p^4\rangle/\langle p^2\rangle \ll 8\pi^2 F_\pi^2 \approx M_\rho^2$ (comp. eq. (25)) before vector mesons are included.

implications of the Lagrangian $\mathcal{L}_Q$ for $\pi\pi$-scattering have been analyzed by Donoghue et al.[15] and Pham and Truong[16]. An analysis of the full Lagrangian (10') including the tachyonic term has recently been performed by Aitchison et al.[17] and by Belkov, Lanjov and Pervushin[18]. Following the latter work let us quote the resulting expression for the Lagrangian of $\pi\pi$-scattering.

$$\mathcal{L}_{\pi\pi} = - \frac{1}{4F_\pi^2} \left[\vec{\pi}^2 (\partial_\mu \vec{\pi}\, \partial^\mu \vec{\pi}) - \beta M_\pi^2 (\vec{\pi}^2)^2 \right] - $$
$$ - \frac{1}{4e^2 F_\pi^4} \left[(\partial_\mu \vec{\pi}\, \partial^\mu \vec{\pi})^2 - (\partial_\mu \vec{\pi}\, \partial_\nu \vec{\pi})^2 \right] + \frac{\gamma}{8e^2 F_\pi^4} (\partial_\mu \vec{\pi}\, \partial^\mu \vec{\pi})^2$$
$$ - \frac{2 M_\pi^2}{3 \Lambda_T^2 F_\pi^4} \left[3 (\partial_\mu \vec{\pi}\, \partial^\mu \vec{\pi}) \vec{\pi}^2 - M_\pi^2 (\vec{\pi}^2)^2 \right] , \qquad (18)$$

where $\beta \sim 1/2$ is a parameter associated with the symmetry-breaking meson mass term. Notice that the tachyonic term (17) does not contribute to the D-wave scattering lengths which are given (in units of M_π^{-5}) by

$$a_2^{I=0} = \frac{\pi}{2} \alpha_0^2 \frac{36\pi^2}{e^2} \frac{1}{15} (2+\gamma) , \quad a_2^{I=2} = - \frac{\pi}{2} \alpha_0^2 \frac{36\pi^2}{e^2} \frac{1}{15} (1-\gamma)$$
$$\alpha_0 \equiv \frac{1}{3} \left(\frac{M_\pi^2}{4\pi^2 F_\pi^2} \right) = 0.019 . \qquad (19)$$

Eqs. (19) enable one to express the parameters e^2, γ by a_2^0 and a_2^2,

$$e^2 = \frac{\pi}{2} \alpha_0^2 \frac{36\pi^2}{5} \frac{1}{a_2^0 - a_2^2} , \quad \gamma = \frac{a_2^0 + 2a_2^2}{a_2^0 - a_2^2} \qquad (20)$$

and using the experimental data[19]

$$a_2^0 = (17 \pm 3) \times 10^{-4} M_\pi^{-5} , \quad a_2^2 = (1.3 \pm 3) \times 10^{-4} M_\pi^{-5} \qquad (21)$$

to get the estimates[18]

$$e^2 = (25.7 \pm 6.7) , \qquad \gamma = (1.2 \pm 0.2) \quad . \tag{22}$$

The value of γ is consistent with the theoretical value $\gamma_{th.} = 1$ which also yields $a_2^2 = 0$, whereas the value for e^2 is somewhat lower than the theoretical value $e^2 = 4\pi^2$. The predictions for the scattering lengths a_0^0, a_0^2 and a_1^1 are quoted in Table 1.

Table 1: $\pi\pi$-scattering lengths[18]

a_ℓ^I	Effective meson Lagrangian		
	without p^4-terms	with p^4-terms	experiment[20]
a_0^0, M_π^{-1}	0.16	0.19	0.23 ± 0.05
a_0^2, M_π^{-1}	-0.045	-0.052	-0.05 ± 0.03
a_1^1, M_π^{-3}	0.030	0.039	0.036 ± 0.010

Finally, using a suitable unitarization scheme it is possible to calculate the various $\pi\pi$ -phase shifts. The corrections to the soft pion results are all quite small but substantially in agreement with experiment[17,18]. The extension of the Lagrangians (16), (17) to flavour SU(3) symmetry is straightforward [9,12]. Belkov, Lanjov and Pervushin have also estimated the πK -scattering lengths using the GMOR-scheme[21] of chiral symmetry breaking. The corresponding results are quoted in Table 2.

Furthermore, the last authors used also a weak current $\times$ current Lagrangian with SU(3) Noether currents to calculate the slope parameters of the decay $K \to 3\pi$.

Table 2: πK -scattering length[18]

	Effective meson Lagrangian				
a_ℓ^I	$\mathcal{L}^{(2)} + \mathcal{L}_{SB}$	$\mathcal{L}_Q$	$\mathcal{L}_T$	Sum	Experiment
$a_0^{1/2}$ M_π^{-1}	0.121	0.013	0.086	0.220	0.335 ± 0.006[22] 0.24 ± 0.02 [23] 0.13 ± 0.09 [24]
$a_0^{3/2}$ M_π^{-1}	-0.085	0.013	-0.060	-0.132	-0.14 ± 0.07 [22] -0.05 ± 0.006[23] -0.13 ± 0.03 [24]
$a_1^{1/2}$ M_π^{-3}	0.0098	0.0038	0.0072	0.0208	0.018 ± 0.002 [23]
$a_1^{3/2}$ M_π^{-3}	0	0.0024	-0.0024	0	

Before concluding this subsection let us mention that there exists an interesting relationship between the Skyrme constant $e = 2\pi$ and the parameters of the ρ meson. Using forward dispersion relations for the $\pi^+\pi^0$-scattering amplitude and saturating its imaginary part by the ρ resonance, Pham and Truong[16] obtained

$$\frac{1}{e^2} \simeq \frac{2F_\pi^2}{M_\rho^2} \quad . \tag{23}$$

Combining this result with the KSFR relation

$$M_\rho^2 = 2 F_\pi^2 g_{\rho\pi\pi}^2 \tag{24}$$

which can be shown to hold in our composite-meson model[9], we obtain the interesting result

$$e \simeq g_{\rho\pi\pi} , \qquad M_\rho^2 = 8\pi^2 F_\pi^2 , \tag{25}$$

where $g_{\rho\pi\pi}$ is the $\rho\pi\pi$-coupling constant. Note that from $\rho \to 2\pi$ decay $g^2_{\rho\pi\pi}/4\pi \sim 3$ so that indeed $g^2_{\rho\pi\pi} \sim 4\pi^2 = e^2$.

b) Vector form factor and electromagnetic radius of the pion

For illustration we shall finally calculate the $\rho\pi\pi$ form factor from (14). To this end recall that direct evaluation of the form factor from a quark triangle diagram yields[25/*)]

$$F_{\rho\pi\pi}(q^2) = g_\rho \left(1 + \frac{q^2 - M_\rho^2}{8\pi^2 F_\pi^2}\right). \tag{26}$$

where q is the four-momentum of the ρ-meson and g_ρ is the ρ-quark coupling constant. In the effective Lagrangian approach it is not difficult to see that the q^2-term in (26) arises from the term linear in $F_{\mu\nu}$ in (14) but gets no contribution from the "Skyrme current" $J_\mu^{(4)i}$ in (12)**). Taking into account (25) we also have

$$F_{\rho\pi\pi}(q^2) = g_\rho \frac{q^2}{M_\rho^2} \tag{27}$$

so that $F_{\rho\pi\pi}$ vanishes at $q^2=0$. Let us now include electromagnetic interactions into the Lagrangian (1)[8,9]. The electromagnetic form factor $F_{\gamma\pi\pi}(q^2)$ gets then contributions from direct $\gamma\pi\pi$ interaction and from a ρ pole term with ρ-γ transition. The result is[25]

$$\begin{aligned} F_{\gamma\pi\pi}(q^2) &= e\left\{1 + \frac{q^2}{8\pi^2 F_\pi^2}\right\} + g_\rho\left\{\left(1 + \frac{q^2 - M_\rho^2}{8\pi^2 F_\pi^2}\right)\frac{e}{g_\rho}\frac{q^2}{M_\rho^2 - q^2}\right\} \\ &= e\left(1 - q^2/M_\rho^2\right)^{-1} = e\left\{1 + \frac{q^2}{8\pi^2 F_\pi^2} + O(q^4)\right\}. \end{aligned} \tag{28}$$

This leads to the following expression for the electromagnetic radius of the pion

$$\langle r^2_{\pi^+}\rangle = \frac{6}{M_\rho^2} = \frac{6}{8\pi^2 F_\pi^2} \tag{29}$$

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*) The form factor (26) fulfils the natural requirement that coupling constant universality $g_\rho \approx g_{\rho\pi\pi}$ holds approximately on the mass shell of the ρ meson.

**) This conclusion differs from the result of ref. 26 but is in agreement with ref. 12, 27.

reproducing the vector meson dominance result without double counting. As has been discussed at length in the literature[17], chiral Lagrangians including ρ and A_1 mesons but no quartic derivative terms may generate Skyrme derivative terms after eliminating the heavy vector mesons. In Lagrangians of the type (14) including quartic derivative terms <u>and</u> heavy mesons, there could then, in principle, arise the problem of double counting. We expect that it is just the suppression of the $\rho\pi\pi$ form factor at $q^2 = 0$ given by (27) which leads to a suppression of the ρ-meson contribution in the derivative expansion of the chiral Lagrangian, thus avoiding double counting.

III.2 Baryon sector

The recent applications of the Skyrme model to the description of baryons[28,29] provide us with another test of the "extended" Skyrme Lagrangian (8). In this context let us first remember that the original Skyrme Lagrangian, although providing a satisfactory qualitative description of the static properties of the nucleon, does not reproduce the important medium range attraction of the nucleon-nucleon force responsible for nuclear binding[30]. On the other hand it is well known that the σ meson plays an essential role in describing the attractive part of the N-N potential. Since the structure of the symmetric term in (16) may be associated with the exchange of a heavy σ particle[16,31] it seems natural to ask whether the new pieces in (16), (17) may lead to the necessary attraction. Before discussing this issue further, let us first mention that the new quartic derivative terms in $\mathcal{L}_Q$ and $\mathcal{L}_T$ do not give a positive definite contribution to the energy. In order to counterbalance their destabilizing effects, one has to include the contribution of the ω-meson[28,29,32]

$$\mathcal{L}_\omega = \mathcal{L}_\omega^0 + \mathcal{L}_{coupl.}, \tag{30}$$

where

$$\mathcal{L}_\omega^0 = -\frac{1}{4}F^\omega_{\mu\nu}F^{\omega\mu\nu} + \frac{M_\omega^2}{2}\omega_\mu^2, \quad F^\omega_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu$$

$$\mathcal{L}_{coupl.} = \beta\,\omega_\mu j^\mu. \tag{31}$$

Here j^μ is the conserved topological current of the chiral field

$$j^\mu = \frac{1}{24\pi^2} \varepsilon^{\mu\nu\rho\sigma} \, tr(L_\nu L_\rho L_\sigma) \tag{32}$$

and $\beta = g_\rho N/2 \sim 9.2$. The theoretical value of β is comparable in magnitude with the ωNN vector coupling constant $g_{\omega NN}$ (phenomenologically $10 \lesssim g_{\omega NN} \lesssim 12$). In the low-energy region $\mathcal{L}_\omega$ may be further reduced to the following current $\times$ current form

$$\mathcal{L}_\omega = -\frac{1}{2} \frac{\beta^2}{M_\omega^2} j_\mu j^\mu . \tag{33}$$

Let us now study the baryon number $N_B = 1$ soliton sector of the extended Skyrme model defined by eqs. (8) and (33). As usual we make the hedgehog ansatz

$$U(x) = e^{i\hat{r}\vec{\tau}\theta(r)} \tag{34}$$

with the boundary conditions $\theta(0) = \pi$ and $\theta(\infty) = 0$ to ensure $N_B = \int d^3x\, j^0(x) = 1$. For the static ansatz (34) the baryon current (32) reduces to

$$j^0(x) = -\frac{1}{2\pi^2} \frac{\sin^2\theta}{r^2}\theta' \quad , \quad \vec{j} = 0 \quad \left(\theta' = \frac{d\theta}{dr}\right) \tag{35}$$

and the Lagrangian becomes

$$\mathcal{L}^{(2)} = -\frac{F_\pi^2}{2}\left(\theta'^2 + 2\frac{\sin^2\theta}{r^2}\right) , \tag{36a}$$

$$\mathcal{L}^{(4)} = \frac{N}{12\pi^2}\Big\{ -\frac{\sin^2\theta}{r^4}\left(r^2\theta'^2 + \frac{1}{2}\sin^2\theta\right) + \frac{1}{8}\left(\theta'^2 + 2\frac{\sin^2\theta}{r^2}\right)^2 + \frac{1}{4r^4}\left(2r\theta' + r^2\theta'' - \sin 2\theta\right)^2 \Big\} , \tag{36b}$$

$$\mathcal{L}_\omega = -\frac{\beta^2}{2M_\omega^2}\left(j^0(x)\right)^2 , \tag{36c}$$

$$\mathcal{L}_{SB} = -F_\pi^2 M_\pi^2 (1-\cos\theta). \tag{36d}$$

To find explicit skyrmion solutions one has in fact to solve the Euler-Lagrange equation following from the Lagrangians (36). For determination of the mass M of the skyrmion it is sufficient to seek for a minimum of the energy functional $E = -\int d^3x\, \mathcal{L}(x)$ by using variational methods. In ref. 29 a variational investigation of the extended Skyrme Lagrangian (8) supplemented by $\mathcal{L}_\omega$ (comp. eq. (33) has been performed which shows that the energy functional has indeed a local minimum for $\beta > 15$ yielding $M \sim 2 M_N$, where M_N is the nucleon mass. Unfortunately, both the values of β and M are however, too large in order to get a realistic description of the nucleon. A further investigation of the existence of physical skyrmion solutions evidently requires the inclusion of <u>all</u> heavy mesons like $\sigma, \omega, \rho, A_1$ into the effective meson Lagrangian. Moreover, it is quite possible that a complete treatment of the coupled meson field equations rather than an expansion in powers of $\partial_\mu \pi$ may be necessary in order to investigate the properties of skyrmions within a meson field theory.

It has been argued that the new derivative terms constained in $\mathcal{L}^{(4)}$ may signal the presence of attractive N-N forces /28/. A detailed analysis of the N-N potential obtained from a modified Skyrme Lagrangian has recently been performed by Lacombe et al. /31/ These authors started with the quartic derivative term $\mathcal{L}_Q$ of eq. (16) supplemented by $\mathcal{L}_\omega$. Linearizing the "symmetric" quartic term by introducing a coupling to the field of the σ -particle and taking M_σ = 800 MeV, they got a good fit to M_N and M_Δ with F_π = 71,5 MeV, β =12, e = 7 and $\gamma/4$ = 0.349. Their calculations confirm the appearance of medium range attractions.

IV. Concluding remarks

In this talk we have illustrated how to derive an effective meson Lagrangian including quartic derivative (Skyrme) terms and the gauged Wess-Zumino term from a superconductor NJL-type of quark model. The implications of the new "fine-structure" terms in the effective meson Lagrangian have then been discussed for both the meson sector ($\pi\pi$- and πK-scattering) and baryon sector. Concerning the baryon sector it unfortunately turns out that the topological soliton of the extended Skyrme Lagrangian has a too large mass ($M \sim 2 M_N$) to be identified with the nucleon/29/. A further investigation of the properties of skyrmions should then start with a complete effective meson Lagrangian including besides pions and ω mesons all other heavy mesons like the σ, ρ and A_1 mesons, too. Moreover, it is quite possible that one must address a complete treatment of the coupled meson field equations including all kinds of mesons. Obviously, one should be able to derive a Skyrme-type Lagrangian directly from QCD, but this is difficult and has only been done indirectly/12,33/, at best.

References:

1. G. 't Hooft, Nucl. Phys. B72, 461 (1974); B75, 461 (1974).
2. E. Witten, Nucl. Phys. B160, 57 (1979).
3. E. Witten, Nucl. Phys. B223, 422, 433 (1983); G. Adkins, C. Nappi and E. Witten, Nucl. Phys. B228, 552 (1983).
4. J. Wess and B. Zumino, Phys. Lett. 37B, 95 (1971).
5. T.H.R. Skyrme, Proc. Roy. Soc. London Ser. A260, 127 (1961); Nucl. Phys. 31, 556 (1962). See also: L.D. Faddeev, Lett. Math. Phys. 1, 289, (1976); A.P. Balachadran et al. Phys. Rev. Lett. 49, 1124 (1982).
6. Ö. Kaymakcalan, S. Rajeev and J. Schechter, Phys. Rev. D30, 594, 2345, (1984). Ö. Kay makcalan and J. Schechter, Phys. Rev. D31, 1109 (1985); Y. Brihaye, N.K. Pak and P. Rossi, Phys. Lett. 149B, 191 (1984); Nucl. Phys. B254, 71 (1985).
7. Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122, 345 (1961); See also: K.Kikkawa, Progr. Theor. Phys. 56, 947 (1976); T. Eguchi, Phys. Rev. D14, 2755 (1976).
8. D. Ebert and M.K. Volkov, Yad. Fiz. 36, 1265 (1982); Z. Phys. C-Part. Fields 16, 205 (1983); M.K. Volkov, Ann. Phys. (NY) 157, 282 (1984).
9. D. Ebert and H. Reinhardt, NBI Prepr. NBI-HE-85-34 (1985) (Nucl. Phys. B, in print). This paper studies a more general NJL model with broken U(n) x U(n) symmetry; Berlin-Zeuthen Prepr. PHE 3 (1986).

10. A.Dhar and S.R. Wadia, Phys. Rev. Lett. 52, 959 (1984); A. Dhar, R. Shankar and S.R. Wadia, Phys. Rev. D 31, 3256 (1985).
11. R.MacKenzie, F. Wilczek and A. Zee, Phys. Rev. Lett. 53, 197 (1984); I.J.R. Aitchison and C.M. Frazer, Phys. Lett. 146B, 63 (1984); Phys. Rev. D31, 2605 (1985); J.A. Zuk, Z. Phys. C-Part. Fields 29, 303 (1985); P. Simic, Phys. Rev. Lett. 55, 40 (1985); R.D. Ball and H. Osborn, Phys. Lett. 165B, 410 (1985).
12. A.A.Andrianov and Yu. V. Novozhilov, Phys. Lett. 153B, 422 (1985); A.A.Andrianov, Phys. Lett. 157B, 425 (1985).
13. W.Bardeen, Phys.Rev. 184, 1848 (1969).
14. K.C.Chou et al. Phys Lett. 134B, 67 (1984); H.Kawai and S.H. Tye, Phys. Lett. 140B, 403 (1984); E.Manes, Nucl. Phys. B250, 369 (1985). See also: N.K.Pak and P.Rossi, Nucl. Phys. B250, 279 (1985); G.C.Rossi, M. Testa and K.Yoshida, Phys. Lett. 134B, 78 (1984).
15. J.F. Donoghue, E. Golowich and B.R.Holstein, Phys. Rev. Lett. 53, 747 (1984).
16. T.N. Pham and Tran N. Truong, Phys. Rev. D31, 3027 (1985).
17. I.J.R. Aitchison, C.M. Frazer and P.J. Miron, Univ. of Oxford Preprint 49/85.
18. A.A.Belkov, A.V. Lanjov and V.N.Pervushin, JINR-Prepr. P2-86-205 (1986)
19. M.M.Nagels et.al., Nucl. Phys. B147, 189 (1979).
20. A.A.Belkov et al., Pion-pion interactions, Moscow, Energoisdat (1985).
21. M.Gell-Mann, R.J. Oakes and B. Renner, Phys. Rev. 175, 2195 (1968).
22. P.Estabrook et. al., Nucl. Phys. B133, 494 (1978).
23. N.Johannesson and G. Nilsson Nuovo Cim. A43, 376 (1978).
24. A.Karabarbounis and G.Shaw, J. Phys.: Nucl. Phys. G6, 583 (1980).
25. A.N. Ivanov, Yad. Fiz. 33, 1679 (1981); D.Ebert and M.K. Volkov, Proceed. of the V Int. Seminar on Problems in High Energy Physics and Quantum Field Theory. Vol. 2, Protvino, 159, (1982).
26. Lai-Him Chan, Phys. Rev. Lett. 55, 21 (1985).
27. J. Gasser and H. Leutwyler, Ann. Phys. (NY) 158, 142 (1984).
28. I.J.R. Aitchison et al., Phys. Lett. 165B, 162 (1985); H.Reinhardt and D.Ebert, NBI Prepr. NBI-HE-85-37 (1985).
29. A.A.Andrianov and V.Yu. Novozhilov, Pisma JETF 42, 469 (1985).
30. A. Jackson, A.D. Jackson and V.Pasquir, Nucl. Phys. A432, 259 (1985).

31. M.Lacombe, B.Loiseau, R.Vinh Mau and W.N. Cottingham. Orsay Prepr. IPNO/TH 85-68 (1985); Phys. Lett. 161B, 31 (1985).
32. G.S.Adkins and C.R. Nappi, Phys. Lett. 137B, 251 (1984).
33. P.Simic, ref. 11. See also: N.I.Karchev and A.A.Slavnov, Theor. Math. Fiz. 65, 192 (1985).

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Эффективные мезонные лагранжианы
и скирмионная физика из кварковой динамики

С помощью метода континуального интеграла обсуждается вывод эффективного кирального лагранжиана для мезонов из кварковой модели сверхпроводящего типа. Этот лагранжиан включает члены с высшими производными типа Скирма, а также член Весса - Зумино. Дискутируются следствия новых производных членов "тонкой структуры" для мезонной физики и для проблемы стабильности скирмиона и NN-потенциала.

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Effective Meson Lagrangians
and Skyrmion Physics from Quark Dynamics

We shall review the path-integral derivation of an effective chiral meson Lagrangian including higher order derivative terms of the Skyrme type as well as the Wess - Zumino term from a superconductor type of quark model. The consequences of the arising new "fine-structure" derivative terms are discussed for meson physics as well as for the problem of the skyrmion stability and the NN-potential.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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