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THE DESTRUCTIVE INTERFERENCE PHENOMENON AS A REASON FOR THE CONFINEMENT IN QED_{1+1}

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1. Introduction

The problem of the quark confinement in its contemporary formulation is based on the deep inelastic scattering experiments. It has been found that the inclusive cross sections, i.e., sums over all hadronic states may be described with the help of the imaginary parts of the quark (parton) diagrams *). From a field-theoretical point of view the quark hadronization implies zero probability for the coloured-particle creation. This statement may be considered as a model-free definition of the confinement.

However, nowadays the interpretation of the confinement problem - its criteria and mechanisms, is rather dependent on the model choice. The most popular criteria are the existence of a linearly-rising potential between the quarks and the increasing of the Wilson-loop area. In their formulation, the Schwinger model - two-dimensional massless quantum electrodynamics, has played an essential role [1,2]. Recent calculations of the coloured particle Green functions have made the confidence in their strictness doubtful [3,4]. It was found that they are compatible with the existence of poles in the quark Green function. Calculation of these poles is one of the standard methods used to determine the elementary excitation spectrum in QFT and statistical physics. The existence of a pole is interpreted as the presence in this spectrum of a particle with quark quantum numbers. From such a point of view the absence of a pole may be considered as a confinement criterion that coincides with the model-independent one mentioned above.

In the present paper we discuss the confinement problem in the Schwinger model in this context. However, the fermionic sector of the model is insufficiently studied due to some difficulties:

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*) Quark diagram hadronization has been called the quark-hadron duality principle which is now the QCD-phenomenology basis and is used successfully in various sum-rules derivation.

1) The bosonization of the theory leads to some additional effects which have to be separated from the dynamical ones;

2) The Green functions are not gauge-invariant, so the results concerning them depend on the gauge-condition choice;

3) The existence of infrared divergencies requires the corresponding regularization scheme.

To elucidate points 1-3 we first consider the free-fermion bosonization in the two-dimensional space-time (section 2). Then, we propose a gauge-invariant method for quantization of the Schwinger model and calculate Green's functions (section 3). In section 4 the topological degeneration of dynamical variables in finite-volume space-time is discussed and it is shown that this degeneration may cause the quark confinement in the massless QED_{1+1}.

2. Bosonization of the Free Fermions

Bosonization provides an adequate method for the description of two-dimensional field-theoretical models with fermions /5,6/. However, considering the equivalent bosonic-theory properties, one usually does not distinguish bosonization effects from the dynamical ones. To do this we shall begin with a brief review of the free-fermion bosonization in two dimensions. In this case the Lagrangian is

$$\mathcal{L}_0(x) = i\bar{\Psi}(x)\gamma^\mu \partial_\mu \Psi(x). \qquad (1)$$

In quantum theory with such a Lagrangian there appears an anomalous term in the current-component commutator

$$[j_{50}(x), j_{51}(y)] = \frac{1}{i\pi}\partial_y \delta(x-y) \qquad (2)$$

$$j_{5\mu}(x) = \bar{\Psi}(x)\gamma_5\gamma_\mu\Psi(x).$$

As is known, the physical reason for this anomaly is the filling of all negative-energy states, i.e., the Dirac sea /7,8/.

A simple substitution

$$j_{5\mu}(x) = \frac{1}{\sqrt{\pi}}\partial_\mu \phi(x) \qquad (3)$$

transforms relation (2) into the scalar field $\phi(x)$ commutator. Then, the current conservation law takes the form of the massless D'Alembert equation

$$\partial^\mu j_{5\mu} = 0 \Rightarrow \partial^\mu\partial_\mu \phi \equiv \Box\phi = 0.$$

Thus, the theory (1) is equivalent to the free massless scalar field one
$$\mathcal{L}_{0,b} = \frac{1}{2}(\partial_\mu\phi)^2.$$

There is only this scalar particle in the spectrum; fermions apparently disappeared. An analogous situation in the Schwinger model has been interpreted in papers /6/ as a manifestation of the confinement that takes place there. Following these papers, one might conclude that the free fermions are confined too. Such a conclusion is obviously wrong, so we need a correct description of the fermions themselves in the bosonized theory. In other words, we have to find the functional dependence of the spinors $\Psi(x)$ on the field $\phi(x)$.

The axial-current component $j_{50}(x)$ is proportional to the canonically conjugated momentum for the field $\phi(x)$

$$j_{50}(x) = \frac{1}{\sqrt{\pi}}\partial_0\phi(x) = \frac{1}{\sqrt{\pi}}\Pi(x).$$

So, the following relations take place:

$$[j_{50}(x), f(\phi(y))] = \frac{1}{\sqrt{\pi}}[\Pi(x), f(\phi(y))] = \frac{1}{i\sqrt{\pi}}\frac{\delta}{\delta\phi(x)} f(\phi(y))$$

that lead us to the equation on $\Psi(x)$:

$$[j_{50}(x), \Psi(\phi(y))] = \frac{1}{i\sqrt{\pi}}\frac{\delta}{\delta\phi(x)}\Psi(\phi(y)) = \delta(x-y)\gamma_5\Psi(\phi(y)).$$

Its solution has the form

$$\Psi(x) = e^{i\sqrt{\pi}\gamma_5\phi(x)}\chi(x)$$

$$\Psi^+(x) = e^{-i\sqrt{\pi}\gamma_5\phi(x)}\chi^+(x),$$

where $\chi(x)$ is a function that does not depend on the field $\phi(x)$. An additional requirement for reproducing the free two-point fermion Green function in this language may be used for defining $\chi(x)$:

$$\langle\Psi(x)\bar{\Psi}(y)\rangle = e^{i\pi\Delta_0(x-y)}\langle\chi(x)\bar{\chi}(y)\rangle$$

(here $\Delta_0(x-y)$ is Green's function of the free massless scalar field). This task may be achieved if we put

$$\chi(x) = e^{i\sqrt{\pi}\gamma_5\Sigma(x)}\chi_0(x),$$

where $\Sigma(x)$ is a free massless scalar field quantized with an indefinite metrics and $\chi_0(x)$ is a free fermion field. Thus, the fermion Green function may be obtained from the generating functional with the action:

$$S_B = \int d^2x \, \mathcal{L}_B \, ,$$

$$\mathcal{L}_B = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}(\partial_\mu \Sigma)^2 + i\bar{\chi}_0 \gamma^\mu \partial_\mu \chi_0 + J_1 \phi + J_2 \Sigma + + \bar{\eta} e^{i\sqrt{\pi}\gamma_5(\phi+\Sigma)} \chi_0 + \bar{\chi}_0 e^{-i\sqrt{\pi}\gamma_5(\phi+\Sigma)} \eta \, . \tag{4}$$

3. The Schwinger Model - Gauge-Invariant Variables and Green's Functions

Let us now turn to the Schwinger model - two-dimensional massless QED:

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\Psi}(i\gamma^\mu \partial_\mu + e\gamma^\mu A_\mu)\Psi \tag{5}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad , \quad \mu,\nu = 0,1 \, .$$

It is well known that the Green function properties depend on the gauge choice in an essential way /9/. So, considering them, it is convenient to formulate the theory only in terms of gauge-invariant quantities.

In the Lagrangian (5) the gauge field component A_0 is not a dynamical one. So, we shall eliminate it before quantization of the theory with the help of the corresponding equation of motion (for instance, a constraint equation)

$$\frac{\delta S}{\delta A_0} = 0 \quad \Rightarrow \quad \partial_1^2 A_0 = \partial_1 \partial_0 A_1 + e j_0 \tag{6}$$

$$S = \int d^2x \, \mathcal{L}(x) \quad , \quad x = (x^0, x^1) \, .$$

Substitution of the formal solution of (6)

$$A_0 = \frac{1}{\partial_1^2}\left(\partial_1 \partial_0 A_1 + e j_0\right) \tag{7}$$

in (5) leads us to the following Lagrangian in terms of the variables $A_1^I(A)$, $\Psi^I(A,\Psi)$:

$$\mathcal{L} = \frac{1}{2}(\partial_0 A_1^I)^2 + i\bar{\Psi}^I \gamma^\mu \partial_\mu \Psi^I - \frac{e^2}{2}(\partial_1^{-1} j_0^I)^2 = \mathcal{L}^I(A^I, \Psi^I), \tag{8}$$

where

$$A_1^I(A) = h(A)(A_1 + \frac{i}{e}\partial_1)h^{-1}(A) = (1 - \partial_1 \frac{1}{\partial_1^2}\partial_1)A_1 \equiv 0$$

$$\Psi^I(A,\Psi) = h(A)\Psi \, . \tag{9}$$

The operator $h(A)$ is determined in accordance with solution (7):

$$h(A) = exp\left\{-ie\int^{x_0} dx_0' \frac{1}{\partial_1^2}\partial_1 \partial_{0'} A_1\right\} = exp\left\{-ie\,\partial_1^{-1} A_1\right\}$$

so as to ensure an U(1)-gauge invariance of the variables (9)

$$\left.\begin{aligned} A_1' &= A_1 + \frac{\partial_1 \lambda(x)}{e} \\ \Psi' &= e^{i\lambda(x)}\Psi \end{aligned}\right\} \Rightarrow \quad \begin{aligned} A_1^I(A') &= A_1^I(A) \\ \Psi^I(A',\Psi') &= \Psi^I(A,\Psi) \, . \end{aligned}$$

We would like to emphasize that the physical variables (9) (which are subsequently used in this paper) are hardly fixed by the dynamics (i.e., by the constraint equation) and by the requirement of gauge invariance. The Lagrangian (8) itself formally coincides with the one in the Coulomb gauge

$$\partial_1 A_1 = 0 \quad , \quad \partial_1 \partial_0 A_1 = 0 \quad \Rightarrow A_1 = 0 \, , \tag{10}$$

i.e.,

$$\mathcal{L}^I(A^I, \Psi^I) = \mathcal{L}_{Coul.}(A,\Psi) = = i\bar{\Psi}\gamma^\mu \partial_\mu \Psi - \frac{e^2}{2}(\partial_1^{-1} j_0)^2 \, . \tag{11}$$

So, the Green-function generating functional which follows from (11) will reflect their gauge-invariant content.

As is known, the equivalent bosonic action for (11) in terms of the scalar field $\phi(x)$ (3)

$$S_B = \int d^2x \left[\frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}\frac{e^2}{\pi}\phi^2\right]$$

points out the existence of a neutral particle with mass $m = e/\sqrt{\pi}$ in the spectrum of the model. Taking into account fermionic degrees of freedom as well, we find the total action in the form

$$S_{tot}[\bar{\eta},\eta,J] = \int d^2x \left[\frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}(\partial_\mu \Sigma)^2 - \frac{m^2}{2}\phi^2 + + i\bar{\chi}_0 \gamma^\mu \partial_\mu \chi_0 + J_1 \phi + J_2 \Sigma + + \bar{\eta} e^{i\sqrt{\pi}\gamma_5(\phi+\Sigma)} \chi_0 + \bar{\chi}_0 e^{-i\sqrt{\pi}\gamma_5(\phi+\Sigma)} \eta\right] .$$

The corresponding generating functional for the Green functions is then written as

$$Z[\bar\eta,\eta,J] = \int D\phi\, D\mathcal{I}\, D\chi_0\, D\bar\chi_0 \exp(i S_{tot}). \tag{12}$$

Thus, we find for the fermion two-point Green function

$$G(x-y) = \frac{\delta^2 Z[\bar\eta,\eta,J]}{\delta\bar\eta(x)\,\delta\eta(y)}\Bigg|_{\bar\eta,\eta=0} = \tag{13}$$

$$= \exp\left\{-i\pi\left[\Delta_m(x-y)-\Delta_0(x-y)\right]\right\} G_0(x-y),$$

where $G_0(x-y)$ is the free-fermion Green function and $\Delta_m(x-y)$ is the massive scalar field one. The asymptotic behaviour of function (13) in the momentum space is

$$G(p)_{p\to\infty} = \frac{\hat p}{p^2}, \qquad G(p)_{p^2\to 0} \sim \frac{\hat p}{(p^2+i\varepsilon)^{5/4}} \tag{14}$$

(see Appendix A and also paper /11/). It follows from (14) that the probability to find a particle with quark quantum numbers is not equal to zero

$$|\Psi|^2 = \lim_{\hat p^2\to 0} \hat p\, G(p) \neq 0,$$

despite the linearly rising potential between the "quarks". Note that the analogous quantity in QED_{3+1}

$$|\Psi|^2 = \lim_{\hat p^2\to m^2} (\hat p - m)\, G(p)$$

equals one in the frames of the same minimal-quantization scheme $(\partial_i A_i^T = 0)$ /12/.

So, the validity of Wilson's criterion (an existence of a linearly-rising potential) does not lead automatically to the confinement. This was first realized in paper /3/; here we only give an example with an exactly solvable model.

4. Topological Degeneration of the Vacuum and of the Dynamical Variables in a Finite Space-Time

The starting point for the minimal quantization of gauge theories is the construction of dynamical gauge-invariant variables by an explicit solution of the constraint equations. However, a mathematically correct formulation of a consistent quantum theory may be

presented only in a finite volume space-time (remind that all physical observables in the field theory are normalized just in this way). The transition to a finite space-time

$$-\frac{T}{2} \le x_0 \le \frac{T}{2}$$

$$-\frac{R}{2} \le x_1 \le \frac{R}{2}$$

is crucial for our further considerations because of the nontrivial consequences of the functional ambiguity in eq.(6) which has not been taken into account. This ambiguity is connected with the solution of the corresponding homogeneous equation - the zero mode $G(x)$ of the operator ∂_1^{-2}

$$\partial_1^2 G(x) = 0, \tag{15}$$

that has to be included into the general solution for A_0. Choosing $G(x)$ in the form

$$G(x) = \frac{1}{e}\partial_0\lambda(x) \tag{16}$$

we are led to the following expression for A_0:

$$A_0 = \frac{1}{\partial_1^2}\left(\partial_1\partial_0 A_1 + e j_0\right) + \frac{1}{e}\partial_0\lambda, \tag{17}$$

where $\lambda(x)$ satisfies

$$\partial_1^2\partial_0\lambda(x) = 0, \qquad \partial_1^2\lambda(x) = 0. \tag{18}$$

In this case the additional term in (17) (when compared with (7)) may be regarded as a dynamical analogy of the Gribov ambiguity /13/*).

Zero mode $G(x)$ changes gauge-invariant variables (9):

$$(A_1^I)^\lambda = g(\lambda)\left(A_1 + \frac{i}{e}\partial_1\right)g^{-1}(\lambda) = A_1^I + \frac{\partial_1\lambda}{e} \tag{19}$$

$$(\psi^I)^\lambda = g(\lambda)\,\psi^I,$$

*) The Gribov ambiguity is based on the existence of gauge transformations which leave invariant the gauge-condition equations. In our case Gribov's equations for the gauge (10) coincide with the dynamical equations (18) for the function $\lambda(x)$.

where the operator $g(\lambda)$ is determined in the same way as $h(A)$ *)

$$g(\lambda)=\exp\left\{ie\int^{x_0}dx_0'\,\frac{\partial_{0'}\lambda}{e}\right\}=\exp\{i\lambda(x)\}. \tag{20}$$

The boundary conditions for the phase $g(\lambda)$ have to reflect the absence of sources in the space, so the relations

$$\lim_{|x_1|\to R/2} g_\pm(x_1)=\pm 1, \tag{21}$$

$$g_\pm(x_1)=g(x_0,x_1)\Big|_{x_0=\pm T/2}$$

should take place.

Now the problem is whether there exist nontrivial solutions of equations (18) in the class of smooth functions (20) that satisfy these boundary conditions.

The functions $g_\pm(x_1)$ determine a map of the space $R(1)$ onto the group U(1) (at the time-interval end points). In fact, condition (21) is a smoothness condition for this map that may be written as

$$\frac{1}{2\pi}\int_{-R/2}^{R/2}dx_1\, i g_\pm\partial_1 g_\pm^{-1}=\frac{1}{2\pi}\int_{-R/2}^{R/2}dx_1\,\partial_1\lambda(x)=n_\pm=0,\pm1,\pm2,\dots \tag{22}$$

Such nontrivial solutions exist and have the form

$$\lambda(x_1|N(x_0))=2\pi N(x_0)\frac{x_1}{R}, \tag{23}$$

where $N(x_0)$ is a smooth function with integer boundary values

$$N(\pm T/2)=n_\pm. \tag{24}$$

This situation implies a degeneration of the "classical" vacua

$$A_\pm = i g_\pm\partial_1 g_\pm^{-1}$$

*) The operator h determining the gauge-invariant variables depends on the terms in the solution of constraint equations that are not connected with the interaction Lagrangian. That is why the additional term in (17) changes its form $h\to h(A)\,g(\lambda)$ and it is just the operator $g(\lambda)$ that describes the phase ambiguity of the fields caused by the zero mode.

with parameters $n_\pm$ *). The "topological" variable $N(x_0)$ describes the nontrivial vacuum dynamics of the two-dimensional Abelian gauge field (Appendix B).

5. Topology and Nonobservation of the "Quarks" in QED_{1+1}

Quantization of the Schwinger model action according to the minimal scheme in finite-volume space-time revealed the structure of the vacuum topological degeneration. As a consequence of this degeneration there appear nonunique phases in the "coloured" field sources in the generating functional (12)

$$\eta^\lambda(x_1|N)\equiv g(x_1|N)\,\eta(x)=e^{i\lambda(x)}\eta(x)=e^{2\pi i N(x_0)\frac{x_1}{R}}\eta(x) \tag{25}$$
$$\bar\eta^\lambda(x_1|N)=\bar\eta(x)\,e^{-2\pi i N(x_0)\frac{x_1}{R}}.$$

Though the function $g(x_1|N)$ itself is a smooth one, after taking an average over degeneration, that it describes (i.e., over the topological number n), there appears a singularity

$$\langle g(x_1|N)\rangle_0=\lim_{L\to\infty}\frac{1}{L}\sum_{n=-L/2}^{L/2}\langle n|n+\frac{x_1}{R}\rangle=\delta_{\frac{x_1}{R},0} \tag{26}$$

where $\delta_{\frac{x_1}{R},0}$ is the Groneker symbol.

This singularity does not affect the two-current correlator structure because the phase factors extinguish each other. There remains a pole at the point $p^2=e^2/\pi$, representing the existence of a massive scalar particle in the spectrum.

At the same time the fermion Green function

$$G(x-y)=\frac{\delta^2 Z_{conf}}{\delta\bar\eta\,\delta\eta}\Big|_{\bar\eta,\eta=0}$$

changes in an essential way.

According to (25), (26) the functional $Z_{conf.}$ is defined as

*) In QED_{1+1} the existence of solution (23), (24) follows straightforwardly from the fact that the map of the coordinate space onto the group one is classified by an integer degree of mapping: $\pi_1(U(1))=Z$, hence eqs.(18) are consistent with the boundary condition (21), (22). In QED_{3+1} such a topological condition does not take place ($\pi_3(U(1))=0$); so both the boundary conditions: three-dimensional analogy of (21) and the usual one

$$\lim_{|x|\to R/2}\lambda(x)=0,$$

lead to one and the same (trivial) result/14/.

$$Z_{conf}[\bar\eta,\eta,J] = \lim_{R,T\to\infty} \lim_{L\to\infty} \frac{1}{L} \sum_{n=-L/2}^{L/2} Z_{R,T}[\bar\eta^{\lambda},\eta^{\lambda},J] .$$

Then, for the representation of the Green function $G(x-y)$ in the momentum space we find

$$G(p)_{p^2\to 0} = \lim_{R,T\to\infty} \lim_{L\to\infty} \frac{1}{L} \int d^2x\, d^2y\, e^{ip(x-y)} e^{-i\pi[\Delta_m(x-y)-\Delta_0(x-y)]} \times$$

$$\times G_0(x-y) \sum_{n=-L/2}^{L/2} \sum_{s=-\infty}^{\infty} \langle n|n+s-\tfrac{x_1}{R}\rangle\langle n-s+\tfrac{y_1}{R}|n\rangle = \tag{27}$$

$$= \lim_{R,T\to\infty} \int d^2x\, d^2y\, e^{ip(x-y)} e^{-i\pi[\Delta_m(x-y)-\Delta_0(x-y)]} G_0(x-y)\, \delta_{\frac{x_1}{R},\frac{y_1}{R}} \equiv 0 .$$

The function $G(p)$ vanishes because of the interference of the phase factors $g(x_1|N)$, that then may be called a destructive one. However, the identity (27) represents the existence of confinement in the Schwinger model in the sense of the model-independent definition from the Introduction. We have to emphasize the way the limit procedures in (27) (on R,T and on L) follow one another (the correct way being the same as in quantum statistics /9/) since the opposite choice leads us to the old result (see section 3).

Conclusions

We have tried to analyze the reasons for confinement by an example of Schwinger's model. In its conventional interpretation the charged-particle confinement is problematic because of the singularity in the "quark" Green function that allows the existence of excitations with quark quantum numbers. In this sense the Wilson criterion is not a criterion for confinement.

In this paper an approach to the problem is proposed where the theory is formulated in terms of dynamical gauge-invariant quantities and is quantized in a finite-volume space-time. As a result, there appears a topological degeneration of the gauge vacuum and the physical field phases. After taking an average over this degeneration the quark Green function vanishes, but the neutral-current correlator (in the limit of an infinite volume) coincides with the one in the standard approach to the model /15/.

Thus, the destructive interference of the topological phases of the physical fields may be considered as a possible reason for the confinement in the two-dimensional massless QED. We would like to emphasize that the topological structure of the Schwinger model coincides with the one of a non-Abelian gauge theory in four dimensions /16/. So, the conclusion about the existence of confinement takes place in QCD too /12/.

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Appendix A

Let us remind the form of the (right) fermion Green function in two-dimensional momentum space

$$\underset{\sim}{G}_{0R}(p) = \frac{\theta(p_+)\,\theta(-p_-)}{p_- + i\varepsilon} + \frac{\theta(-p_+)\,\theta(p_-)}{p_- - i\varepsilon} = \frac{p_+}{p_+p_- + i\varepsilon}, \tag{A.1}$$

where the "cone" components are defined as usual

$$p_\pm = p_0 \pm p_1 .$$

We shall now calculate the exact Green function of the (right) "quark" in the Schwinger model using relation (13). As we are going to discuss the confinement problem, we are interested in the behaviour of this function when $p^2 \to 0$. With the corresponding asymptotics of propagators $\Delta_0(x)$ and $\Delta_m(x)$ (entering into eq.(13)) taken into account, we find

$$\underset{\sim}{G}_R(p^2\to 0) = -\frac{m^{-1/2}}{4\pi} \int_{-\infty}^{\infty} dx_+ \int_{-\infty}^{\infty} dx_-\, e^{i(p_+x_- + p_-x_+)/2} \frac{(x_+x_-)^{1/4}}{(x_- - i\varepsilon)} =$$

$$= -\frac{m^{-1/2}}{4\pi}\Bigg[\int_0^{\infty} dx_+\, x_+^{1/4} e^{ip_-x_+/2} \int_{-\infty}^{\infty} dx_-\, \frac{x_-^{1/4}}{x_- - i\varepsilon/x_+} e^{ip_+x_-/2} + \tag{A.2}$$

$$+ \int_0^{\infty} dx_+ (-x_+)^{1/4} e^{-ip_-x_+/2} \int_{-\infty}^{\infty} dx_-\, \frac{x_-^{1/4}}{x_- + i\varepsilon/x_+} e^{ip_+x_-/2} \Bigg] .$$

The integrals in the first term of (A.2) can easily be calculated

$$J_1 = \int_0^\infty dx_+ \, x_+^{1/4} e^{ip_- x_+/2} = \left(\frac{2i}{p_- + i\varepsilon}\right)^{5/4} \Gamma\left(\tfrac{5}{4}\right) \theta(-p_-)$$

$$J_2 = \int_{-\infty}^{\infty} dx_- \frac{x_-^{1/4}}{x_- - i\varepsilon} e^{ip_+ x_-/2} = (-2i)^{1/2} \left(\frac{2i}{p_+ + i\varepsilon}\right)^{1/4} \cdot \Gamma\left(\tfrac{1}{4}\right) \theta(p_+)$$

that finally gives us

$$\underset{\sim}{G}_{R(1)}(p^2 \to 0) = C\theta(p_+)\theta(-p_-)\left(\frac{1}{p_- + i\varepsilon}\right)\left(\frac{1}{p_+ p_- + i\varepsilon}\right)^{1/4} \tag{A.3}$$

$$C = \frac{\Gamma^2\left(\frac{1}{4}\right)}{4\pi m^{1/2}} .$$

In an analogous way for the second term we obtain

$$\underset{\sim}{G}_{R(2)}(p^2 \to 0) = C\theta(-p_+)\theta(p_-)\left(\frac{1}{p_- - i\varepsilon}\right)\left(\frac{1}{p_+ p_- + i\varepsilon}\right)^{1/4} . \tag{A.4}$$

Comparing (A.3), (A.4) with (A.1) we are led to the conclusion that

$$\underset{\sim}{G}_R(p^2 \to 0) = C \, \underset{\sim}{G}_{0R}(p) \left(\frac{1}{p^2 + i\varepsilon}\right)^{1/4} .$$

Appendix B

The dynamics of the topological degeneration of the gauge-field vacuum is described by the action

$$S_T = \frac{1}{2}\int_{-T/2}^{T/2} dx_0 \int_{-R/2}^{R/2} dx_1 \left(\frac{\partial_1 \partial_0 \lambda}{e}\right)^2 = \frac{I}{2}\int_{-T/2}^{T/2} dx_0 \dot{N}^2 \tag{B.1}$$

$$I = \frac{1}{R}\left(\frac{2\pi}{e}\right)^2 .$$

Quantization of the action (B.1) is not difficult:

$$\mathcal{K} = \frac{\delta L_T}{\delta \dot{N}} = \dot{N} I \quad , \quad [\mathcal{K}, N] = i \, ,$$

where

$$L_T = \frac{\dot{N}^2 I}{2} .$$

The topological momentum $\mathcal{K}$ spectrum is easily found /17/ by taking into account the physical equivalence of the states

$$\langle p | N \rangle = \exp\{-ipN\} \quad \text{and} \quad \langle p | N+n \rangle = \exp\{-ip(N+n)\} .$$

The real state represents Bloch's wave that is an average over this degeneration with a weight $\exp\{in\theta\}$

$$\langle \mathcal{K} | N \rangle = \lim_{L \to \infty} \frac{1}{L} \sum_{n=-L/2}^{L/2} e^{in\theta} e^{-i\mathcal{K}(N+n)} =$$

$$= \begin{cases} e^{-i(2\pi k + \theta)N} , & \mathcal{K} = 2\pi k + \theta \\ 0 , & \mathcal{K} \neq 2\pi k + \theta \end{cases} \tag{B.2}$$

$$k = 0, \pm 1, \pm 2, \ldots \quad ;$$

$$|\theta| \leq \pi .$$

The spectrum (B.2) gives us for the free Hamiltonian

$$H_0 = \frac{\mathcal{K}^2}{2I} = (2\pi k + \theta)^2 \frac{e^2 R}{8\pi^2} = \frac{R}{2} \hat{E}^2 ,$$

where

$$\hat{E} = \frac{1}{i} \frac{\delta}{\delta A_1}$$

is a constant electric field /7/. Its minimal value (in modulo)

$$E_{min} = \frac{e\theta}{2\pi}$$

coincides with Coleman's constant electric field be introduced /10/ to explain the θ-vacuum in the Schwinger model. He considered θ as a simple parameter. In our approach θ is connected with the new topological variable, so it has a dynamical content as a characteristic of these constant electric fields that represent the real infrared vacuum of the theory.

The explicit expression for $N(x_0)$

$$N(x_0) = \frac{x_0}{T}(n_+ + n_-) + \frac{1}{2}(n_+ - n_-)$$

allows one to rewrite (B.1) in the form

$$S_T = \frac{I}{2T}\left(n_+ - n_-\right)^2 = \frac{I\nu^2}{2T} ,$$

where

$$\nu = \frac{e}{4\pi}\int d^2x\, \varepsilon_{\mu\nu} F^{\mu\nu} = n_+ - n_-$$

is the Pontryagin index [18]. Thus, the dynamical interpretation of this quantity is a peculiarity of the approach proposed in this paper.

So, at the end points of the time interval the so-called "classical vacua" of the theory [19] are purely gauge fields

$$A_{1\pm} = \frac{1}{e}\partial_1 \lambda_\pm(x_1)$$

and the gauge field

$$A_1(x) = \frac{1}{e}\partial_1 \lambda(x_1|N(x_0))$$

interpolates between them.

References

1. Casher A., Kogut J. Phys.Rev., 1974, D10, p. 732.
2. Wilson K.G. Phys.Rev., 1974, D10, p. 2445.
3. Arbuzov B.A. Phys.Lett., 1983, 125B, p. 197.
4. Zinoviev Yu.M. Teor.Mat.Fiz., 1982, 50, p. 207.
5. Klaiber B. Lectures in Theoretical Physics. Boulder Lectures, 1967. Gordon and Breach, New York, 1968, p. 141.
6. Kogut J., Susskind L. Phys.Rev., 1975, D11, p. 3594. Coleman S., Jackiw R., Susskind L. Ann.Phys. (N.Y.), 1975, 93, p. 267.
7. Ilieva N.P., Pervushin V.N. Yad.Fiz., 1984, 39, p. 1011.
8. Kiskis J. Phys.Rev., 1985, D31, p. 2006.
9. Bogolubov N.N., Bogolubov N.N., Jr. Introduction to Quantum Statistical Mechanics. Singapore, 1982.
10. Coleman S. Ann.Phys. (N.Y.), 1976, 101, p. 239.
11. Nekrasov M., Rochev V. Proc. of the 6th Seminar on the Problems of High Energy Physics and QFT. Protvino- 1982, vol. 1, p. 235.
12. Pervushin V.N. Preprint JINR, P2-85-520, Dubna, 1985.
13. Gribov V.N. Nucl.Phys., 1978, B139, p. 1.
14. Ilieva N.P. JINR Commun., E2-85-559, Dubna, 1985.

15. Tavkhelidze A.N., Tokarev V.F. Phys. of Elementary Particles and Atomic Nuclei, 1985, 16, p. 973.
16. Pervushin V.N. Riv. Nuovo Cim., 1985, 8, N. 10.
17. Ilieva N.P., Pervushin V.N. JINR Commun., E2-85-355, Dubna, 1985.
18. Belavin A. et al. Phys.Lett., 1975, 59B, p. 85.
19. Jackiw R. Lecture Notes. Les Houches, France, 1983.

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Явление деструктивной интерференции как причина конфайнмента в КЭД$_{1+1}$

Безмассовая КЭД$_{1+1}$ рассматривается в терминах динамических калибровочно-инвариантных переменных. На примере вычисления фермионной функции Грина демонстрируется, что линейный рост потенциала не исключает существования в спектре модели возбуждений с квантовыми числами "кварка" (т.е. выполнение критерия Вильсона еще не означает существования конфайнмента). Обсуждается топологическое вырождение фазы физических полей в конечном пространстве-времени. Показано, что причиной конфайнмента может быть деструктивная интерференция фазовых множителей.

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The Destructive Interference Phenomenon as a Reason for the Confiniment in QED$_{1+1}$

Two-dimensional massless QED is considered in terms of gauge-invariant dynamical variables. By an example of the fermion Green function it is shown that the linearly rising potential allows the existence of excitations with quark quantum numbers in the spectrum of the model (so the validity of Wilson's criterion does not lead automatically to the confinement). The topological generation of the physical-fields phase in a finite-volume space-time is considered. The destructive interference of the phase factors is pointed out as a possible reason for the confinement.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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