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NONLOCAL CONDENSATES AND QCD SUM RULES FOR PION WAVE FUNCTION

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An important problem of the theory of strong interactions is the calculation, "from the first principles of QCD", of the parton distribution functions and hadronic wave functions $\varphi_\pi(x)$, $\psi_N(x_1, x_2, x_3)$ ... accumulating information about the nonperturbative aspects of the quark-gluon dynamics. A very promising method to calculate the lowest moments of these functions is the QCD sum rule (SR) approach /1/. For example, the zeroth moment of $\varphi_\pi(x)$ (i.e., the f_π constant) was obtained in ref. /1/ with 5% accuracy. In ref. /2/ the SR for f_π were formally generalized for next moments of the function $\varphi_\pi(x)$. Information about the nonperturbative dynamics within the QCD SR method is accumulated by a power series over the vacuum expectation values (VEV's) of local operators which determines the magnitude of hadronic characteristics. Note, however, that $\varphi_\pi(x)$ is the function parametrizing matrix elements of a nonlocal operator [x)]

$$\langle 0|\bar{u}(0)\gamma_5\gamma_\mu d(z)|P\rangle\Big|_{z^2=0} = i P_\mu \int_0^1 e^{i(Pz)x} \varphi_\pi(x)\,dx. \qquad (1)$$

Thus, there arises the question whether it is possible to get reliable information about the essentially nonlocal objects within the standard version of the SR method restricted to the simplest local VEV's $\langle \bar{q}(0)q(0)\rangle$, $\langle G_{\mu\nu}(0)G_{\mu\nu}(0)\rangle$, etc., or it is necessary to take into consideration nonlocal VEV's $\langle\bar{q}(y)q(z)\rangle$, $\langle G_{\mu\nu}(y)G_{\mu\nu}(z)\rangle$... Moreover, the latter are in fact the initial objects of all calculations within the QCD SR approach, while the local VEV's emerge from them after expansion into the Taylor series.

To study the bilocal VEV's, it is convenient to introduce the parametrization[xx)]

$$\langle\bar{q}(0)q(z)\rangle = \langle\bar{q}q\rangle \int_0^\infty e^{\nu z^2/4}\,\Phi(\nu)\,d\nu \qquad (2)$$

having the structure of the α-representation for a propagator. The expansion of $\langle\bar{q}(0)q(z)\rangle$ over local operators corresponds to that

x) Here and in what follows we take fields of quarks u, d and gluons A_μ in the Fock-Schwinger gauge [3] $z^\mu A_\mu(z)=0$ where the covariant derivatives D_μ coincide with ordinary ones ∂_μ.

xx) Deriving a QCD SR one can always perform the Wick rotation $z_0 \to i z_0$, i.e., to treat all coordinates as Euclidean, with $z^2<0$.

of $\Phi(\nu)$ over $\delta^{(n)}(\nu)$: $\Phi(\nu) = \delta(\nu) - (\lambda^2/2)\,\delta'(\nu) + \ldots$
where $\lambda^2 \equiv \langle \bar q D^2 q\rangle / \langle \bar q q\rangle$ is the average virtualness of the vacuum quarks.

For the bilocal VEV containing a γ-matrix

$$\langle \bar q(0)\gamma_\mu q(z)\rangle = z_\mu \int_0^\infty e^{\nu z^2/4}\, \Psi(\nu)\, d\nu , \qquad (3)$$

the zeroth moment of $\Psi(\nu)$ is zero in the limit of massless quarks and that is why the $\delta^{(n)}(\nu)$ — expansion for $\Psi(\nu)$ starts with the $\delta'(\nu)$ term:

$$\Psi(\nu) = A\left[\delta'(\nu) - \frac{57}{80}\lambda^2 \delta''(\nu)\right] \quad \text{where} \quad A = \frac{2}{81}\pi\alpha_s\langle \bar q q\rangle^2 .$$

The contribution proportional to $\alpha_s\langle \bar q q\rangle^2$ results also from the "trilocal" condensates $\langle \bar q(0)\gamma_\mu \binom{1}{\gamma_5} A_\nu(y) q(z)\rangle$ which can be parametrized by a triple integral representation of the same type.

The role of the functions Φ, Ψ becomes especially clear, if one writes the SR directly for the wave function ($\bar x \equiv 1-x, \ldots$)

$$f_\pi \varphi_\pi(x) = \frac{3}{4\pi^2}\, x\bar x\, M^2\left(1-e^{-s_0/M^2}\right) + 4\,\Psi(xM^2) + \qquad (4)$$
$$\frac{16}{9}\pi\alpha_s\langle \bar q q\rangle^2 \left\{ \int_0^1 \bar x \bar y\, dy \int_0^1 da \int_0^1 db\; \Phi\!\left(\frac{xM^2}{a}\right) \Phi\!\left(\frac{yM^2}{b}\right) \cdot \right.$$
$$\left. \frac{\theta(x>\bar y)\,\theta(a<\bar b) + \theta(x<\bar y)\,\theta(a>\bar b)}{|xy\bar a\bar b - \bar x\bar y a b|} \right\} +$$
$$+ \ (\text{trilocals}) + O(\langle GG\rangle) + x \leftrightarrow \bar x .$$

Thus, the longitudinal momentum distribution of the quarks inside the pion is related to the virtualness distribution of the vacuum fields.

The standard SR [1,2] results from eq. (4) if one takes the first term of the $\delta^{(n)}$ — expansion for $\Phi(\nu)$, $\Psi(\nu)$. This model is, evidently, too crude if the average virtualness of the vacuum quarks λ^2 (and/or) gluons) is not small compared to the typical hadronic scale $s_0^{(N=0)} \simeq 0.75$ GeV^2. Existing estimates [5] yield $\lambda^2 = (0.4\pm0.1)$ GeV^2 $\sim s_0^{(N=0)}$. In such a situation, instead of the standard expansion over the local VEV's, one should use an expansion in which the large virtualness of the vacuum field has been taken into

account just in the first term. In other words, for function $M(z^2) = \langle \bar q(0) q(z)\rangle$ with finite correlation length $\sim 1/\mu$ of the vacuum fluctuations it is much more preferable to use the expansion of $\Phi(\nu)$ over $\delta^{(n)}(\nu-\mu^2)$ the first term of which takes into account the main effect caused just by the finite width of the function $M(z^2)$, while the subsequent terms describe those due to deviation of its form from the Gaussian one. That is why we take $\Phi(\nu)$ equal to $\delta(\nu-\lambda^2/2)$ and $\Psi(\nu)$ equal to $A\,\delta'(\nu-\frac{57}{80}\lambda^2)$. Values of the shifts are determined, obviously, by the second terms of the $\delta^{(n)}$ - expansions for $\Phi(\nu)$, $\Psi(\nu)$. In a similar way one can construct model δ-functions for the trilocal and gluonic VEV's. As a result, we obtain the following SR for the moments of the pion wave function:

$$4\pi^2 f_\pi \langle \xi^N\rangle = \frac{3\,M^2}{(N+1)(N+3)}\left(1-e^{-s_0/M^2}\right) + \frac{\pi\alpha_s}{3M^2}\langle GG\rangle\,\delta_G^N + \qquad (5)$$
$$\frac{64}{81}\pi^3\alpha_s\langle \bar q q\rangle^2 \left\{ \frac{1}{M^4}\sum_{i=0}^{2} \Delta_i\,\delta_i^N\left(1+2N\frac{\Delta_i}{\delta_i}\right)\theta(\delta_i>0) + \right.$$
$$\left. \frac{18}{\lambda^4}\left[\frac{1-\delta_1^{N+1}}{N+1} - \frac{1-\delta_1^{N+2}}{N+2}\right]\right\},$$

where $\xi = x-\bar x$, $\delta_i = 1 - a_i\frac{\lambda^2}{M^2}$, $\Delta_i = 1 - a_i\frac{\lambda^2}{2M^2}$, $a_0 = \frac{57}{80}$, $a_1 = 1$, $a_2 = \frac{4}{3}$.

If one takes $\lambda^2 = 0$, then eq. (5) coincides with the Chernyak – Zhitnitsky (CZ) SR [2], while the $O(\lambda^2)$ term in the λ^2 – expansion of eq. (5) gives (in a model-independent way) the magnitude of the $\alpha_s\langle \bar q D^2 q\rangle\langle \bar q q\rangle$ – contribution. It should be noted that the latter completely cancels, for $M^2 = 0.6$ GeV^2, the $\alpha_s\langle \bar q q\rangle^2$ contribution.

In the CZ SR [2] the relative contribution of $\alpha_s\langle \bar q q\rangle^2$ and $\alpha_s\langle GG\rangle$ corrections rapidly grows with that of N. As a consequence, the scale S_0 for N=2 (4) is 2.25 (3) times as high as that for N=0. That is why the values of $\langle \xi^N\rangle$ obtained in ref. [2] are by factors 2.25, 3 ... higher than the "asymptotic" values $3/(N+1)(N+3)$ corresponding to the asymptotic [6] wave function

In the SR (5) the coefficient in front of the numerically most important $\alpha_s \langle \bar q q \rangle^2$ -contribution decreases with N by almost the same law as the perturbative contribution, and as a consequence, fitting the SR (5) one obtains for the lowest moments the values

$$\langle \xi^2 \rangle = 0.25 \pm 0.01, \qquad \langle \xi^4 \rangle = 0.13 \pm 0.01, \qquad \langle \xi^6 \rangle = 0.07 \pm 0.02 \tag{6}$$

only slightly differing from the asymptotic ones. It is not surprising that the model wave function $\varphi_\pi(x) = f_\pi \frac{8}{\pi} \sqrt{x(1-x)}$ reproducing the values (6) is also close to φ_π^{as}. It should be emphasized here that the overestimate of $\langle \xi^2 \rangle$, $\langle \xi^4 \rangle$ in ref. /2/ is a direct consequence of the approximations $\Phi(\nu) \sim \delta(\nu)$, $\Psi(\nu) \sim \delta'(\nu)$. For any functions $\Phi(\nu)$, $\Psi(\nu)$ securing the "observed" value $\lambda^2 = 0.4$ GeV^2 of the ratio $\langle \bar q D^2 q \rangle / \langle \bar q q \rangle$ the results for $\langle \xi^N \rangle$ always will be close to those displayed in eq. (6).

Explicit form of the functions $\Phi(\nu)$, $\Psi(\nu)$ etc., in principle, can be obtained from a specific model (or, ideally, the theory) of the QCD vacuum. More practicable, however, seems a way based on the fact that the SR's similar to eq. (5) can be obtained for other wave functions and also for quark and gluon distribution functions which are known experimentally. This opens a possibility of formulating the inverse problem, i.e., that of finding the "vacuum distribution functions" $\Phi(\nu)$, $\Psi(\nu)$ (that are universal for all the hadrons!) from the given functions $f_{u/p}(x)$, $f_{d/p}(x)$, $f_{u/\pi}(x)$, etc.

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Михайлов С.В., Радюшкин А.В. E2-86-259
Нелокальные конденсаты и КХД правила сумм для волновой функции пиона

Обнаружено, что КХД правила сумм /ПС/для моментов волновой функции пиона $\phi_\pi(x)$ весьма чувствительны к форме координатной зависимости "нелокальных конденсатов" $\langle \bar q(0) q(z) \rangle \equiv M(z^2)$, $\langle \bar q(0) \gamma_\mu q(z) \rangle$ и т.п. Получены модифицированные ПС и найден явный вид $\phi_\pi(x) = \frac{8}{\pi} f_\pi \sqrt{x(1-x)}$ для распределений $M(z^2)$, имеющих ширину, диктуемую стандартным значением $\lambda^2 = 0{,}4$ ГэВ^2 отношения $\lambda^2 = \langle \bar q D^2 q \rangle / \langle \bar q q \rangle$.

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Mikhailov S.V., Radyushkin A.V. E2-86-259
Nonlocal Condensates and QCD Sum Rules for Pion Wave Function

It is shown that QCD sum rules (SR) for the moments of the pion wave function $\phi_\pi(x)$ are very sensitive to the z-dependence of the "nonlocal condensates" $\langle \bar q(0) q(z) \rangle \equiv M(z^2)$, $\langle \bar q(0) \gamma_\mu q(z) \rangle$, etc. We discuss a modified SR and obtain the explicit form of $\phi_\pi(x) = \frac{8}{\pi} f_\pi \sqrt{x(1-x)}$ corresponding to a distribution $M(z^2)$ which has the width dictated by the standard value $\lambda^2 = 0.4$ GeV^2 of the ratio $\lambda^2 = \langle \bar q D^2 q \rangle / \langle \bar q q \rangle$.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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