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THE METHOD OF UNIQUENESS: MULTILOOP CALCULATIONS IN QCD

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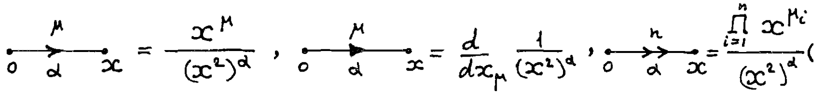
§ 1. Introduction

A continued quantitative test of quantum chromodynamics (QCD) stimulates the calculation of radiative corrections of higher orders. This in turn leads to the appearance of new methods of calculation/1-3/. In recent years in a number of papers/4-7/ a new method of calculation of massless Feynman diagrams has been developed. It was called the method of uniqueness. Its possibilities have been demonstrated by fiveloop calculations in the φ^4 model. The difference of gauge theories consists in the presence of derivatives (or momenta) on the lines of diagrams. For instance, when calculating the moments of structure functions of deep-inelastic lepton-hadron scattering (DIS) there appear the diagrams with n derivatives (momenta) for the n-th moment/8/. In this paper the method of uniqueness is generalized to the diagrams containing propagators with an arbitrary number of momenta on the lines. As usual, the object of calculation is the coefficient function depending on $\varepsilon=(4-D)/2$, where D is the space-time dimension. The point of interest is the coefficients of negative power and few coefficients of positive power of ε.

The paper is organized as follows. In sect.2 we present the main formulae and the methods used. In sect. 3 these formulae are applied to calculate some typical diagrams needed for further work. In sect. 4 one of the diagrams contributing to α_s-correction to the longitudinal structure function of DIS is calculated. At the end we give the total result for the α_s-correction.

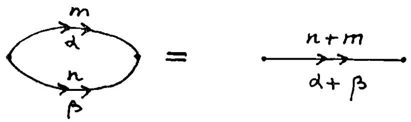
§ 2. Main Formulae

All the calculations are performed in the coordinate representation. The lines of graphs are associated with powers of the type $1/(x^2)^\alpha$, α being called the index of the line; the arrow with subscript μ corresponds to a vector x^μ, the black arrow corresponds to derivative $\partial/\partial x_\mu \equiv \partial_\mu$, two arrows (black arrows) with subscript n correspond to the product of n vectors $x^{\mu_1}\dots x^{\mu_n}$ (derivatives $\partial_{\mu_1},\dots\partial_{\mu_n}$). If the subscript is in brakets (n), this means a traceless product of n vectors (derivatives)

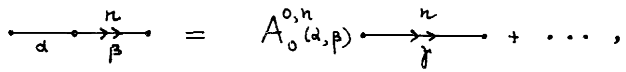

(1)

Consider now the calculation rules.

1. Contribution of a simple loop with some vectors on the lines is an ordinary product

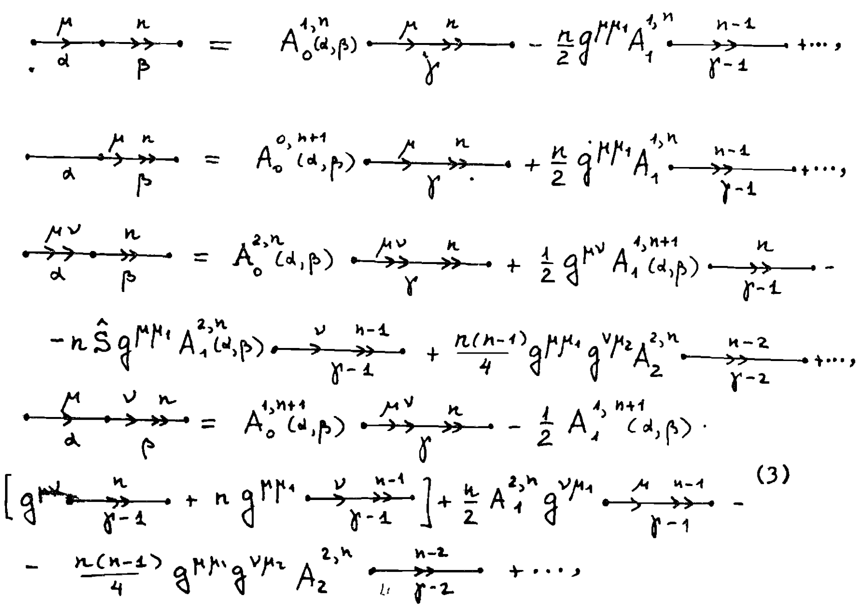

(2)

2. The chains are integrated as follows. If there are n vectors with "blind" subscripts $x^{\mu_1}\dots x^{\mu_n}$, then we will neglect the terms $g^{\mu_i\mu_j}$. In the case of "marked" vectors $x^{\mu}x^{\nu}\dots$ it is necessary to know all the structures. The integration formulae are the following:



where

$$A_i^{n,m}(\alpha,\beta)=\frac{a_n(\alpha)\,a_m(\beta)\,\pi^D}{a_{n+m-i}(\gamma-i)},\quad a_n(\alpha)=\frac{\Gamma(n+\frac{D}{2}-\alpha)}{\Gamma(\alpha)},\quad \gamma=\alpha+\beta-\frac{D}{2},$$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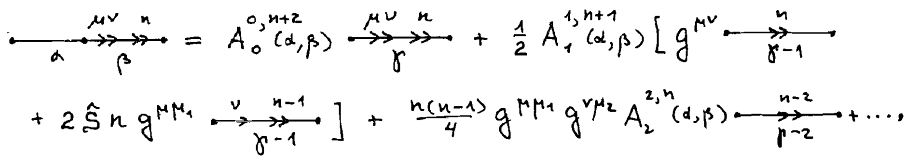
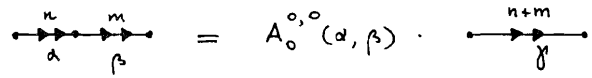

(3)

where the symmetrizer $\hat{S}$ is defined as

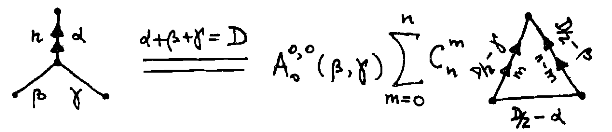
$$\hat{S}\,g^{\mu\mu_1}x^{\nu}=\frac{1}{2}\left(g^{\mu\mu_1}x^{\nu}+g^{\nu\mu_1}x^{\mu}\right).$$

The result of integration for a larger number of marked vectors is straightforward.

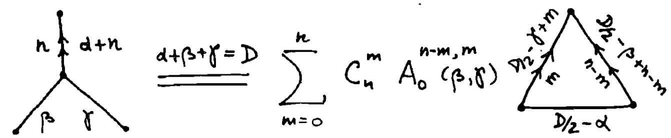


In the last chain there could be the product of marked derivatives because the structures $\sim g^{\mu_i\mu_j}$ do not appear.

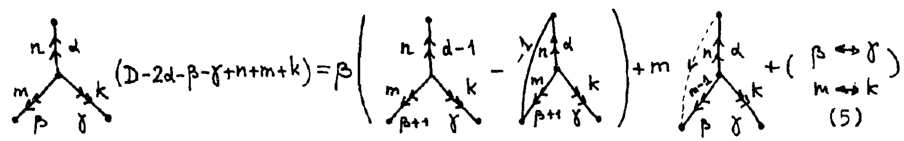
3. Differentiating the identity connecting the unique vertex ($\sum\alpha_i = D$) with unique triangle ($\sum\alpha_i = D/2$) (see, e.g., ref. /6/) we get


(4a)

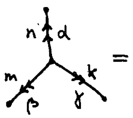
It we look for the vectors, then

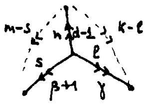
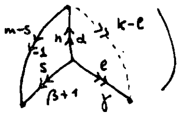

(4b)

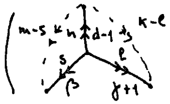
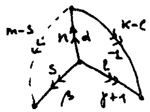
4. Integrating by parts with account of $\partial_\mu(x-y)^\mu = D$ we get the following equation for the vertex with an arbitrary index


(5)

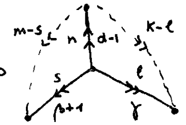
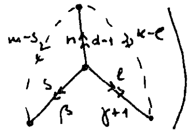
Repeating this operation ($m+k$) times we come to

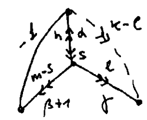
 $= \frac{\Gamma(k+1)\Gamma(m+1)}{\Gamma(D-2d-\beta-\gamma+k+m+n+1)} \sum_{s=0}^{m}\sum_{\ell=0}^{k} \frac{\Gamma(D-2d-\beta-\gamma+n+s+\ell)}{\Gamma(s+1)\Gamma(\ell+1)} \cdot$

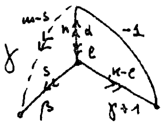
$\Big\{ \beta \Big($  $-$  $\Big) +$ (6a)

$+ \gamma \Big($  $-$  $\Big) \Big\} =$

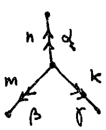
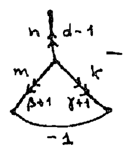
$= \frac{\Gamma(k+1)\Gamma(m+1)}{\Gamma(D-2d-\beta-\gamma+k+m+n+1)} \Big\{ \sum_{s=0}^{m}\sum_{\ell=0}^{k} \frac{\Gamma(D-2d-\beta-\gamma+n+s+\ell)}{\Gamma(s+1)\Gamma(\ell+1)} \cdot$

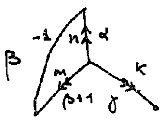
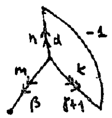
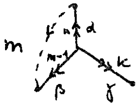
$\Big(\beta$  $+ \gamma$  $\Big) -$

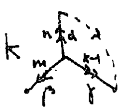
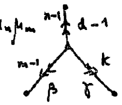
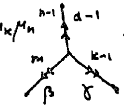
$- \sum_{s=0}^{m}\sum_{\ell=0}^{k} \frac{\Gamma(D-2d-\beta-\gamma+n+\ell)\Gamma(D-2d-\beta-\gamma+n+\ell+m+1)}{\Gamma(\ell+1)\Gamma(m-s+1)\Gamma(D-2d-\beta-\gamma+n+\ell+s+1)} \beta$ 

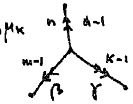
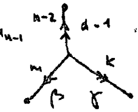
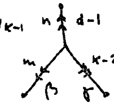
$- \sum_{s=0}^{m}\sum_{\ell=0}^{k} \frac{\Gamma(D-2d-\beta-\gamma+n+s)\Gamma(D-2d-\beta-\gamma+n+s+k+1)}{\Gamma(s+1)\Gamma(k-\ell+1)\Gamma(D-2d-\beta-\gamma+n+\ell+s+1)} \gamma$  $\Big\}$ (6b)

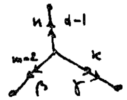
Integrating by parts and taking into account $\partial_\mu^2 (x-y)^2 = 2D$ we get, as compared to eq.(5), a more useful in some cases equation:

 $\frac{d+\beta+\gamma-D/2}{1+\beta+\gamma-D/2}(D-1-d-\beta-\gamma+n+m+k) = \frac{\beta\gamma}{1+\beta+\gamma-D/2}$  $-$

$-\beta$  $-\gamma$  $+ \frac{d+\beta+\gamma-D/2}{1+\beta+\gamma-D/2} \Big[m$  $+$

$+k$  $\Big] + \frac{1}{4(1+\beta+\gamma-D/2)} \Big[2nm\, g^{\mu_n\mu_m}$  $+ 2nk\, g^{\mu_k\mu_n}$  $+$

$+ 2mk\, g^{\mu_m\mu_k}$  $+ n(n-1)\, g^{\mu_n\mu_{n-1}}$  $+ k(k-1)\, g^{\mu_k\mu_{k-1}}$ 

$+ m(m-1)\, g^{\mu_m\mu_{m-1}}$  $\Big]$. (7)

5. While calculating the moments of structure functions one deals with traceless products of vectors. The use of traceless products $x^{(\mu_1\cdots\mu_n)}$ enables us to ignore the terms $\sim g^{\mu_i\mu_j}$ because they are easily reconstructed from the general form of traceless products. Hence during the integration one has to look for the coefficient of the main term $x^{\mu_1}\cdots x^{\mu_n}$ (see 2.). The formulae for the traceless products follow from their connection with Gegenbauer polynomials

$$x^{\mu_1}\cdots x^{\mu_n} = \hat{S}\sum_{p=0}^{[\frac{n}{2}]} \frac{\Gamma(n-2p+D/2)\,2^{-2p}\,n!}{\Gamma(n-2p+1)\Gamma(n-p+D/2)} g^{\mu_1\mu_2}\cdots g^{\mu_{2p-1}\mu_{2p}} \times x^{(\mu_{2p+1}\cdots\mu_n)}(x^2)^p, \quad (8a)$$

$$x^{(\mu_1\cdots\mu_n)} = \hat{S}\sum_{p\geq 0} \frac{\Gamma(n-p+\frac{D}{2}-1)\,2^{-2p}\,n!}{\Gamma(n-2p+1)\,p!\,\Gamma(n+\frac{D}{2}-1)} g^{\mu_1\mu_2}\cdots g^{\mu_{2p-1}\mu_{2p}} \times x^{\mu_{2p+1}}\cdots x^{\mu_n}(x^2)^p. \quad (8b)$$

For the product of vectors one has

$$z^{(\mu_1\cdots\mu_n)}\, x^{(\mu_1\cdots\mu_n)} = \frac{n!\,\Gamma(1-\varepsilon)}{2^n\,\Gamma(n+1-\varepsilon)} C_n^{1-\varepsilon}(\hat{xz})(x^2 z^2)^{n/2}, \quad (9)$$

where $C_n^{1-\varepsilon}(\cos\theta)$ is the Gegenbauer polynomial.

With the help of eqs. (8)-(9) one can obtain the following equalities

$$y^\mu \frac{d}{dx_\mu} x^{(\mu_1\cdots\mu_n)} = \frac{n}{2}\,\frac{n-3+D}{n-2+D/2}\, y^{\mu_n} x^{(\mu_1\cdots\mu_{n-1})},$$

$$x^{\mu_n} x^{(\mu_1 \cdots \mu_{n-1})} = \frac{2(n-2+D/2)}{n-3+D} x^{(\mu_1 \cdots \mu_n)},$$

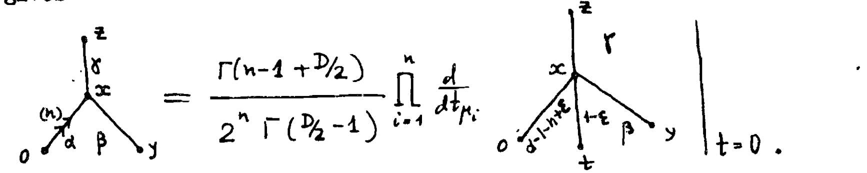
$$y^{\mu_{n+1}} x^{(\mu_1 \cdots \mu_{n+1})} = (yx)\, x^{(\mu_1 \cdots \mu_n)} - \frac{n}{4} \frac{n-3+D}{(n-1+\frac{D}{2})(n-2+D/2)} \cdot y^{\mu_n} x^{(\mu_1 \cdots \mu_{n-1})} x^2, \qquad (10)$$

$$x^{\mu_{n+1}} x^{(\mu_1 \cdots \mu_{n+1})} = \frac{n+D-2}{2(n-1+D/2)} x^{(\mu_1 \cdots \mu_n)} x^2 .$$

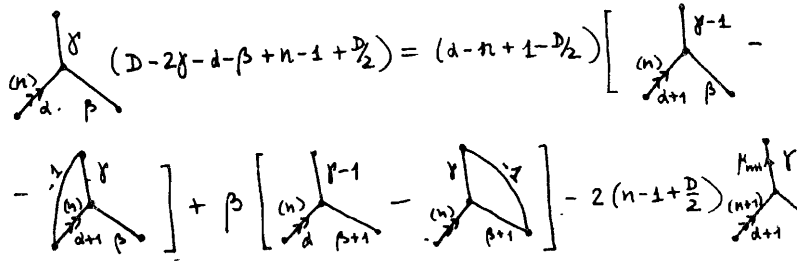
The first three eqs. are valid being multiplied by $x^{\mu_1} \ldots x^{\mu_n}$. Equation (9) leads also to

$$\frac{x^{(\mu_1 \cdots \mu_n)}}{(x^2)^n} = \frac{\Gamma(n-1+D/2)}{2^n \Gamma(\frac{D}{2}-1)} \prod_{i=1}^{n} \frac{d}{dt_{\mu_i}} \frac{(x^2)^{D/2-1}}{[(x-t)^2]^{D/2-1}} \Bigg|_{t=0} \qquad (11)$$

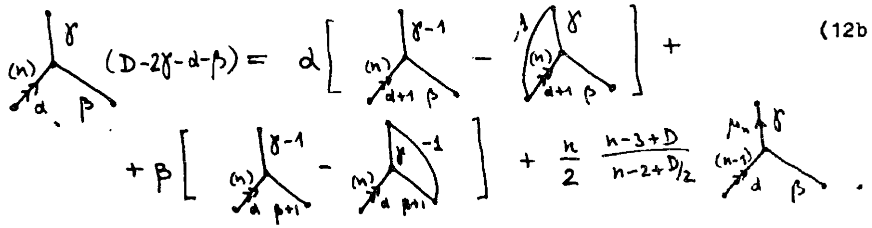
For the vertex containing the traceless product of n vectors this gives



Integrating by parts in the r.h.s. we have

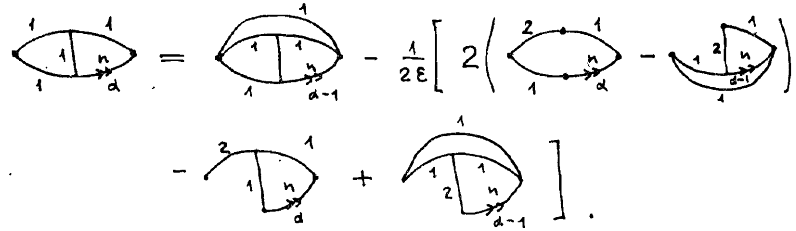


(12a)



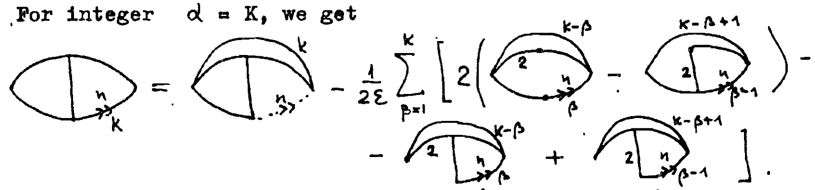
(12b)

6. To calculate complicated diagrams, it is often convenient to use functional relations analogous to those of ref./7/. This sometimes simplifies the calculations. For example,



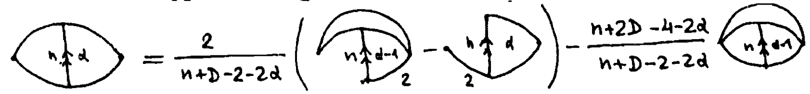
(13a)

For integer $\alpha = K$, we get



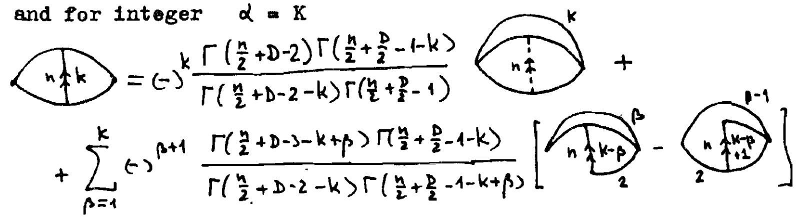
(13b)

For another typical diagram we have (for even n)



(14a)

and for integer $\alpha = K$



(14b)

§ 3. Examples of the Diagram Evaluation

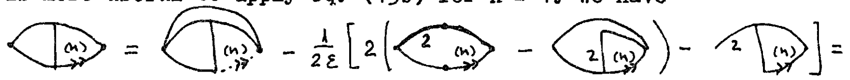
We demonstrate the efficiency of the proposed method calculating a number of typical diagrams taken from practical calculations. We proceed in x - space. Any diagram of p-space can be transformed into x-space either via the Fourier transform

$$\int \frac{d^D p\, e^{ipx} p^{\mu_1} \cdots p^{\mu_n}}{(p^2)^\nu} = \pi^{D/2} 2^{D-2\nu} (-i)^n \frac{d}{dx_{\mu_1}} \cdots \frac{d}{dx_{\mu_n}} \frac{a_0(\nu)}{(x^2)^{D/2-\nu}}$$

or considering the dual diagram. The last way is preferable due to the absence of any multipliers and changes of indices. The dual diagram arises from the initial one replacing all p_i by x_i with the diagram - integral correspondence as in x - space.

1. We start with the already discussed diagrams (13), (14). The first diagram arises in all complicated diagrams giving contribution to the nonsinglet structure function of DIS (see sect. 4). The second one appears in the nonplanar case.

To evaluate the first diagram, one can expand over the upper vertex (eq.(5)) or lower vertex (eq. (6)). However, for our purposes it is more useful to apply eq. (13b) for K = 1. We have



$$= \sum_{k=0}^{n} C_n^k A_0^{0,k}(1,1) A_0^{0,k}(1,1+\varepsilon) - \frac{1}{2\varepsilon}\Big[2A_0^{0,n}(1,2)\big(A_0^{0,n}(1,1) - A_0^{0,n}(1,1+\varepsilon)\big) - A_0^{0,n}(1,1) A_0^{0,n}(1+\varepsilon,2)\Big] \qquad (15)$$

Using now the expansion of Γ -functions

$$\frac{\Gamma(n+1+a\varepsilon)}{n!\,\Gamma(1+a\varepsilon)} = 1 + S_1(n)a\varepsilon + [S_1^2(n) - S_2(n)]\frac{a^2\varepsilon^2}{2} + [S_1^3(n) - 3S_2(n)S_1(n) + 2S_3(n)]\frac{a^3\varepsilon^3}{3!} + \dots, \qquad (16)$$

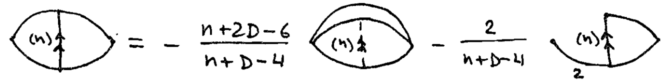
$$\Gamma(1+a\varepsilon) = \exp\Big[-\gamma a\varepsilon + \sum_{n=2}^{\infty} \frac{\zeta(n)}{n}(-a)^n\varepsilon^n\Big],$$

where $S_i(n) = \sum_{k=1}^{n} 1/k^i$, $\zeta(n) = S_n(\infty)$ is the Riemannian ζ -function and γ is the Euler constant. We finally get

 $= \frac{1}{n+1}\Big[S_3(n) + S_2(n)S_1(n) - 2T(n) + 6\zeta(3)\Big] + O(\varepsilon)$,

where $T(n) = \sum_{k=1}^{n} \frac{S_1(k)}{k^2}$. (17)

For the second diagram we proceed in the same way. Applying eq. (14b) for $k = 1$ we come to



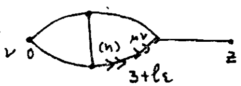
$$= -\frac{n+2D-6}{n+D-4}\sum_{k=0}^{n} C_n^k (-)^k A_0^{0,k}(1,1) A_0^{0,n-k}(1,1) - \frac{2}{n+D-4} A_0^{0,n}(1,1) A_0^{0,n}(2,1+\varepsilon). \qquad (18)$$

Using eq.(16) we find

 $= [1 + (-1)^n]\frac{1}{2}\Big[\frac{(1-\delta_n^0)4K_2(n)}{n(n+1)} + \delta_n^0 6\zeta(3)\Big] + O(\varepsilon)$, (19)

where $K_i(n) = \sum_{k=1}^{n}\frac{(-)^{k+1}}{k^i}$.

2. When calculating DIS structure functions by the technique of "gluing"/8/ one comes to diagrams of the type

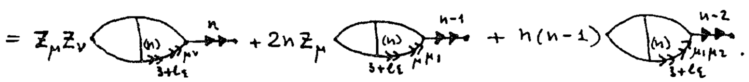
$I_n(\ell) = z_\mu z_\nu$  , (20)

where the limit

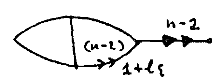
$$I_n = \lim_{\ell \to -3} (\ell+3)\varepsilon(n+1) I_n(\ell) \qquad (21)$$

gives the desired contribution. Here also the kinematics of DIS should be taken into account, which means that $z^2 = 0$, i.e., one neglects the proton mass.

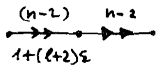
The complexity of calculations is due to the presence of a large number of various structures of μ and ν which should be contracted with $z^\mu z^\nu$. To avoid this and to reduce the diagram (20) to the calculated one (17), we use the following trick. Differentiating (20) with respect to $(d/dz_{\mu_i})^n$ and summing over $\mu_1 \dots \mu_n$, we get

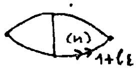
$\left(\frac{d}{dz_{\mu_i}}\right)^n I_n(\ell) =$ 

The first two diagrams equal zero due to the kinematic restrictions ($z^2 = 0$), and the third one with the help of eq.(10) is reduced to

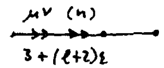
$\left(\frac{d}{dz_{\mu_i}}\right)^n I_n(\ell) \overset{z^2=0}{=} n(n-1)\frac{(n+D-3)(n+D-4)}{(n+\frac{D}{2}-2)(n+\frac{D}{2}-3)}$ 

This diagram can be easily calculated because all vectors are now under the traceless product. We have

$$\left(\frac{d}{dz_{\mu}}\right)^n I_n(\ell) \overset{z^2=0}{=\!=} n(n-1)\frac{(n+D-3)(n+D-4)}{(n+\frac{D}{2}-2)(n+D/2-3)} D_{n-2}(\ell)$$  ,

where $D_n(\ell)$ is the coefficient function of the diagram 

Hence the coefficient function of interest with account of kinematics is reduced to the diagram considered before:

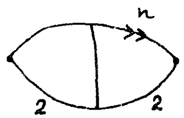
$$I_n(\ell) \overset{z^2=0}{=\!=} D_{n-2}(\ell)\, z_\mu z_\nu$$  .

Taking now the limit (21) and putting $\varepsilon = 0$ we finally arrive at

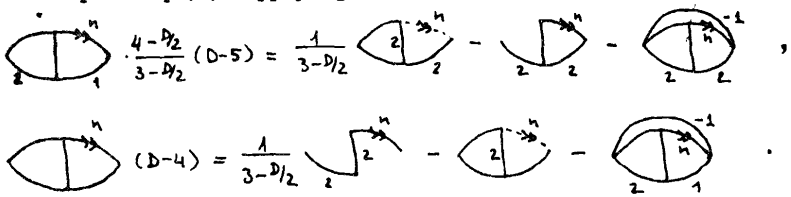
$$I_n = D_{n-2}$$

where D_n is given by eq. (17).

3. To demonstrate the efficiency of eq.(7) as compared to eq.(5), we consider the diagram, which is complicated not only because of its topological structure but due to the poles in ε .

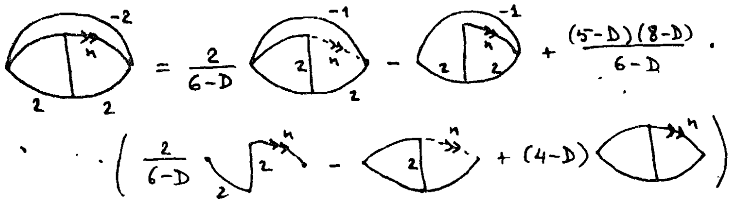


Transforming the lower vertex we come to cumbersome sums which should be expanded over ε . However, these problems can be separated with the help of eq.(7). Applying the latter to the lower vertex we have



(22)

Comparing these two expressions we find the following representation of the initial diagram



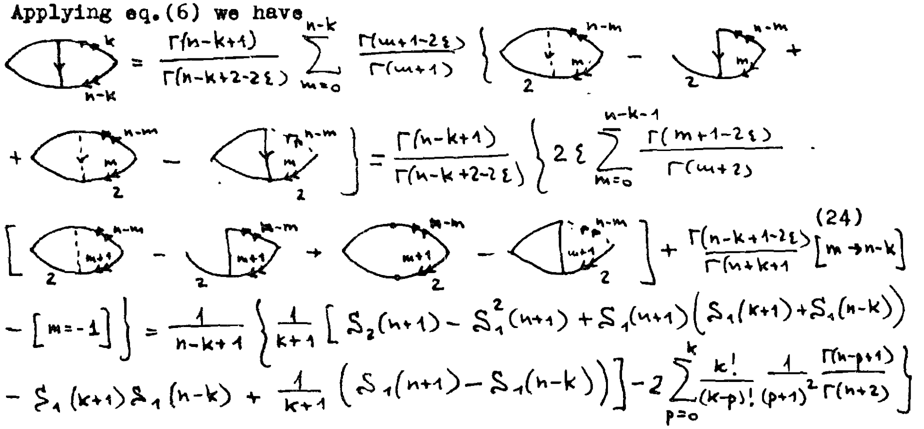
(23)

The resulting diagram is now easily calculated. The only complicated diagram in the r.h.s. is multiplied by $4-D = 2\varepsilon$ and contributes only to $O(\varepsilon)$ terms.

4. As an example demonstrating the usefulness of eq.(6) we consider the integral

$$\sum_{k=0}^{n} C_n^k \frac{(-)^{k+1}}{k+1}$$  .

Applying eq.(6) we have

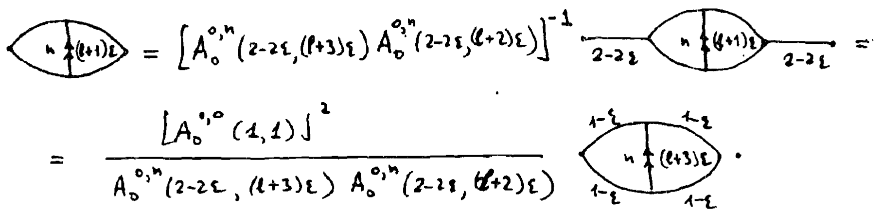


(24)

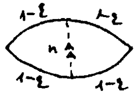
5. As an example of calculation by the uniqueness relation we consider the integral

$$I_n = \lim_{\ell\to-3}(\ell+3)\,\varepsilon(n+1)$$  . (25a)

We have



Hence

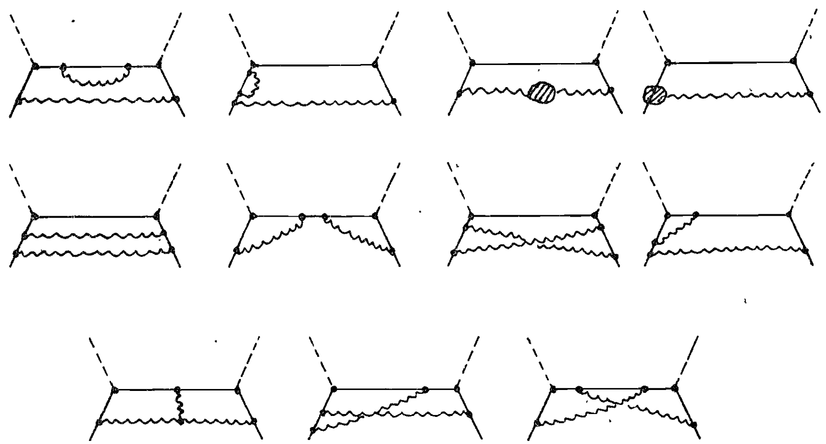
$$I_n = -2\varepsilon(n+1)\left[\frac{\Gamma(1-\varepsilon)}{\Gamma(1)}\right]^4 \frac{\Gamma(n+2+\varepsilon)}{\Gamma(n+2-\varepsilon)}$$  =

$$= -2\varepsilon(n+1)\left[\frac{\Gamma(1-\varepsilon)}{\Gamma(1)}\right]^4 \frac{\Gamma(n+2+\varepsilon)}{\Gamma(n+2-\varepsilon)} \sum_{m=0}^{n} C_n^m (-)^m A_1^{0,m}(1-\varepsilon,1-\varepsilon)\, A_1^{0,n-m}(1-\varepsilon,1-\varepsilon) =$$

$$= -\frac{2}{\varepsilon}\,\frac{1+(-1)^n}{n+2}\,\frac{\Gamma(n+2+\varepsilon)}{\Gamma(n+2-\varepsilon)} \qquad (25b)$$

§ 4. Deep-Inelastic Scattering: α_s-Correction to the Longitudinal Structure Function

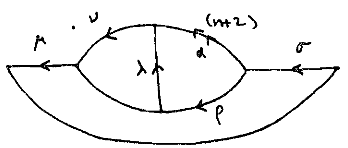
In this section we present the diagrams contributing to the two-loo correction to the longitudinal structure function of DIS **(see the figure).**



Here the solid, wavy and dotted lines mean quark, gluon and photon, respectively.

To illustrate the usefulness of the method of uniqueness, we consider one of complicated diagrams (No. 8). To get the longitudinal part, we multiply the diagram by a projector $p^\mu p^\nu/q^2$, where p and q are quark and photon momenta. We are interested in the cocoefficient function of n-th moment. To find it, we use the technique of "gluing"/8/. For this purpose we multiply the result by $q^{(\mu_1\cdots\mu_n)}/(q^2)^d$, where $d=3+\ell\varepsilon$ and integrate over q. As a result, we have

$$I^L_{n+2}(\ell) = 2(2-D)\, Sp\left(\hat p\,\gamma_\mu \gamma^\tau \gamma_\nu \hat p\, \gamma_\lambda \gamma \gamma_\rho^\tau \hat p\, \gamma_\sigma\right) C_F^2 \cdot \qquad (26)$$



$d = 3+\ell\varepsilon$

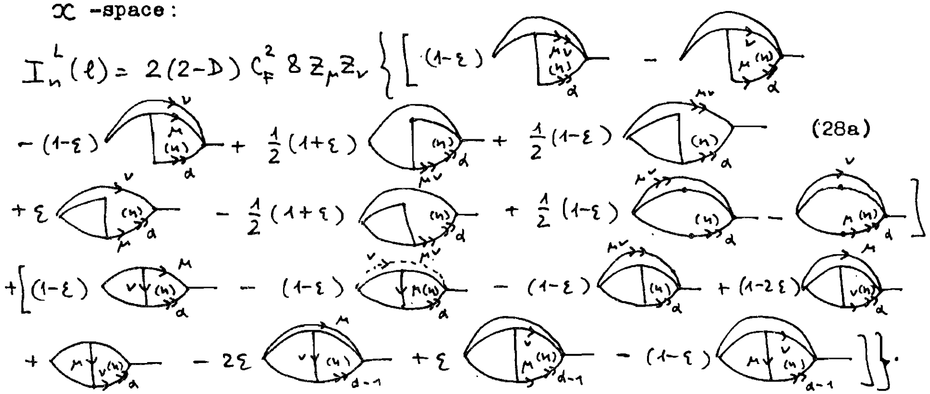
We are interested in the contribution

$$KR' I^L_{n+2} = \sum_{a=2,3} \lim_{\ell\to -a} (\ell+a)\, K\left(I^L_{n+2}(\ell) - \Delta I^L_{n+2}(\ell)\right)\Big|_{4(n+1)}, \qquad (27)$$

where ΔI means the integral with a counterterm instead of a divergent subgraph.

Evaluating the trace and passing to a dual diagram, we obtain in x-space:

$$I^L_n(\ell) = 2(2-D)\, C_F^2\, 8\, z_\mu z_\nu \Big\{ \Big[\ldots \qquad (28a)$$

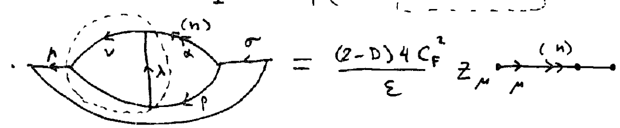


In eq. (28a) in the first square bracket all the diagrams are simple, and in the second one they are more difficult. Calculation of one of them has been done in sect. 3. The others are evaluated in the same way; the result is

$$K I^L_n = \frac{(2-D)\,2^3 C_F^2}{(n-1)\varepsilon}\Big[-\frac{1}{\varepsilon} + 4\Big(\frac{1}{n} - \frac{2}{n+1}\Big)\big[S_3(n-2) + S_2(n-2)\cdot S_1(n-2) - 2T(n-2) + 6\zeta(3)\big] + S_2(n-2)\Big(-\frac{4}{n} + \frac{12}{n+1}\Big) - 2S_1(n-2)\Big(1 + \frac{1}{n-1} + \frac{2}{n} - \frac{5}{n+1}\Big) - \frac{5}{2} + \frac{1}{n-1} - \frac{1}{n} + \frac{3}{n+1}\Big]. \qquad (28b)$$

The contribution of the diagram with a counterterm is

$$K\,\Delta I^L_n = K\,\frac{(2-D)\,2C_F^2}{1}\, Sp\left(\hat p\,\gamma_\mu \gamma^\tau \gamma_\nu \hat p\, \gamma_\lambda \gamma^\tau \gamma_\rho \hat p\, \gamma_\sigma\right)\cdot \qquad (28c)$$



$$= \frac{(2-D)\,4C_F^2}{\varepsilon}\, z_\mu \ldots = \frac{(D-2)\,8C_F^2}{(n-1)\varepsilon^2}\Big[1 + \varepsilon\Big(2S_1(n-2) + \frac{1}{n-1} + \frac{1}{n} + \frac{1}{n+1} + 2\Big)\Big].$$

Eventially, we find the following contribution of the given diagram to the coefficient function

$$KR' I^{L}_{n+2} = \frac{2^4 C_F^2}{(n+1)\varepsilon} \Big\{ 4\left(\frac{2}{n+3} - \frac{1}{n+2}\right)\Big[S_3(n) + S_2(n) S_1(n) - 2T(n) + 6\zeta(3)\Big] + 4 S_2(n)\left(\frac{1}{n+2} - \frac{3}{n+3}\right) + 2S_1(n) \cdot \left(\frac{1}{n+1} + \frac{2}{n+2} - \frac{5}{n+3}\right) + \frac{1}{2} - \frac{2}{n+1} + \frac{2}{n+3} \Big\}. \tag{28d}$$

The total contribution of all the diagrams, fig. 5, can be represented in the form

$$C^L_n\left(\frac{Q^2}{\mu^2}, \alpha_s\right) = \frac{\alpha_s}{4\pi}\frac{4C_F}{n+1}\left(1 + \frac{\alpha_s}{4\pi}\Big[\left(-\beta_0 - \frac{\gamma_0^n}{2}\right)\left(\ln\frac{Q^2}{\mu^2} - \ln 4\pi + \gamma\right) + R^L_n(\overline{MS})\Big]\right),$$

where

$$R^L_n(\overline{MS}) = (2C_F - C_A)\Big[8K_2(n)S_1(n) - 8Q(n) + 4K_3(n) - 4S_3(n) + 12\zeta(3) - \frac{6}{5}\frac{1-\delta_n^2}{n-2}\left(4K_2(n) - 3\right) - 4\delta_n^2\left(\frac{9}{5}\zeta(3) - \frac{21}{10}\right) - 8K_2(n)\left(1 + \frac{1}{n} - \frac{1}{n+1} - \frac{3}{5}\frac{1}{n+3}\right) - S_1(n)\cdot\frac{23}{3} - \frac{215}{18} + \frac{11}{3}\frac{1}{n} + \frac{11}{3}\frac{1}{n+1} - \frac{18}{5}\frac{1}{n+3}\Big] + 2C_F\Big[S_1^2(n) - S_2(n) + S_1(n)\left(\frac{19}{6} - \frac{1}{n} - \frac{1}{n+1}\right) + \frac{277}{36} - \frac{7}{6}\frac{1}{n} - \frac{19}{6}\frac{1}{n+1} + \frac{1}{n^2} - \frac{1}{(n+1)^2}\Big] - \frac{4}{3}T_F\left(S_1(n) + \frac{19}{6} - \frac{1}{n} - \frac{1}{n+1}\right), \qquad Q(n) = \sum_{k=1}^{n}\frac{(-)^{k+1}S_1(k)}{k^2}.$$

This result has a simpler analytical form than that of ref./11/ and coincides with it numerically. The numerical difference with the calculations of ref./10/ corresponds to that mentioned in ref./11/. Note that the use of the method of uniquencess enables us to find the result without computer. Physical applications of eq.(29) will be discussed in a separate publication.

§ 5. Conclusion

We have demonstrated possibilities of the method of uniqueness for the evaluation of Feynman diagrams in QCD. Though the formulae are more cumbersome as compared to scalar theories, all calculations of complicated diagrams are reduced to a number of steps and all integrations are performed algorithmically without a direct integral evaluation or expansion in an infinite series. These features determine the advantages and wide possibilities of the method of uniqueness.

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Казаков Д.И., Котиков А.В. E2-86-204

Метод уникальностей: многопетлевые вычисления в КХД

Метод уникальностей распространен на диаграммы с произвольным числом производных /или импульсов/ на линиях. Получен ряд формул, удобных для вычисления в калибровочных теориях. Приводятся примеры вычисления многопетлевых диаграмм, встречающихся в КХД. Метод применяется к вычислению α_s-поправки к продольной структурной функции глубоконеупругого рассеяния лептонов на адронах.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1986

Kazakov D.I., Kotikov A.V. E2-86-204

The Method of Uniqueness: Multiloop Calculations in QCD

The method of uniqueness is generalized to diagrams with an arbitrary number of derivatives (or momenta) on the lines. A number of useful formulae is obtained which can be used for calculations in gauge theories. Some examples of multiloop calculations in QCD are given. The method is applied to the calculation of α_s-correction to the longitudinal structure function of deep-inelastic lepton-hadron scattering.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1986