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ON THE RADIATIVE CORRECTIONS TO THE NEUTRINO DEEP INELASTIC SCATTERING

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2. DERIVATION OF THE FORMULAE FOR CC AND NC DEEP-INELASTIC νN-SCATTERING

The calculation of the inclusive cross-sections $d^2\sigma^{\nu_\ell(\bar\nu_\ell)}/dxdy$ (x and y are usual scaling variables) for the reactions

$$\nu_\ell(\bar\nu_\ell) + N \to \ell(\bar\ell) + X, \quad \nu_\ell(\bar\nu_\ell) + N \to \nu_\ell(\bar\nu_\ell) + X \tag{4}$$

up to one-loop electroweak radiative corrections was carried out in the following way [2]:

- Within the unique renormalization scheme calculate, up to one-loop corrections, the differential cross-section $[d^2\sigma^{\nu_\ell(\bar\nu_\ell)}_{CC(NC)}]_i$ for scattering of $\nu_\ell(\bar\nu_\ell)$ off an individual quark "i"

$$\begin{aligned} &\nu_\ell(\bar\nu_\ell)(k_1) + q_i(p_1) \to \ell(\bar\ell)(k_2) + q_f(p_2) \\ &\nu_\ell(\bar\nu_\ell)(k_1) + q_i(p_1) \to \nu_\ell(\bar\nu_\ell)(k_2) + q_f(p_2). \end{aligned} \tag{5}$$

  The expressions for $d^2\sigma_i$'s were obtained in the extreme relativistic regime, i.e., all masses were neglected in comparison with amplitude invariants s,t,u except in the logarithmic terms.
- In the expressions for $d^2\sigma_i$'s replace the initial parton momentum by the following quantity

$$p_1^{(i)} \to \xi p_N, \tag{6}$$

  where ξ is the fraction of nucleon 4-momentum p_N carried by i-th quark.
- Multiply the cross-section by the parton distribution functions $f_i(\xi)$ of the i-th kind of quark, integrate over ξ and sum incoherently cross sections for all types of quarks involved in reaction (4).
- Average the cross sections over the proton and neutron to obtain the cross sections for scattering off an iso-scalar nucleon.

The derived $d^2\sigma^{\nu_\ell(\bar\nu_\ell)}_{CC(NC)}$ is a result of a complete treatment of the lowest order electroweak radiative corrections within the quark-parton model. This result is therefore model-dependent, due to the model assumed, parton distributions (structure functions) used, and due to the dependence of the one-loop formulae on initial and final quark masses.

As far as the structure functions are concerned, the model dependence caused by them can be minimized if they are taken from the experiment. If the structure functions will be changed

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sizebly after application of the radiative correction procedure, the latter can be repeated iteratively. Moreover, usually the influence of the structure functions on the radiative correction defined as

$$\delta^{\nu_\ell(\bar{\nu}_\ell)}_{CC(NC)}(E,x,y) = \frac{[d^2\sigma^{\nu_\ell(\bar{\nu}_\ell)}_{CC(NC)}/dx\,dy]^{1\ell}}{[d^2\sigma^{\nu_\ell(\bar{\nu}_\ell)}_{CC(NC)}/dxdy]^{0\ell}} - 1 \qquad (7)$$

(1ℓ means calculated up to "one-loop", 0ℓ means "zero-loop" or Born approximation) is reduced substantially due to cancellation of the dependence on structure functions in the cross section ratio (7).

Indeed, a comparison of the correction to $d^2\sigma_{CC}/dxdy$ computed using the recent CDHS parametrization of the structure functions/20/ with the corrections computed with Barger-Phillips/21/ structure functions shows that the dependence on the input functions is rather weak.

Radiative corrections are known to depend on the masses of particles involved. Contrary to the lepton masses, the choice of quark masses is quite arbitrary, leading to some uncertainties in the corrections. The most natural choice of the initial quark mass is

$$m_1 = \xi M_N . \qquad (8)$$

The variation of the final quark masses between 1 MeV and 1 GeV changes the corrections in absolute values less than by one per cent and, moreover, only at extreme x ($x \approx 0$ and $x \approx 1$).

So, our main conclusion is that the radiative corrections obtained are almost model-independent, i.e., this dependence is small in comparison with the correction itself and with the experimental errors.

3. THE LIST OF FORMULAE FOR CC NEUTRINO-QUARK SCATTERING

3.1. $\nu_\ell + q_i \to \ell^- + q_f$ $(\bar{\nu}_\ell + \bar{q}_i \to \ell^+ + \bar{q}_f)$

$$\frac{d^2\sigma^{1\ell}}{dxdy} = \frac{[G_F^{(\mu)}]^2 S|K_{if}|^2}{\pi}\{x f_i(x)(1+\frac{\alpha}{\pi}\delta_i^{CC}) + \frac{\alpha}{\pi}\int_x^1 d\xi f_i(\xi)\Phi_i^{CC}(\xi) + \frac{\alpha}{\pi}\int_x^1 d\xi(\xi-x)^{-1}[\xi f_i(\xi) I_i(\xi,x) - x f_i(x) I_i(x,x)]\} . \qquad (9)$$

Here $S = 2E_\nu \cdot M_N$, with E_ν being the initial lab. ν_μ energy

$$I_i(\xi,x) = \frac{\xi}{\xi - y(\xi-x)}\ln\frac{S_\tau^2}{\tau m_\ell^2} - 2 + f_i[2\ln\frac{S_\tau U}{\tau S_U} - 2 + \frac{y(\xi-x)}{\xi - y(\xi-x)}\ln\frac{S_\tau^2}{\tau m_\ell^2}] + f_i^2(\ln\frac{U^2}{\tau m_1^2} - 2)$$

with K_{ij} - corresponding Kobayshi-Maskawa matrix element, m_ℓ - final lepton mass, m_1 - initial quark mass (set equal to $x\cdot M_N$ or $\xi \cdot M_N$ in integrals over ξ), m_2 - final quark mass, $U = S\cdot y\cdot\xi$, $\tau = Sy(\xi-x)+m_2^2$, $S_\tau = S[\xi - y(\xi-x)]$, $S_U = S\xi(1-y)$, $I_i(x,x) = I_i(\xi = x, x)$, with $m_1 = xM_N$,

$$f_i \text{ is the initial quark charge } = \begin{cases} +2/3 \text{ for all "up" quarks } (u,c,t) \\ -1/3 \text{ for all "down" quarks } (d,s,b) \end{cases}$$

$$\delta_i^{CC} = I_i(x,x)\ln\frac{Sy(1-x)}{m_2^2} + \frac{3}{4} + (\frac{7}{4} - \ln\frac{S_q}{m_\ell^2})\ln\frac{S_q}{m_2^2} - \frac{1}{2}\ln^2\frac{S_q}{m_2^2} + \frac{\pi^2}{2} - \frac{3}{2}\ln\frac{S_q}{M_Z^2} + \frac{3}{4}\ln\frac{S_q}{m_\ell^2} + f_i[\frac{7}{4} + (\frac{3}{2} + 2\ln(1-y))\ln\frac{Q^2}{m_2^2} - \ln^2\frac{Q^2}{m_2^2} + \frac{\pi^2}{2} - \frac{3}{2}\ln\frac{S_q(1-y)}{M_Z^2} - \ln y\ln(1-y)] + f_i^2[-1 + (\frac{7}{4} - \ln\frac{Q^2}{m_1^2})\ln\frac{Q^2}{m_2^2} - \frac{1}{2}\ln^2\frac{Q^2}{m_2^2} + \frac{3}{4}\ln\frac{Q^2}{m_1^2}],$$

with $S_q = Sx$ and $Q^2 = Sxy$,

$$\Phi_i^{CC}(\xi) = (1+f_i)^2\frac{Q^2}{4\tau}(1 - \frac{m_2^2}{\tau}) + \frac{1}{4} + \frac{3}{4}y - \frac{y}{2}[1 + \frac{\xi}{\xi - y(\xi-x)}]\ln\frac{S_\tau^2}{\tau m_\ell^2} + f_i[2 - \frac{y\xi}{\xi - y(\xi-x)}\ln\frac{S_\tau^2}{\tau m_\ell^2} + y(y - 2 - y\frac{x}{\xi})\ln\frac{S_\tau U}{\tau S_U} + (\frac{1}{2} + y)\frac{x}{\xi}] + f_i^2[2 - (1 + \frac{x}{2\xi} + \frac{x^2}{2\xi^2})\ln\frac{U^2}{\tau m_1^2} + (2 - \frac{y}{2} + \frac{y^2}{4})\frac{x}{\xi} + (\frac{3}{2} - \frac{y}{2} - \frac{y^2}{4})\frac{x^2}{\xi^2}].$$

These formulae as well as the corresponding set for the $\bar{\nu}$ case presented below are essentially the same as in ref./2/ They are corrected only in the following two points:

i) they are rewritten using the renormalization scheme discussed in the Introduction,

ii) some terms of the order m_2^2/τ^2 improperly omitted in /2/ are re-established here.

Within the given renormalization framework all genuine weak loops are essentially absorbed into $G_F^{(\mu)}$. Nevertheless, terms containing $\ln S_q/M_Z^2$ are to some extent connected with the unification of interaction (M_Z can be considered as a correct cut-off following from $SU(2)_L \otimes U(1)$). This is the reflection of a theorem proved in ref./4/.

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$$+\frac{\alpha}{\pi}\int_x^1 d\xi(\xi-x)^{-1}[\xi f_i(\xi) I_i(\xi,x) - x f_i(x) I_i(x,x)]\}.$$

Here $S = 2E_\nu \cdot M_N$, with E_ν being the initial lab. ν_μ energy

$$I_i(\xi,x) = \frac{\xi}{\xi - y(\xi-x)}\ln\frac{S_\tau^2}{\tau m_\ell^2} - 2 +$$

$$+ f_i[2\ln\frac{S_\tau U}{\tau S_U} - 2 + \frac{y(\xi-x)}{\xi - y(\xi-x)}\ln\frac{S_\tau^2}{\tau m_\ell^2}] + f_i^2(\ln\frac{U^2}{\tau m_1^2} - 2)$$

with K_{ij} - corresponding Kobayshi-Maskawa matrix element, m_ℓ - final lepton mass, m_1 - initial quark mass (set equal to $x\cdot M_N$ or ξM_N in integrals over ξ), m_2 - final quark mass, $U = S\cdot y\cdot\xi$, $\tau = Sy(\xi-x) + m_2^2$, $S_\tau = S[\xi - y(\xi-x)]$, $S_U = S\xi(1-y)$, $I_i(x,x) = I_i(\xi = x, x)$, with $m_1 = xM_N$.

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$$\delta_i^{CC} = I_i(x,x)\ln\frac{Sy(1-x)}{m_2^2} + \frac{3}{4} + (\frac{7}{4} - \ln\frac{S_q}{m_\ell^2})\ln\frac{S_q}{m_2^2} - \frac{1}{2}\ln^2\frac{S_q}{m_2^2} + \frac{\pi^2}{2} - \frac{3}{2}\ln\frac{S_q}{M_Z^2} +$$
$$+\frac{3}{4}\ln\frac{S_q}{m_\ell^2} + f_i[\frac{7}{4} + (\frac{3}{2} + 2\ln(1-y))\ln\frac{Q^2}{m_2^2} - \ln^2\frac{Q^2}{m_2^2} + \frac{\pi^2}{2} - \frac{3}{2}\ln\frac{S_q(1-y)}{M_Z^2} -$$
$$-\ln y\ln(1-y)] + f_i^2[-1 + (\frac{7}{4} - \ln\frac{Q^2}{m_1^2})\ln\frac{Q^2}{m_2^2} - \frac{1}{2}\ln^2\frac{Q^2}{m_2^2} + \frac{3}{4}\ln\frac{Q^2}{m_1^2}],$$

with $S_q = Sx$ and $Q^2 = Sxy$,

$$\Phi_i^{CC}(\xi) = (1+f_i)^2\frac{Q^2}{4\tau}(1-\frac{m_2^2}{\tau}) + \frac{1}{4} + \frac{3}{4}y - \frac{y}{2}[1 + \frac{\xi}{\xi - y(\xi-x)}]\ln\frac{S_\tau^2}{\tau m_\ell^2} +$$
$$+ f_i[2 - \frac{y\xi}{\xi - y(\xi-x)}\ln\frac{S_\tau^2}{\tau m_\ell^2} + y(y-2-y\frac{x}{\xi})\ln\frac{S_\tau U}{\tau S_U} + (\frac{1}{2}+y)\frac{x}{\xi}] +$$
$$+ f_i^2[2 - (1 + \frac{x}{2\xi} + \frac{x^2}{2\xi^2})\ln\frac{U^2}{\tau m_1^2} + (2 - \frac{y}{2} + \frac{y^2}{4})\frac{x}{\xi} + (\frac{3}{2} - \frac{y}{2} - \frac{y^2}{4})\frac{x^2}{\xi^2}].$$

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For this reason the radiative corrections to the charge current ν-reactions are in practice "almost QED" corrections. The usage of structure functions requires some comments.

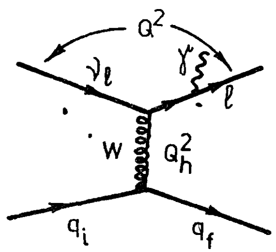
Into the formulae presented in the Appendix the quark distributions $f_i(x)$ (or $f_i(\xi)$) should enter, strictly speaking, in the scaling approximation because these formulae are integrated over the photon phase space at fixed ξ.

Some people/10/, in an attempt to take into account the scaling violation, replace

$$f_i(\xi) \to f_i(\xi, Q^2) \tag{10}$$

in the above integrands ($Q^2 = Sxy$). We also did the same in our programs.

Actually, this is only partially correct. It is true for diagrams with emission of γ's from quark legs where W(Z) probes the nuclear structure with fixed Q^2. But for the diagram



W probes the nuclear structure with $Q_h^2 \neq Q^2 = Sxy$. And Q_h^2 is equal to Q^2 only if $\xi = x$ when the emitted photon has zero energy, otherwise Q_h^2 is a function of photon kinematical variables and instead of replacement (10) we should have

$$f_i(\xi) \to f_i(\xi, \bar{Q}^2), \tag{11}$$

where $\bar{Q}^2$ is some average value.

But, due to the weak dependence of $f_i(\xi, Q^2)$ on Q^2 and due to the weak dependence of δ's on $f_i(\xi, Q^2)$ we expect that the bias introduced by the partially correct replacement (10) will be an effect of the second order. For the NC ν-reaction we obviously do not face with this problem because no protons can be emitted from the leptonic current.

In the scaling approximation eq.(9) can be integrated analytically over x yielding the simpler formula for the y-distribution:

$$\frac{d\sigma_i^{1\ell}}{dy} = \frac{[G_F^{(\mu)}]^2 S|K_{if}|^2}{\pi} \int_0^1 dx \cdot x \cdot f_i(x) [1 + \frac{\alpha}{\pi} \delta_i^{(1)}(x, y)], \tag{12}$$

where

$$\delta_i^{(1)}(x, y) = \frac{1}{2} + \frac{5}{12}\pi^2 + \frac{3}{4}\ln\frac{S_q}{m_\ell^2} - \frac{3}{2}\ln\frac{S_q}{M_Z^2} - \frac{1}{2}\Phi(1-y) - 2\Phi(y) -$$
$$- \frac{1}{2}\ln^2 y - \frac{1}{2}\ln^2(1-y) - \frac{1}{2}\ln(1-y)\ln\frac{S_q}{m_\ell^2} + y(\frac{5}{4} - \frac{1}{2}\ln\frac{S_q}{m_\ell^2}) +$$
$$+ (-\frac{7}{4} + \frac{y}{2} + \ln\frac{S_q}{m_\ell^2})\ln y + (1-y)\ln(1-y) + f_i[3 - \frac{3}{2}\ln\frac{S_q}{M_Z^2} +$$
$$+ \Phi(1-y) - \Phi(y)] + f_i^2[\frac{23}{72} - \frac{\pi^2}{6} - \frac{2}{3}\ln\frac{Q^2}{m_1^2} - \frac{5}{12}y + \frac{1}{24}y^2],$$

and $\Phi(y) = -\int_0^y \frac{\ln|1-t|}{t} dt$ is the Spence function. We tested that $\delta_i^{(1)}(x,y)$ coincides analytically at $f_i = -1/3$ with $g^\nu(y, S)$ from Appendix F of ref./6/ if one subtracts from the latter quantity the following constant

$$C = \ln\frac{M_W^3}{\mu M_Z^2} + \frac{1}{4}, \tag{13}$$

which reflects the choice of the $\overline{MS}$ renormalization scheme.

At last, expression (12) can be integrated analytically over y yielding the simple result of ref./5/ for the correction to the total cross section

$$\sigma_i^{1\ell} = \frac{[G^{(\mu)}]^2 S|K_{if}|^2}{\pi} \int_0^1 dx \cdot x \cdot f_i(x) \{1 + \frac{\alpha}{\pi}[(3 - \frac{3}{2}\ln\frac{S_q}{M_Z^2})(1 + f_i) +$$
$$+ f_i^2(\frac{19}{24} - \frac{\pi^2}{6} - \frac{2}{3}\ln\frac{S_q}{m_1^2})]\}, \tag{14}$$

which confirms not only the correct shape of distributions but also the correct normalization to $G_F^{(\mu)}$ within the chosen renormalization framework.

From Eq.(12) there follows trivially the y-distribution for purely leptonic inverse μ-decay: $\nu_\mu + e \to \mu^- + \nu_e$. One should only set $f_i = -1$, $S_q = S = 2m_e E_\mu$, $Q^2 = Sy$ and delete the integration over x and factors $|K_{if}|^2$ and $x \cdot f_i(x)$. In this case the term $\ln s/M_Z^2$ drops out.

Finally, we tested numerically for several incident energies and different values of y that:

$$\frac{d\sigma_i^{1\ell}}{dy} = \int_0^1 dx \frac{d^2\sigma_i^{1\ell}}{dxdy}. \tag{15}$$

This chain of comprehensive tests convinced us that we did not make mistakes either in formulae derivation (which was done with the aid of the analytic manipulations system SCHOONSCHIP [22]) or in their programming.

At last, in our programs we have neglected the charmed sea contributions, replaced the KM matrix by the simple Cabibbo matrix and neglected the effects of the charm excitation threshold (ξ_C - operator in [10]). All these effects are expected to be also a second-order effect for δ, as affecting both the numerator and denominator in (7). We have examined the effect of the charm threshold on the radiative correction and we have seen that it is very small indeed.

3.2. $\bar{\nu}_\ell + q_i \to \ell^+ + q_f$ ($\nu_\ell + \bar{q}_i \to \ell^- + \bar{q}_f$)

For this case we present only the full list of formulae, all comments made in Sect.3.1 being valid.

$$\frac{d^2\bar{\sigma}_i^{1\ell}}{dxdy} = \frac{[G_F^{(\mu)}]^2 S|K_{if}|^2}{\pi}\{(1-y)^2 x f_i(x)(1+\frac{\alpha}{\pi}\bar{\delta}_i^{CC}) + \frac{\alpha}{\pi}\int_x^1 d\xi f_i(\xi)\bar{\Phi}_i^{CC}(\xi) +$$

$$+\frac{\alpha}{\pi}(1-y)^2 \int_x^1 d\xi(\xi-x)^{-1}[\xi f_i(\xi)\bar{I}_i(\xi,x) - x f_i(x)\bar{I}_i(x,x)]\}, \qquad (16)$$

$$\bar{I}_i(\xi,x) = I_i(\xi,x)|_{f_i\to -f_i},$$

$$\bar{\delta}_c^{CC} = \bar{I}_i(x,x)\ln\frac{Sy(1-x)}{m_2^2} - 1 + (\frac{7}{4} - \ln\frac{S_q}{m_\ell^2})\ln\frac{S_q}{m_2^2} - \frac{1}{2}\ln^2\frac{S_q}{m_2^2} + \frac{\pi^2}{2} + \frac{3}{4}\ln\frac{S_q}{m_\ell^2} +$$

$$+ f_i[\frac{7}{4} - (\frac{3}{2} + 2\ln(1-y))\ln\frac{Q^2}{m_2^2} + \ln^2\frac{Q^2}{m_2^2} - \frac{\pi^2}{2} - \frac{3}{2}\ln\frac{S_q}{M_Z^2} + \ln y\ln(1-y)] +$$

$$+ f_i^2[-1 + (\frac{7}{4} - \ln\frac{Q^2}{m_1^2})\ln\frac{Q^2}{m_2^2} - \frac{1}{2}\ln^2\frac{Q^2}{m_2^2} + \frac{3}{4}\ln\frac{Q^2}{m_1^2}],$$

$$\bar{\Phi}_i^{CC}(\xi) = (1-f_i)^2\frac{Q^2}{4\tau}(1-\frac{m^2}{\tau})(1-y)^2 - \frac{1}{4}y + \frac{1}{4}(1-y)^2 +$$

$$+\frac{y}{4}[1+2(1-y)-7(1-y)^2]\frac{\xi}{\xi-y(\xi-x)} + y(1-y)[\frac{1}{2}-2(1-y)]\frac{\xi^2}{[\xi-y(\xi-x)]^2} -$$

$$-\frac{3}{2}y(1-y)^2\frac{\xi^3}{[\xi-y(\xi-x)]^3} + \frac{1}{2}y(1-y)^2\frac{\xi^2}{[\xi-y(\xi-x)]^2}[1+\frac{\xi}{\xi-y(\xi-x)}]\ln\frac{S_\tau^2}{\tau m_\ell^2} +$$

$$+f_i\{-\frac{1}{2} - (1-y) - \frac{1}{2}(1-y)^2 + (-\frac{1}{2}+y)\frac{x}{\xi} + \frac{3}{2}y(1-y)^2\frac{\xi}{\xi-y(\xi-x)} + \frac{1}{2}y(1-y)^2\frac{\xi^2}{[\xi-y(\xi-x)]^2} +$$

$$+y(1-y)^2\frac{\xi}{\xi-y(\xi-x)}\ln\frac{S_\tau^2}{\tau m_\ell^2}\} + f_i^2\{2(1-y)^2 - (1-y)^2(1+\frac{x}{2\xi}+\frac{x^2}{2\xi^2})\ln\frac{U^2}{\tau m_1^2} +$$

$$+[\frac{1}{4}+\frac{7}{4}(1-y)^2]\frac{x}{\xi} + [-\frac{1}{4}+(1-y)+\frac{3}{4}(1-y)^2]\frac{x^2}{\xi^2}\}.$$

For y-distributions one has

$$\frac{d\bar{\sigma}_i^{1\ell}}{dy} = \frac{[G_F^{(\mu)}]^2 S|K_{if}|^2}{\pi}\int_0^1 dx\cdot x\cdot f_i(x)[(1-y)^2 + \frac{\alpha}{\pi}\bar{\delta}_i^{(1)}(x,y)], \qquad (17)$$

where

$$\bar{\delta}_i^{(1)}(x,y) = -\frac{5}{4} + y + (1-y)^2\frac{\pi^2}{2} - 2(1-y)^2\Phi(y) - (1-y)^2\Phi(1-y) -$$

$$-\frac{1}{2}(1-y)^2\ln^2 y - (1-y)^2\ln^2(1-y) + (1-y)^2\ln\frac{y}{1-y}\ln\frac{S_q}{m_\ell^2} +$$

$$+(-\frac{7}{4}+\frac{5}{2}y-y^2)\ln\frac{y}{1-y} + (\frac{3}{4}-\frac{y}{2})\ln\frac{S_q}{m_\ell^2} +$$

$$+f_i\{\frac{1}{2}-\frac{5}{2}y+\frac{7}{4}y^2-\frac{3}{2}(1-y)^2\ln\frac{S_q(1-y)}{M_Z^2} + (1-y)^2[\Phi(y)-\Phi(1-y)]\} +$$

$$+f_i^2[\frac{23}{72} - (1-y)^2\frac{\pi^2}{6} - \frac{2}{3}(1-y)^2\ln\frac{Q^2}{m_1^2} - \frac{2}{9}y - \frac{1}{18}y^2].$$

Again, it was tested numerically that

$$\frac{d\bar{\sigma}_i^{1\ell}}{dy} = \int_0^1 dx\frac{d^2\bar{\sigma}_i^{1\ell}}{dxdy} \qquad (18)$$

and it was tested that $\bar{\delta}^{(1)}(x,y)$ at $f_i = +2/3$ coincides with $g^{\bar{\nu}}(x,S)$ from ref. [6] if from the latter one subtracts $(1-y)^2 C$, with C given by eq.(13).

4. THE LIST OF FORMULAE FOR NC NEUTRINO-QUARK SCATTERING

The NC neutrino-fermion scattering cross-section is represented following ref. [23] in terms of the induced $\rho(Q^2)$ and $\kappa(Q^2)$ factors and the QED correction (see also ref. [14] for the case of elastic $\nu_\mu e$-scattering).

4.1. $\nu_\ell + q_i \to \nu_\ell + q_f \quad (\bar\nu_\ell + \bar q_i \to \bar\nu_\ell + \bar q_f)$

$$\frac{d^2\sigma_i^{1\ell}}{dxdy} = \frac{[G_F^{(\mu)}\rho_{NC}^{(\nu_\ell;f_i)}(Q^2)]^2}{\pi} x f_i(x) \{[g_L^{(\nu_\ell;f_i)}(Q^2)]^2 +$$

$$+ [g_R^{(\nu_\ell;f_i)}(Q^2)]^2(1-y)^2 - \frac{2m_1^2 y}{S_q} g_L g_R \} + \frac{d\sigma_i^{QED}}{dxdy}, \tag{19}$$

$$\frac{d\sigma_i^{QED}}{dxdy} = \frac{[G_F^{(\mu)}]^2 S\alpha}{\pi^2} f_i^2 \{[g_L^2 + g_R^2(1-y)^2][\delta_i^{NC} x f_i(x) +$$

$$+ \int_x^1 d\xi(\xi-x)^{-1}(\xi f_i(\xi) I_i^{(0)}(\xi,x) - x f_i(x) I_i^{(0)}(x,x))] + \int_x^1 d\xi f_i(\xi)[g_L^2 H_L(\xi) + g_R^2 H_R(\xi)]\}.$$

Here

$$g_L = -\frac{1}{2} + |f_i| \sin^2\theta_W, \quad g_R = |f_i| \sin^2\theta_W,$$

$$g_L^{(\nu_\ell;f_i)}(Q^2) = -\frac{1}{2} + |f_i| \sin^2\theta_W^{(\nu_\ell;f_i)}(Q^2),$$

$$g_R^{(\nu_\ell;f_i)}(Q^2) = |f_i| \sin^2\theta_W^{(\nu_\ell;f_i)}(Q^2),$$

with

$$\sin^2\theta_W^{(\nu_\ell;f_i)}(Q^2) = \kappa^{(\nu_\ell;f_i)}(Q^2)\sin^2\theta_W, \quad \sin^2\theta_W = 1 - \frac{M_W^2}{M_Z^2}.$$

In the above expression $\rho^{(\nu_\ell,f_i)}(Q^2)$ and $\kappa^{(\nu_\ell;f_i)}(Q^2)$ are some quantities induced by weak loops, i.e.,

$$\rho^{(\nu_\ell;f_i)}(Q^2) = 1 + \frac{\alpha}{\pi}\rho_1^{(\nu_\ell;f_i)}(Q^2),$$

$$\kappa^{(\nu_\ell;f_i)}(Q^2) = 1 + \frac{\alpha}{\pi}\kappa_1^{(\nu_\ell;f_i)}(Q^2).$$

The explicit expressions for ρ_1 and κ_1 will be presented below, but we start with all quantities in the QED cross section $d^2\sigma_i^{QED}$:

$$I_i^{(0)}(\xi,x) = \ln\frac{U^2}{\tau m_1^2} - 1 - \frac{m_2^2}{\tau},$$

$$\delta_i^{NC} = I_i^{(0)}(x,x)\ln\frac{Sy(1-x)}{m_2^2} + \ln\frac{Q^2}{m_2^2}\left(1 - \ln\frac{Q^2}{m_1^2}\right) + \frac{3}{2}\ln\frac{Q^2}{m_1 m_2} - 1 - \frac{1}{2}\ln^2\frac{Q^2}{m_2^2},$$

$$H_L(\xi) = 1 - \frac{m_2^2 Q^2}{4\tau^2} - \frac{3Q^2}{4\tau} - \left(1 + \frac{x}{2\xi} + \frac{x^2}{2\xi^2}\right)\ln\frac{U^2}{\tau m_1^2} +$$

$$+ \left[\frac{1}{4}(1-y)^2 + \frac{7}{4}\right]\frac{x}{\xi} + \left[-\frac{1}{4}(1-y)^2 + (1-y) + \frac{3}{4}\right]\frac{x^2}{\xi^2},$$

$$H_R(\xi) = \left[1 - \frac{m_2^2 Q^2}{4\tau^2} - \frac{3Q^2}{4\tau} - \left(1 + \frac{x}{2\xi} + \frac{x^2}{2\xi^2}\right)\ln\frac{U^2}{\tau m_1^2}\right](1-y)^2 +$$

$$+ \left[\frac{1}{4} + \frac{7}{4}(1-y)^2\right]\frac{x}{\xi} + \left[-\frac{1}{4} + (1-y) + \frac{3}{4}(1-y)^2\right]\frac{x^2}{\xi^2}.$$

The cross section (19) can be integrated over x, and the result reads

$$\frac{d\sigma_i^{1\ell}}{dy} = \frac{[G_F^{(\mu)}]S}{\pi}\int_0^1 dx [\rho_{NC}^{(\nu_\ell;f_i)}(Q^2)]^2 x f_i(x) \{[g_L^{(\nu_\ell;f_i)}(Q^2)]^2 + \tag{20}$$

$$+ [g_R^{(\nu_\ell;f_i)}(Q^2)]^2(1-y)^2 - \frac{2m_1^2 y}{S_q} g_L g_R\} + \frac{d\sigma_i^{QED}}{dy},$$

where

$$\frac{d\sigma_i^{QED}}{dy} = \frac{[G_F^{(\mu)}]^2 S\alpha}{\pi^2} f_i^2 \int_0^1 dx \cdot x \cdot f_i(x)[g_L^2 h_L^{(1)}(x,y) + g_R^2 h_R^{(1)}(x,y)], \tag{21}$$

$$h_L^{(1)}(x,y) = -\frac{1}{18} + \frac{1}{3}(1-y) + \frac{1}{24}(1-y)^2 - \frac{2}{3}\ln\frac{Q^2}{m_1^2} - \frac{\pi^2}{6},$$

$$h_R^{(1)}(x,y) = -\frac{1}{18}(1-y)^2 + \frac{1}{3}(1-y) + \frac{1}{24} - \frac{2}{3}(1-y)^2\ln\frac{Q^2}{m_1^2} - (1-y)^2\frac{\pi^2}{6},$$

with $Q^2 = Sxy$ and $m_1 = xM_N$.

The integrand of (21) as a function of y coincides completely with the expressions of ref./14/ for the case of elastic $\nu_\mu e$-scattering. When integrated over y, eq.(21) leads to the simple result of ref./4/ for the QED part of the correction to the total cross section:

$$\sigma_i^{QED} = \frac{[G_F^{(\mu)}]^2 S\alpha}{\pi^2} f_i^2 \int_0^1 dx \cdot x \cdot f_i(x)\left[g_L\left(\frac{19}{24} - \frac{\pi^2}{6} - \frac{2}{3}\ln\frac{S_q}{m_1^2}\right) + \right. \tag{22}$$

$$\left. + g_R^2\frac{1}{3}\left(\frac{43}{24} - \frac{\pi^2}{6} - \frac{2}{3}\ln\frac{S_q}{m_1^2}\right)\right].$$

4.2. $\bar{\nu}_\ell + q_i \to \bar{\nu}_\ell + q_f (\nu_\ell + \bar{q}_i \to \nu_\ell + \bar{q}_f)$

For this case all the expressions of Sect.4.1 are valid with the simple replacement

$$g_L \leftrightarrow g_R, \quad g_L^{(\nu_\ell ; f_i)}(Q^2) \leftrightarrow g_R^{(\nu_\ell ; f_i)}(Q^2). \tag{23}$$

4.3. The explicit expression for $\rho_1^{(\nu_\ell ; f_i)}(Q^2)$ and $\kappa_1^{(\nu_\ell ; f_i)}(Q^2)$

The form factors $\rho^{(\nu_\ell ; f_i)}(Q^2)$ and $\kappa^{(\nu_\ell ; f_i)}(Q^2)$ induced by loops are slightly different from those presented in ref.[14] for ν_μe-elastic scattering

$$\rho_1^{(\nu_\ell ; f_i)}(Q^2) = \frac{1}{4(1-R)}\{\frac{3}{4}\frac{\ln R}{1-R} + \frac{9}{4} + \frac{3}{4} r_{HZ}[\frac{\ln r_{HZ}}{R(1-r_{HZ})} + \frac{\ln(R/r_{HZ})}{R - r_{HZ}}] - \tag{24}$$
$$- \frac{3}{2}(1+s_i) - \frac{3}{2R}s_i[\frac{1}{2} - 2(1-R)|f_i| + 4(1-R)^2 f_i^2] +$$
$$+ 3\frac{m_t^2}{M_W^2}[\frac{1}{2}I_0(Q^2, m_t^2, m_t^2) - I_1(Q^2, m_t^2, 0)]\}$$
$$R = \frac{M_W^2}{M_Z^2}, \quad r_{HZ} = \frac{M_H^2}{M_Z^2} \qquad s_i = f_i/|f_i|.$$

At $f_i = -1$ (24)* coincides with $\rho^{(\nu;e)}$ given by eq.(11) of ref.[14]

$$\kappa_1^{(\nu_\ell ; f_i)}(Q^2) = \frac{1}{4(1-R)}\{\frac{R}{1-R}[Z(-1) - W(-1) + Z^t(-1) - W^t(-1)] +$$
$$+ \frac{3}{2}(1+s_i) - 5 - \frac{2}{3}R + \frac{3}{2R}s_i[\frac{1}{2} - 3(1-R)|f_i| + 4(1-R)^2 f_i^2] + \tag{25}$$
$$+ 4I_3(Q^2, m_\ell^2, m_\ell^2) + \sum_f C_f[8(1-R)Q_f^2 - 2|Q_f|]I_3(Q^2, m_f^2, m_f^2)\}.$$

The quantities $Z(-1), W(-1), Z^t(-1), W^t(-1)$ and I_i are presented in ref.[14]. At $f_i = -1$ expression (25) coincides with (12) for the ν_μe-case[14]. We tested that at $f_i = -1, m_t = 0$ (24) and (25) reproduce numerically Tables 1 and 2 from ref.[23]. We tested also

* With our method of derivation of cross sections within the QPM we use ρ and κ for individual quarks of the i-th kind rather than quantities like $\rho^{(\nu,h)}$ used in ref.[4].

that m_t-terms coincide analytically with those given in ref.[17] if in the latter a simple misprint is corrected in formula (1.10) (factor $(4/3\, s_\theta^2/c_\theta^2 - 1)$ in the first line should be replaced by $(4/3\, s_\theta^2 - 1)$). At last, we tested that our form factors ρ and κ completely reproduce Table 2 from ref.[17] for the dependence on m_t of $\sin^2\theta_W$ extracted from the NC/CC neutrino cross sections ratio.

So, we see that any part of NC and CC cross sections $d\sigma^{1\ell}/dy$ coincides either analytically or numerically at least with one of the other-authors independent calculations. Hence, we may conclude that our expressions for $d^2\sigma^{1\ell}/dxdy$ presented here for the first time within the Sirlin framework are tested so comprehensively that we may claim them to be correct.

5. FINAL REMARKS

The presented formulae were realized in three FORTRAN programs available at CERN and JINR

1) NUDIS1 calculates $d\sigma^{1\ell}/dy$
2) NUDIS2 calculates $d^2\sigma^{1\ell}/dxdy$
3) NUDIST checks numerically that

$$\frac{d\sigma^{1\ell}}{dy} = \int_0^1 dx \frac{d^2\sigma^{1\ell}}{dxdy}, \tag{26}$$

which provides an additional test of the correctness of FORTRAN codes in our programs.

The abbreviation "NUDIS" stands for NeUtrino Deep Inelastic Scattering, "1" and "2" means $d\sigma$ and $d^2\sigma$ and "T" means Test.

All these programs take an average of cross sections for protons and neutrons (the isoscalar target). As the individual cross sections for scattering off the i-th type of quark are abailable there too, it is easy to obtain corrections for an arbitrary mixture of protons and neutrons. Comments in the body of the program should make this task easy.

As an example of numerical results derived with the aid of these programs we present in the table the radiative corrections to

$$R^\nu = \frac{\sigma^\nu_{NC}}{\sigma^\nu_{CC}} \tag{27}$$

and caused by them shifts in $\sin^2\theta_W$ estimated by the formula

$$\Delta\sin^2\theta_W = \frac{\frac{1}{2} - \sin^2\theta_W + \frac{20}{27}\sin^4\theta_W}{1 - \frac{40}{27}\sin^2\theta_W}(\delta R_\nu^{NC} + \delta R_\nu^{CC}), \tag{28}$$

where $\delta R_{\nu}^{NC} + \delta R_{\nu}^{CC}$ is the total electroweak radiative correction to R_{ν}. In the numerical calculation we use the following parameters: $\langle E_{\nu} \rangle = 80$ GeV, $E_h > 10$ GeV, $M_Z = 93.8$ GeV, $M_H = 100$ GeV, $\sin^2\theta_W = 0.227$.

Table

Dependence on m_t *of* δR_{ν}^{NC} *and* $\Delta \sin^2\theta_W$

m_t, GeV	δR_{ν}^{NC}, %	δR_{ν}^{CC}, %	$\Delta \sin^2\theta_W$
30	0.12		-0.0099
45	0.31		-0.0090
60	0.46		-0.0083
90	0.05	-2.23	-0.0102
120	-0.12		-0.0110
180	-0.21		-0.0114
240	-0.18		-0.0113

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Бардин Д.Ю., Докучаева В.А. E2-86-260

О радиационных поправках
к глубоконеупругому рассеянию нейтрино

Представлен единый набор формул для радиационных поправок к дваждыдифференциальным сечениям глубоконеупругого рассеяния нейтрино в каналах заряженного (CC) и нейтрального (NC) токов. Вычисления проведены в рамках простой кварк-партонной модели в схеме перенормировок на массовой поверхности. Показано, что эти формулы, будучи проинтегрированы до однократного сечения $d\sigma/dy$ или до полного сечения воспроизводят многие существующие в литературе результаты.

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On the Radiative Corrections
to the Neutrino Deep Inelastic Scattering

A unique set of formulae is presented for the radiative corrections to the double differential cross section of CC and NC deep inelastic neutrino scattering within a simple quark parton model. It is shown that these cross sections when being integrated up to the one-dimensional distribution with respect to y or up to the total cross section reproduce many results existing in the literature.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1986