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PRESENT STATUS OF HARMONIC SUPERSPACE

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1. Preliminaries. Harmonic superspace (SS) has been invented by us two years ago /1,2/ in searching for a manifestly invariant description of theories with extended SUSY. The first successes of this approach (unconstrained superfield (SF) formulations of matter hypermultiplets and N=2 super Yang-Mills (SYM) theory, finding analytic SS group and prepotentials of the Einstein N=2 supergravity (SG), the first off-shell formulation of N=3 SYM) have been reported at the preceding conference /3/. During the next two years, an essential progress has been achieved both in understanding the basics of the method and in its further applications. In this talk we review these developments and outline the problems to be solved.

Recall the basic motivation which has led us to harmonic SS. We have realized in 1984 that an ordinary SS with finite sets of auxiliary fields is inadequate to extended SUSY's. There were several indications of this. First, one faced the familiar "no-go" theorems /4/. Second, there were serious difficulties in constructing unconstrained SF formulations of N=2 theories (analogous to formulations known in the case N = 1).

The way out we suggested was as follows /1-3/. One has to extend a conventional SS $(x^{\alpha\dot\alpha}, \theta^{\alpha}_{i}, \bar\theta^{\dot\alpha i})$ by adding new purely internal coordinates $u^{(a)}_{i}$ which parameterize a coset G/H, G being the automorphism group of the relevant superalgebra, H its certain subgroup. These coordinates are lowest harmonics on G/H. The SF's defined on harmonic SS $\mathbb{HR}$ thus constructed contain infinite sets of conventional SF's appearing as coefficients of harmonic expansions in powers of $u^{(a)}_{i}$. Among these general harmonic SF's there can be found SF's of a lower Grassmann dimension, the analytic SF's. These live as unconstrained objects on an analytic subspace $\mathbb{AR}$ of $\mathbb{HR}$. Just the analytic SF's provide us with manifestly invariant unconstrained geometric formulations of SUSY theories with $N \geqslant 2$. Such formulations became possible due to the following radically new property. The number of auxiliary and/or gauge degrees of freedom in analytic SF's (appearing in their harmonic expansions) is infinite. This is just the point where the above no-go theorems fail.

Now we turn to listing the main results obtained within the harmonic SS approach for the last two years.

2. The most general self-coupling of N=2 matter. One of the important problems in SUSY is to identify the "ultimate" off-shell representation which yields the most general matter self-coupling.

In the case N=1, it is a chiral multiplet described by unconstrained SF on a complex N=1 SS $\mathbb{C}^{4|2} = (x_L^{\alpha\dot\alpha}, \theta^\alpha)$. The genuine N=2 analogue of N=1 chiral SF is a complex analytic N=2 SF $q^+(\zeta, u)$ living as an unconstrained function on analytic N=2 SS /1/ ($\pm$ denote U(I)-charge):

$$\mathbb{R}^{4+2|4} = (\zeta, u) \equiv (x^{\alpha\dot\alpha}, \theta^{+\alpha}, \bar\theta^{+\dot\alpha}, u^\pm_i),\ u^{+i}u^-_i = 1,\ u^\pm_i \in SU(2),\ (i=1,2) \qquad (1)$$

The most general q^+ -action is /5,6/

$$S_q = \frac{1}{\varkappa^2}\int d\zeta^{(-4)} du\, \mathcal{L}^{(+4)}(q^+_A, \bar q^+_A, \dots, u^+_i, u^-_i);\ [\varkappa] = cm^2,\ [q^+] = cm^0 \qquad (2)$$

where integration goes over analytic SS(1). An analytic density $\mathcal{L}^{(+4)}$ arbitrarily depends on $q^+_A, \bar q^+_A$ (A=1,2,...), on its harmonic derivatives of any order (these are dimensionless and preserve analyticity) and may include explicitly harmonics u^+_i, u^-_i .

There were many attempts to describe N=2 matter by off-shell multiplets having a finite number of auxiliary fields (familiar tensor and relaxed multiplets and some newly proposed ones /6-9/). All these multiplets are most easily represented by some analytic SF's /6/. In /10-12,6/ we have defined an N=2 duality transformation and have shown with the help of it that all the self-couplings of above multiplets are equivalent to certain subclasses of the general q^+ - action (2) /12,6/ (e.g., having special isometries, etc.). Thus, the latter presumably describes the most general matter self-coupling in rigid N=2 SUSY [x]. This is apparently due to an infinite number of auxiliary fields in q^+ . As has been shown recently by Howe, Stelle and West /8/, infiniteness of the number of auxiliary fields is unavoidable when extending off-shell a complex form of a hypermultiplet.

According to Alvarez - Gaumé and Freedman /14/ any N=2 matter action produces a hyper-Kähler σ model in the physical boson sector. Correspondingly, we may call $\mathcal{L}^{(+4)}$ in (2) the "hyper-Kähler potential" /6/ (by analogy with the Kähler potential of N=1 case) /15/ . An analytic SS formulation suggests a new way of explicit calculation of hyper-Kähler metrics. Given an arbitrary $\mathcal{L}^{(+4)}$, one may eliminate auxiliary fields by their equations of motion to obtain a metric on the manifold of physical bosons which is guaranteed to be hyper-Kähler. The simplest example is a familiar Taub-NUT manifold which is coded in the action /5/

$$S_{TN} = \frac{1}{\varkappa^2}\int d\zeta^{(-4)} du\, [\bar q^+ D^{++} q^+ + \lambda (q^+)^2 (\bar q^+)^2]. \qquad (3)$$

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[x] Extension to nonzero central charges /1/ or to $d = 6$ /8,13/ is straightforward.

In ref. /16/ we have found analytic SF actions leading to other interesting metrics. These are the Eguchi - Hanson and multi-Eguchi - Hanson metrics (manifolds of dimension 4), Calabi and multi-Calabi ones (dimension 4n), metrics on the cotangent bundle of a 2nm-dimensional Grassmann manifold (dimension 4 nm, m $\geq$ 1).

Thus, we arrive at a suggestive idea of classifying hyper-Kähler metrics according to relevant hyper-Kähler potentials $\mathcal{L}^{(+4)}$. Intriguing questions are what is the precise mathematical meaning of $\mathcal{L}^{(+4)}$ and how the latter is connected with the primary principles of the hyper-Kähler geometry.

3. A closed form of the N=2 SYM action. An interesting development of our geometric formulation of N=2 SYM /1/ has been made by Zupnik /13/ . He has found a closed compact form for the action in terms of the analytic harmonic connection $V^{++}(\zeta, u)$. It heavily uses harmonic distributions introduced in /17/ (analogs of the N=0 distributions $1/x^n$, $\delta(x)$) and is written as /13/ (in the central basis of $\mathbb{HR}^{4+2|8}$):

$$S_{SYM_2} = -\frac{1}{g^2}\int d^{12}z\, \mathrm{Tr}\, \ln[1 + K_V](z)\ ,\ [g] = cm^0, \qquad (4)$$

where

$$K_V^{(-1,1)}(1,2) = \frac{V^{(++)}(z, u_2)}{u_1^{+i} u^+_{2i}} \qquad (5)$$

and Tr is taken both over discrete indices of the adjoint representation of gauge group and harmonic arguments u_1, ... u_n (the latter are regarded as continuous matrix indices with the harmonic integration over them instead of ordinary summation). The n-th term in the expansion of (4) in powers of V^{++} is as follows:

$$S^n_{SYM_2} = \frac{1}{g^2}\frac{(-i)^n}{n}\int d^{12}z\, du_1 \dots du_n \frac{\mathrm{Tr}\, V^{++}(z, u_1)\dots V^{++}(z, u_n)}{(u_1^+ u_2^+)\dots(u_n^+ u_1^+)}. \qquad (6)$$

In eqs. (5), (6), $1/u_1^+ u_2^+$ is a particular example of harmonic distributions, viz. the harmonic Green function:

$$D_1^{++}\frac{1}{u_1^+ u_2^+} = \delta^{(1,-1)}(u_1, u_2),\quad D^{++} = u^{+i}\frac{\partial}{\partial u^{-i}}$$

where $\delta^{(1,-1)}(u_1, u_2)$ is one of the variety of harmonic δ -functions /17/. As in the N=1 case, the N=2 SYM action is non-polynomial. Essentially new features are the nonlocality in harmonics and absence of spinor derivatives in interaction vertices. Harmonic nonlocalities do not create problems when quantizing N=2 SYM and are not present in final answers for SF amplitudes [18]. Note that the Lagrangian density in (4) is gauge invariant only up to full harmonic derivatives and is thus of the Chern-Simons type (in contradistinction to the tensor density in the standard representation of N=2 SYM action via the constrained chiral strength).

4. Quantization in harmonic N=2 SS. One of the main incentives to construct the harmonic SS formulations of theories with extended SUSY was the desire to have a manifestly supersymmetric quantization scheme in terms of unconstrained SF's [1-3]. Now this problem is completely solved for N=2 matter and SYM theories: in papers [17,18] we have given an extensive exposition of relevant harmonic superspace Green functions and Feynman rules as well as the first examples of manifestly N=2 supersymmetric quantum calculations [x)]. The main lesson is that these SF techniques are not more difficult than those in the case N=1. Let us quote, e.g., the SF propagators of q^+-hypermultiplet and of V^{++} (in the Feynman gauge and in the central basis of $\mathbb{HR}^{4+2|8}$):

$$\langle \bar{q}^+_a(1)\, q^+_b(2)\rangle \sim \frac{i}{p^2}\,\frac{(D_1^+)^4(D_2^+)^4}{(u_1^+u_2^+)^3}\,\delta^8(\theta_1-\theta_2)\,\delta_{ab} \qquad (7)$$

$$\langle V^{++}_a(1)\, V^{++}_b(2)\rangle \sim \frac{i}{k^2}\,(D_1^+)^4\,\delta^8(\theta_1-\theta_2)\,\delta^{(-2,2)}(u_1,u_2)\,\delta_{ab},$$

where the operators $(D^+)^4$ ensuring analyticity and harmonic distributions $1/u_1^+u_2^+$ and $\delta^{(-2,2)}(u_1,u_2)$ appear. Expressions for vertices are also simple.

Now we shall sketch the most important features of harmonic supergraph techniques.

A. Harmonic coordinates (in contrast, e.g., to extra coordinates in Kaluza - Klein theories) do not lead to new divergences. The reason is that only nonpropagating (auxiliary or gauge) degrees of freedom are associated with them, not the physical ones.

B. Harmonic nonlocalities disappear if external legs of a diagram are placed on-shell. All the harmonic integrals can be computed by simple algebraic manipulations.

C. Quantum corrections can always be written as integrals with the full Grassmann measure $d^8\theta$.

D. No ghosts-for-ghosts are needed when quantizing N=2 SYM.

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x) Some of these studies have been performed in parallel with us by Kubota and Sawada and Ohta and Yamaguchi [19] (in the latter paper for a nonzero central charge).

The fact that the effective action is an integral of the type $d^8\theta$ is known to yield significant improvements in the ultraviolet behaviour. The most striking application [18] is a simple general proof of off-shell finiteness of hyper-Kähler supersymmetric d=2 σ models corresponding to the d=2 reduction of the general q^+ action (2). Indeed, in d=2 $[x]=cm^0$ and $[q^+(p,\theta,u)] = cm^2$ so the n-particle contribution to the effective action has the generic form:

$$\Gamma_n = \int d^8\theta\, du\, (d^2p)^{n-1}\,[q(p,\theta,u)]^n\, I(p) \qquad (8)$$

in accord with the property C. One easily observes that $[I(p)] = cm^2$, and hence $I(p)$ is convergent.

Now we are concerned with quantization of N=3 SYM [2] along similar lines. It seems especially urgent to work out a proper background field method.

5. N=2,3 supergravities in harmonic SS. In ref. [1] we have defined the harmonic SS group of Einstein N=2 SG (in its first version) and corresponding unconstrained analytic SG pre-prepotentials. Now we know these for conformal N=2,3 SG too [20]. The gauge groups of the latter preserve the fundamental concepts of analytic subspace and $U(1)$-charges, as well as the unitarity and unimodularity conditions of harmonics. These are local extensions of rigid conformal supergroups SU(2,2/2) and SU(2,2/3). In the case N=2 [20]:

$$\delta\zeta^M = \lambda^M(\zeta,u)\,,\ \delta u^+_i = \lambda^{++}(\zeta,u)\,u^-_i\,,\ \delta u^-_i = 0 \qquad (M = m, \mu+, \dot{\mu}+). \qquad (9)$$

The fundamental unconstrained geometric quantities which represent the Weyl multiplet are ++ components of the analytic vielbein $H^{++M}(\zeta,u)$, $H^{++++}(\zeta,u)$ covariantizing the derivative D^{++}:

$$\mathcal{D}^{++} = \partial^{++} + H^{++M}(\zeta,u)\,\partial_M + H^{++++}(\zeta,u)\,\partial_{++}\,,\quad \partial^{++} = u^{+i}\frac{\partial}{\partial u^{-i}}\,,\ \partial_{++} = u^{-i}\frac{\partial}{\partial u^{+i}}.$$

Generalization to N=3 is straightforward; one has only to take into account the presence of two independent complex analytic directions in SU(3) / U(I) x U(I) [2] instead of one (++) in SU(2)/U(I).

Following the standard conpensation ideology we may, in principle, construct action for any version of the minimal Einstein N=2 SG as a sum of actions of compensating Maxwell $(V^{++5}(\zeta,u))$ and matter N=2 multiplets in the conformal SG background [21]. At present we dispose of the analytic SS description of all the matter compensators known before [1,20,12]. So, to construct the complete SF actions for all the versions of minimal Einstein SG, it remains to find the action for $V^{++5}(\zeta,u)$, and this is in progress now. A new possibility is to use as a compensator the basic unconstrained analytic SF q^+ (or ω)[20]. The corresponding version of Einstein SG will contain an infinite number of auxiliary fields. We expect it to be very promising.

An interesting problem ahead is to construct off-shell Einstein N=3 SG. A component consideration [22] implies that it can be obtained by coupling three Maxwell N=3 multiplets (or one SO(3) - N=3 SYM multiplet) to conformal N=3 SG. Thus, what one needs is to extend the analytic SF action of N=3 SYM [2] to local conformal SUSY.

6. New trends. A most modern area of applications of the harmonic SS approach is the superparticle and superstring theories. Recently the "light-cone harmonic SS" has been constructed [23] which extends ordinary N=1 SS in D dimensions by adding harmonics on the coset SO(1,D-1)/ SO(1,1)xSO(D-2), SO(1,D-1) being the Lorentz group of M^D. In such a harmonic SS there is an invariant analytic subspace involving half the original spinor variables. The superparticle action can be reformulated in this light-cone analytic subspace so that no local fermionic invariance is needed and the Lorentz symmetry is preserved (due to the presence of new harmonic variables which carry Lorentz vector indices). Another application is the 10-dimensional SYM theory. Its on-shell constraints can be rewritten as the integrability conditions for the existence of the light-cone analytic SF's. There are, however, serious difficulties with the off-shell formulation (as distinct from the N=3, d=4 SYM [2]). We would like to point out that other harmonizations of d=10 N=1 SS are also possible, corresponding to different choices of the coset of SO(1,9). These conceal potentialities which may have utterly unexpected manifestations, e.g., in superstrings and d=10, N=1 (d=4, N=4) SYM theories. An important point is that the invariant analytic subspaces arising with this type of harmonization contain reduced numbers both of Grassmann and ordinary bosonic (x) coordinates, as is illustrated already by the simple example of ref.[23].

Another line of extending the harmonic SS business has been proposed by Kallosh[24,25]. She gave some reasonings [25] that the harmonic SS with the even part $M^{10}\times(E_8\times E_8/U_1(1)\times\ldots\times U_{16}(1))$ presumably can be used for the off-shell formulation of d=10 $E_8\times E_8$ Yang - Mills - supergravity, associated with the heterotic string. Also, a complete tensor apparatus of N=3 SYM theory in the harmonic SS [2] has been constructed [24,25].

For these two years, some mathematical developments of the harmonic SS approach have been made. In particular, Rosly and Schwarz in ref.[26] treated it with the accent on its affinity with twistors

(recall that our geometric formulation of N=2 SYM essentially incorporates the interpretation of N=2 SYM constraints given for the first time by Rosly[27]).

We end this rather schematic survey by indicating the principal direction of further studies. The most urgent problem now is, in our opinion, to find a closed off-shell geometric formulation of the most intriguing of d=4 SYM theories, the N=4 SYM theory. It cannot be achieved by a simple prolongation of formulations found by us for N=2 and N=3 theories (see analysis in [28]) and thus seems to require an essentially new look. We believe that the solution of this task will naturally lead us to superstrings, keeping in mind a familiar correspondence between N=4, d=4 and N=1, d=10 SYM theories.

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1. Введение. 2. Наиболее общее самодействие N = 2 материи и гиперкэлеров потенциал. 3. Замкнутое выражение для N = = 2 ЯМ действия в терминах аналитической гармонической связности. 4. Квантование в гармоническом суперпространстве. Доказательство конечности d = 2 - гиперкэлеровых суперсимметричных σ-моделей. 5. Конформные и эйнштейновские N = 2,3 супергравитации в гармоническом суперпространстве. 6. Новые направления: применения к суперчастице и суперструне.

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Present Status of Harmonic Superspace

1. Preliminaries. 2. The most general self-coupling of N = 2 matter and introducing hyper-Kahler potential. 3.A closed form of N = 2 SYM action in terms of analytic harmonic connection. 4. Quantization in harmonic superspace. A proof of finiteness of d = 2 hyper-Kahler supersymmetric σ models. 5. Conformal and Einstein N = 2,3 supergravities in harmonic superspace. 6. New trends: applications to superparticle and superstring.

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