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# CONSIDERATION OF THE VACUUM OF QCD IN A COMPOSITE QUARK MODEL. NONSTRANGE HADRONS

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## 1. Introduction

Composite quark models are successfully applied to describe static properties of hadrons: masses, magnetic moments and so on. However, their connection with QCD looks quite problematic. This became clear after the method of QCD sum rules /1/ was suggested. Among the applications of that method there are also the calculations of static properties of hadrons /1-5/.

Out of QCD sum rules it was recognised that these characteristics are basically attributed to the existence of some condensates in the QCD vacuum: quark $\langle\bar{Q}Q\rangle$, $\langle G^2\rangle$, etc. It seems likely that the most consistent attempt to consider the QCD vacuum structure in composite models was undertaken in quark bag models /6/. So, in the MIT version of the model it is supposed that the vacuum is completely destroyed within the bag where quark fields exist.

However, this hypothesis was not confirmed within the QCD sum rules /7/. The bag constant B characterizing the degree of vacuum destruction appeared to be considerably smaller than the "depth" of nonperturbative vacuum calculated from the QCD sum rules. So, neglect of nonperturbative effects in the bag does not look grounded.

The aim of this paper is to show that the quark model consistent with independent "experimental" calculations of sum rules may be constructed only if the MIT hypothesis on the vacuum fully destroyed by colour fields is ruled out. We shall propose that the vacuum in the bag is not destroyed and the nonperturbative effects of the quark interaction are the most important in this region.

## 2. The Model Hamiltonian

As a starting point we choose the QCD Lagrangian

$$\mathcal{L}_{QCD} = -\frac{1}{4}\left(G^{a}_{\mu\nu}\right)^2 + \sum_{F,i,j} \bar{\psi}_{F,i}\left(i\nabla_\mu^{ij} - m_F\delta^{ij}\right)\psi_{F,j} \tag{1}$$

and the appropriate equations of motion following from it

$$\begin{aligned} i\gamma^\mu \nabla_\mu \psi &= m\psi, \\ \mathcal{D}_\mu G_a^{\mu\nu} &= g\bar{\psi}\gamma^\nu \frac{\lambda_a}{2}\psi , \end{aligned} \tag{2}$$

where

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu ,$$
$$i\nabla_\mu = i\partial_\mu + \frac{g}{2}\lambda^a A^a_\mu ,$$

$\mu, \nu$ are the Lorentz indices; $a, b$ are colour indices, $F$ are spin-unitary indices.

Then, the following suppositions are made concerning the structure of a solution that corresponds to the choice of a zero approximation to the theory. First, we think that the low-frequency and high-frequency behaviour of solutions are independent of each other with a high degree of accuracy.

Second we suppose the high-frequency $\omega \sim \omega_q$ (valence) field components are described by solutions of the bag-model equations. Eventually, we might choose any other reasonable equations defining localization of valence fields, and the quality of this supposition wouldn't change.

Low frequency (condensate) components of the fields $\omega \ll \omega_q$ are solutions of the QCD equations and are characterized by a set of numbers, different vacuum condensate quantities.

Under such suppositions the model Hamiltonian of the interaction of valence components with condensates is restored uniquelly.

So, to take into account the interaction of valence quarks with vacuum fields, we make in (1) the transformation

$$\Psi(x) = q(x) + Q(x), \qquad (3)$$
$$A(x) = A_T(x) + A_{vac}(x).$$

Here, low-frequency vacuum fields satisfy the equations of motion

$$i\gamma^\mu \nabla_\mu Q = mQ , \qquad (4)$$
$$\mathcal{D}_\mu G^{\mu\nu}_{vac} = g\bar{Q}\gamma^\nu \frac{\lambda}{2} Q ,$$

where $\nabla_\mu$ and $\mathcal{D}_\mu$ are covariant derivatives with respect to $A_{vac}(x)$. In this work only nonstrange quarks are considered, i.e. we choose $m_u = m_d = m_q = 0$.

In expression (3) $q(x)$ and $A_T(x)$ are localized components of quark and gluon fields and, as proposed, they are approximated by solutions of the MIT bag model equations



$$\left.\begin{array}{l} i\gamma^\mu \nabla^T_\mu q = 0 , \\ \mathcal{D}^\mu_T F^a_{\mu\nu} = \frac{g}{2}\bar{q}\gamma_\nu \lambda^a q , \end{array}\right\} \text{ in bag,} \qquad (5)$$
$$\left.\begin{array}{l} i\gamma_\mu n^\mu q = q , \\ n^\mu F_{\mu\nu} = 0 . \end{array}\right\} \text{ on surface.}$$

Here $\nabla^T_\mu$, $\mathcal{D}^T_\mu$ are covariant derivatives with respect to $A_T(x)$. To take into account the interaction of quarks with the field $A_T(x)$ in the bag, the QCD perturbation theory is used. In the static cavity approximation for massless quarks the solutions of equations (5) of zero order in $g$ are [6]

$$q_n(x) = \frac{N}{\sqrt{4\pi}} \begin{pmatrix} i j_0(x_n r) U \\ -j_1(x_n r)(\vec{\sigma}\hat{\vec{r}}) U \end{pmatrix} e^{-i\omega_n t} , \qquad (6)$$

where $\omega_n = x_n/R$ are single-particle energies of massless quarks in the bag with radius R.

The Hamiltonian of interaction of the valence fields $q(x)$ with the vacuum ones $Q(x)$ and $A_{vac}(x)$ follows from (1)-(6):

$$H_{int} = \frac{\omega}{2}(\bar{q}\gamma_0 Q + \bar{Q}\gamma_0 q) + \frac{i}{2}(\bar{q}\gamma_0 \partial^0 Q - \partial^0 \bar{Q}\gamma_0 q) - \qquad (7)$$
$$- \frac{g}{4}(\bar{q}\gamma_\mu \lambda^a Q + \bar{Q}\gamma_\mu \lambda^a q) A^{vac\,\mu}_a - \frac{g}{2}\bar{q}\gamma_\mu \lambda^a q A^{vac\,\mu}_a .$$

Thus, in our model a hadron is a set of subvacuum excited states with quark quantum numbers localised in the bag. These valence quarks are quasifree and interact with nonperturbative vacuum through (7) and with each other by the QCD perturbative theory. The latter interaction characterises the colour-magnetic interaction of quarks and the former defines, in particular, the effective quark mass in the bag.

3. <u>The Condensates Contribution to the Hadron Energy</u>

A correction to the hadron energy caused by the interaction is defined by the expression:

$$\Delta E = \frac{\langle \Phi | H_{int} | \Psi \rangle}{\langle \Phi | \Psi \rangle} = \langle \Phi | H_{int} | \Psi \rangle_c , \qquad (8)$$

where $|\Phi\rangle$ is a nonperturbated hadron wave function, the symbol $C$ means the connected diagrams

$$|\Psi\rangle = U(0,-\infty)|\Phi\rangle , \tag{9}$$

$$U(0,-\infty)=\sum_{n=0}^{\infty}\frac{(-i)^n}{n!}\int_{-\infty}^{0}dt_1\ldots\int_{-\infty}^{0}dt_n\, T[H_{int}(t_1)\ldots H_{int}(t_n)].$$

To carry out calculations, we shell use the fixed-point gauge[8] $a_\mu A^\mu(a)=0$ . In this gauge the fields are expressed as a series ( $a_\mu=0$ ):

$$Q(x)=Q(0)+x^{\sigma}\nabla_{\sigma}Q(0)+\frac{x^{\sigma}x^{\tau}}{2}\nabla_{\sigma}\nabla_{\tau}Q(0)+\ldots, \tag{10}$$

$$A_{\mu}^{vac\,a}(x)=\frac{x^{\rho}}{2}G^{a}_{\rho\mu}+\frac{x^{\rho}x^{\sigma}}{3}(\mathcal{D}_{\sigma}G_{\rho\mu})^{a}+\ldots$$

For instance, from (9) we have in the first-order perturbation theory:

$$\Delta E=-i\int_{-\infty}^{0}\langle\Phi|H_{int}(0)H_{int}(t)|\Phi\rangle\, dt . \tag{11}$$

In the case of massless quarks, from (7), (8), (10), and (11) we obtain a correction proportional to the quark condensate

$$\Delta E=-\frac{\langle\bar{Q}Q\rangle\omega_0}{48}\langle\Phi|\int q(\vec{x})q(\vec{y})\,d\vec{x}\,d\vec{y}\,|\Phi\rangle = -N\frac{\pi\langle\bar{Q}Q\rangle R^2}{24(\varkappa_0-1)}, \tag{12}$$

where $N$ is the number of quarks in a hadron.

Other contributions from (8) are depicted in the diagrams of fig.1.

In this work we shall consider only the space-homogeneous condensates. According to our basic assumptions, the vacuum fluctuations with frequencies that are much larger than the characteristic frequencies of valence quarks in the bag ( $\omega \gg \omega_q$ ) are taken into account independently by introducing the interactions with instantons. These effects will be discussed in the next section.

Under the above assumptions the energy of interaction of quarks with condensates in the bag is defined by the contributions from diagrams a) and f) of fig.1 [x]

[x] As was shown in the QCD sum rules [1,3] it is these diagrams that define the spectrum of light mesons.



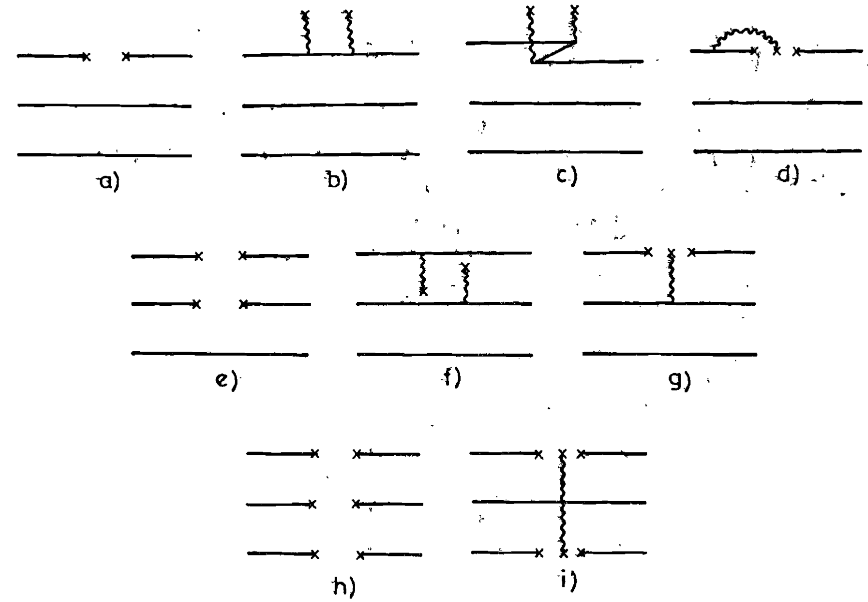


Fig.1

Diagrams that contribute to the hadron energy: a)-d) - one-particle, e)-g) - the pair interactions of quarks, h) - i) - the three-particle interactions. Notation: ——x - quarks "dives" in the condensate, ~~~x the same for gluon.

$$\Delta E_{vac}=-N\frac{\pi}{24}\frac{\langle\bar{Q}Q\rangle R^2}{\varkappa_0-1}+\frac{\pi^2\langle\bar{Q}Q\rangle^2R^5}{2^7 3^2\varkappa_0(\varkappa_0-1)}M_0 , \tag{13}$$

where for baryons $M_0=12$ and for mesons $M_0=4$ .

It follows from expression (13) that in our model, stability of the bag is achieved in a self-consistent manner due to the interaction of quarks with physical vacuum. So, in contrast with MIT model it is not necessary to introduce into the mass formula geometrical terms such as the volume (surface) energy $BR^3(\sigma R^2)$ and the Casimir energy $-\frac{Z_0}{R}$ . Such contributions are still indefinite in nature, and, as the calculations of $Z_0$ show [20], unimportant numerically.

By taking into account the correction due to the center of mass motion [10] the hadron mass is defined by the expression:

$$M^2 = \left(\frac{N\varkappa_0}{R} + \Delta E_G + \Delta E_{vac}\right)^2 - N\left(\frac{\varkappa_0}{R}\right)^2 , \tag{14}$$

where $N\varkappa_0/R$ is the kinetic energy of quarks in the bag, $\Delta E_G$ is due to one-gluon exchange (of the QCD perturbation theory in the bag [9]). Choosing the standard values [1]

$$\alpha_s = 0,7 \quad ; \quad \langle \bar{Q}Q \rangle = -(250 \text{ MeV})^3 \tag{15}$$

and using the bag stability condition

$$\frac{dM^2}{dR} = 0, \tag{16}$$

one obtains from (14):

$$m_\Delta = 1228 \text{ MeV}, \quad m_\rho = 735 \text{ MeV},$$
$$m_N = 1133 \text{ MeV}, \quad m_\pi = 591 \text{ MeV}.$$

When standard parameters (15) are used the mass formula well reproduces energies of the $\Delta$ -isobar and $\rho$-meson and gives essentially larger values of the nucleon and $\pi$-meson masses. In the next section we proceed to consider the mechanism that has an essential influence on forming the energy spectrum of particles, the wave function of which contains scalar spin channels.

## 4. Instantons

In the QCD vacuum there are also fluctuations with $\omega_{vac} \ll \omega_0$, for which the series (13) already has no sense, and all expansion terms have to be considered. Instantons are an example of such fluctuations [11]. The presence of instantons in the QCD vacuum was shown[12,13] to result in a specific interaction between quarks that may be responsible for a spontaneous breaking of chiral symmetry in QCD and also for solving the U(I)-problem. Moreover, it is clear from the sum-rule analysis [14,15] that instantons render an essential influence on the meson-mass spectrum. Recently, the effects of instantons of a small size on hadron masses have been estimated within the MIT model. Our basic assumption is that a nonperturbative vacuum is not destroyed in the bag. Therefore, for massless quarks the effective Lagrangian induced by instantons has the form [16,17]

$$\mathcal{L}_{eff} = \eta_{int}\Big[\bar{u}_R u_L \bar{d}_R d_L + \frac{3}{32}\big(\bar{u}_R \lambda^a u_L \bar{d}_R \lambda^a d_L - \tag{17}$$
$$- \frac{3}{4}\bar{u}_R \sigma_{\mu\nu}\lambda^a u_L \bar{d}_R \sigma_{\mu\nu}\lambda^a d_L\big) + (R \to L)\Big],$$

where $q_{R,L} = \frac{1}{2}(1 \pm \gamma_5)q$ is the Dirac field operator of a quark and $\eta = 4\pi^2\rho_c^2/3$, $\rho_c$ is the characteristic size of an instanton in the QCD vacuum. Evaluating (17) with the bag model wave functions, we obtain the contributions of the direct instantons to energies of the $\pi$-meson and nucleon [16]:

$$\Delta E_\pi = -\frac{\eta}{R_\pi^3}, \quad \Delta E_N = -\frac{3}{4}\frac{\eta'}{R_N^3}, \tag{18}$$

where

$$\eta = \frac{\pi\rho_c^2\varkappa_0}{4(\varkappa_0 - 1)^2 j_0^4(\varkappa_0)} \int_0^1 dx\, x^2 \left[j_0^2(\varkappa_0 x) + j_1^2(\varkappa_0 x)\right]^2$$

(the contributions are equal to zero for the $\rho$-meson and $\Delta$-isobar because of the chirality selection rule [14,15]).

From (14) and (18) for $\rho_c = 2 \text{ GeV}^{-1}$ [15] we obtain in the first order in $\rho_c^2/R^2$

$$m_\pi = 117 \text{ MeV}, \quad m_N = 953 \text{ MeV}. \tag{19}$$

It should be noted that the value of the $\pi$-meson mass is very sensitive to the parameter $\rho_c$. Increasing this size only by 5% results in a massless $\pi$-meson. This fact likely indicates a decisive role of instantons in restoring the chiral symmetry of the model.

## 5. Discussion of the Results

We have constructed the composite quark model in which the interaction of quarks with the QCD vacuum condensates plays a dominant role. Moreover, the long-wave fluctuations define the effective quarks mass that depends essentially on hadron quantum numbers and is characterized by quark vacuum condensates. On the other hand, due to inclusion of the interaction of quarks with the short-wave part of vacuum fluctuations allows us to a great extent to explain the mass splitting between the hadron $SU_f(3)$ multiplets. Total contributions well describe the masses of hadrons composed of $u$- and $d$-quarks.

Being a success in describing static characteristics of hadron ground states, the MIT model became very popular. At the same time some questions were left unanswered within this version of the bag model: the spectroscopy of excited states of the light hadrons, the states with heavy quarks, an unacceptable large value of the parameter $\alpha_s$, and others.

On the other hand, there are few approaches describing the particle properties, among which the method of QCD sum rules is based most directly on physical "experimental" information. Therefore, a quantum and qualitative adjustment of the parameters of our model with the ones of the obtained QCD sum rules seems to give hope for extending the range applicability of the bag model $^{x}$.

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$^{x}$ In the work /19/ there was also made an attempt to take into account the contribution of long-wave fluctuations to the bag energy. The diagrams considered in /19/, as the analysis shows, give a correction to hadron masses which is less than 5% .

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Учет структуры вакуума КХД в составной кварковой модели. Нестранные адроны

Предлагается метод учета вакуумных конденсатов КХД в рамках составной кварковой модели. Стабильность мешка обеспечивается самосогласованным образом благодаря взаимодействию кварков с конденсатами в мешке. Показано, что инстантонные вклады существенны для спектроскопии скалярных частиц. Предложенная модель удовлетворительно описывает энергетический спектр основных состояний нестранных адронов.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Consideration of the Vacuum of QCD in a Composite Quark model. Nonstrange Hadrons

The method of manifestation of QCD vacuum condensates within quark composite model is proposed. The bag stability is warranted selfconsistently by taking into account interaction between quarks and condensates in the bag. It is shown that instanton contributions are important to construct the spectroscopy of scalar particles. The model proposed describes well the energy spectrum of ground states of nonstrange hadrons.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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